# FINITE ELEMENT MODELING OF DYNAMIC INTERFACIAL RUPTURE AND SLIDING

D. Coker[1], A.J. Rosakis[2] and A. Needleman[1]
[1]Division of Engineering, Brown University, USA
[2]Division of Engineering and Applied Physics, California Institute of Technology, USA

## ABSTRACT

Analyses of fracture and sliding under dynamic loading conditions are discussed. The calculations are carried out within a framework where two constitutive relations are used: a volumetric constitutive relation between stress and strain and a surface constitutive relation that characterizes the fracture and/or frictional behavior of an interface. Attention is confined to two dimensional plane stress or plane strain analyses. Issues that are addressed including limiting rupture and sliding speeds along interfaces and modes of frictional sliding.

## 1 INTRODUCTION

Fracture and sliding along interfaces arises in a variety of applications including, for example, structures subject to impact loading and earthquake dynamics. Although there is a large literature on interfacial fracture and sliding phenomena, there are fundamental issues that remain unresolved. Until relatively recently research in the area of dynamic interfacial fracture mechanics has been focused primarily on the study of the subsonic regime of dynamic crack growth. The first experimental evidence of high speed, subsonic, crack growth in bimaterial interfaces was presented by Tippur and Rosakis (1991) who found that crack tip speeds easily approached the smaller of the two Rayleigh wave speeds, suggesting the possibility of high velocity intersonic crack growth. Intersonic crack propagation along a bimaterial interface was first observed experimentally by Liu et al. (1993). Hence, questions arise concerning the limiting speed for crack propagation, particularly along bimaterial interfaces, and the factors that set this limiting speed.

Frictional sliding along an interface between two deformable solids is a basic problem of mechanics. The classical Amontons-Coulomb description of friction states that the shear stress at an interface is proportional to the normal stress, with the coefficient of proportionality being the coefficient of friction. Surprisingly, Adams (1995) showed that the problem of frictional sliding along an interface between two elastic solids, with sliding governed by Amontons-Coulomb friction, is unstable to perturbations for a wide range of friction coefficients and material properties. Hence, in that regime the boundary value problem is ill-posed. As a consequence, there has been considerable recent interest in developing frictional constitutive relations that are physically based and lead to well-posed boundary value problems.

We will present recent work aimed at addressing fundamental issues concerning fracture and sliding along interfaces under dynamic loading conditions.

## 2 FORMULATION AND NUMERICS

In a finite strain Lagrangian formulation, with the initial undeformed configuration taken as reference and with all field quantities considered to be functions of convected coordinates $y^i$ and time $t$, the principle of virtual work can be written as

$$\int_V \mathbf{S} : \delta\mathbf{E}\, dV - \int_{S_{int}} \mathbf{T}\cdot\delta[\mathbf{u}]\, dS = \int_{S_{ext}} \mathbf{T}\cdot\delta\mathbf{u}\, dS - \int_V \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}\cdot\delta\mathbf{u}\, dV \qquad (1)$$

where **S** is the second Piola-Kirchhoff stress tensor, **u** is the displacement vector, $[\mathbf{u}]$ is the displacement jump across the cohesive surface, $\mathbf{A} : \mathbf{B}$ denotes $A^{ij}B_{ji}$, and $V$, $S_{ext}$ and $S_{int}$ are the volume, external surface area and internal cohesive or frictional surface area, respectively, of the body in the reference configuration. The density of the material in the reference configuration is $\rho$, **T** is the traction vector and the Lagrangian strain, **E**, is given by

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}) \quad , \quad \mathbf{F} = \mathbf{I} + \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \tag{2}$$

with **I** the identity tensor and **x** denoting the position vector of a material point in the reference configuration.

At $t = 0$, the body is stress free and at rest. Impact loading is imposed as sketched in Fig. 1. A cohesive and/or frictional constitutive relation is prescribed along the centerline and the material is taken to be linear elastic. In calculations where an interface frictional constitutive relation is used an initial compressive stress is prescribed, whereas in calculations where a cohesive constitutive relation is used to characterize the interface the initial compressive stress is taken to be zero. The calculations are two dimensional assuming plane stress conditions.

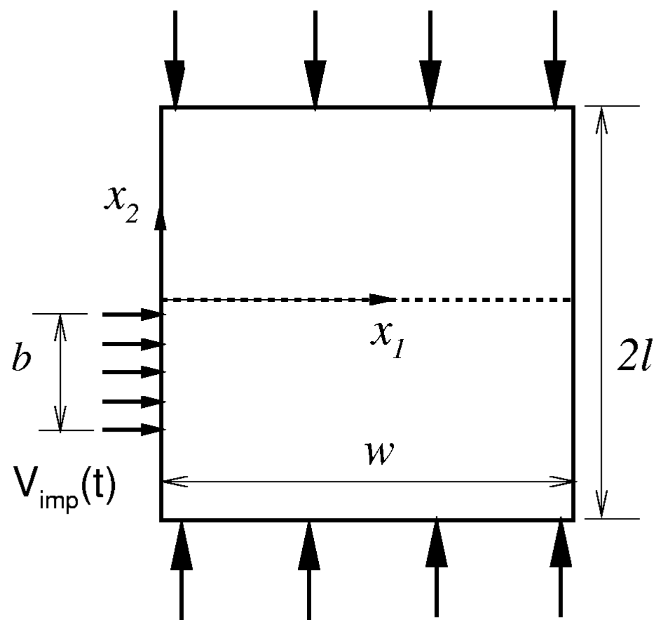


Figure 1: Geometry and loading configuration used in the finite element calculation.

A finite element discretization is used that is based on linear displacement triangular elements that are arranged in a ‘crossed-triangle’ quadrilateral pattern. The equations of motion that result from substituting the finite element discretization into (1) are integrated numerically by an explicit integration procedure, the Newmark $\beta$-method with $\beta = 0$, Belytschko et al. (1976). A lumped mass matrix is used instead of the consistent mass matrix, since this has been found preferable for explicit time integration procedures, from the point of view of accuracy as well as computational efficiency, Krieg and Key (1973).

## 3 RESULTS

In Coker at al. (2003), dynamic fracture along an interface between a fiber reinforced epoxy composite and Homalite was analyzed. Sustained crack growth was found to occur within discrete speed ranges delimited by characteristic elastic wave speeds. The faster longitudinal wave speed appeared to provide the upper limit to the sustainable crack speed. Excellent agreement was found between the numerically computed and experimentally observed attainable regimes of sustained crack speed and between the numerically computed and experimentally observed isochromatic fringe patterns (contours of maximum principal stress difference). Also, under certain conditions a pulse-like normal traction distribution was obtained along the bond line even though a purely elastic cohesive relation was used in the calculations. At the time of this writing, the pulse-like normal traction distribution has not yet been observed experimentally.

In Coker et al. (2004), calculations were carried out to investigate dynamic sliding along the interface of two Homalite plates held together by compressive stresses and subject to impact shear loading. Finite element calculations using rate- and state-dependent friction laws for the interface showed that sliding of an interface can occur in different modes depending on the imposed compressive stress and the impact conditions. Friction modes that involve uniform sliding behind the rupture front, an isolated slip pulse, multiple slip pulses or a combination of these were obtained. It is found that the isolated slip pulse tends to become narrower and steeper with time while the multiple pulses do not. In the calculations, the frictional sliding mode changes from an expanding crack-like mode to a slip pulse with increasing initial compressive stress at constant impact speed and to a train of pulses with increasing impact speed at constant initial compressive stress.


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