PREDICTING BRITTLE FRACTURE USING THE THEORY OF CRITICAL DISTANCES: CONSTRAINT EFFECTS

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ABSTRACT

The Theory of Critical Distances (TCD) is a method for predicting the effects of notches and other stress concentrations on the failure of materials. The approach is a linear-elastic one; the theory reduces to LEFM in the case of a sharp crack. It has previously been used to predict high-cycle fatigue in metals and brittle fracture in composites. This paper considers the use of TCD to predict unstable fracture initiated from notches in metallic materials, with particular attention being paid to the effects of constraint. Data were obtained from the literature showing how the measured fracture toughness of a specimen containing a notch varies with root radius, in steels and aluminium alloys at a range of temperatures. Knowing K_c, TCD can be applied using just one other material constant: the characteristic distance, L. It was found that TCD can give accurate predictions of notch fracture strength under both plane strain and plane stress conditions, though naturally the values of the material constants vary with constraint. For given specimen thickness and notch length, increasing notch root radius can reduce constraint: this was demonstrated for data on steels for which sharp cracks failed under plane strain conditions whilst notches failed in plane stress. It was shown that this effect can be predicted by TCD. The nature of the theory allows it to be very easily applied to stress concentrations with any arbitrary geometry and loading, and therefore to real components analysed using finite element methods.

1 INTRODUCTION

This paper is concerned with the prediction of brittle fracture in metallic materials, and especially with the effect of notches and other stress-concentrating features such as occur in engineering components. Brittle fracture is defined as any failure which occurs as a result of the unstable propagation of a crack, whether the actual mechanism of propagation is brittle cleavage or ductile shearing/void-growth. Linear Elastic Fracture Mechanics (LEFM) is capable of predicting the onset of brittle fracture very accurately, but only under certain conditions: the pre-existing feature must be a crack, of macroscopic length, and the principle of contained plasticity must apply. In addition, difficulties arise with LEFM in situations of varying constraint: the fracture toughness, K_c, varies with constraint level, being at its lowest under plane strain conditions and rising as the constraint is reduced. The Theory of Critical Distances (TCD) describes an approach to the assessment of notches, which has had some success in the prediction of both brittle fracture and fatigue. In particular it has been used frequently to predict brittle fracture in fibre-composite materials (beginning with the work of Whitney and Nuismer [1]); it also forms the basis of the Neuber [2] and Peterson [3] correction factors for high-cycle fatigue behaviour of notches. It has been only rarely used to predict brittle fracture in polymers [4] and almost never applied to metals or ceramics. The author is of the opinion that TCD has considerable potential as a tool for failure prediction, especially given the widespread use of finite element analysis for estimation of elastic stress fields in components. The aims of the present paper are to investigate the use of TCD for predicting the effect of notch geometry on brittle fracture in metals, especially considering the effect of constraint.

The theory has the following elements:

(a) it makes use of the local elastic stress field, i.e. the variation of stress with distance from the notch, the stress being calculated assuming linear elastic behaviour with no yielding. This stress field can be represented by $\sigma(r,\theta)$ where r and θ are polar coordinates centred on the point of maximum stress at the notch root.

(b) two material parameters are defined: a characteristic strength σ_o and a characteristic material distance, L. These are assumed to be material constants;

(c) failure is assumed to occur when some function of the local stress field, $f(\sigma)$, becomes equal to σ_o. This function can take a number of forms, all of which involve the characteristic distance L. The simplest form of TCD, which we call the Point Method (PM), uses the value of the maximum principal stress at the point r=L/2 measured along a line drawn normal to the surface, starting at the point of maximum stress. This can be represented simply as:

$$\sigma(L/2,0) = \sigma_o \tag{1}$$

An alternative function, which we call the Line Method (LM) uses the average value of the stress calculated along the same line, extended to a length 2L, thus:

$$\frac{1}{2L}\int_0^{2L} \sigma(r,0)dr \tag{2}$$

Other functions, for example using the average stress over an area or volume, or using a weighting function, have also been attempted. However, in previous work we have found the simple PM and LM to be very successful and accurate for a range of problems, so only these methods will be considered here. For the simple case of a long, sharp crack, LEFM theory can be used to show that there is a relationship between K_c and the two constants used in TCD, thus:

$$L = (1/\pi).(K_c/\sigma_o)^2 \tag{3}$$

This arises due to the unique relationship between K and $\sigma(r,\theta)$ for a crack.

2 BRITTLE FRACTURE IN PLANE STRAIN

Fig.1 shows the results of using TCD to predict the effect of notch root radius on the measured K_c value of specimens of steel tested by Wilshaw et al [5] at low temperatures. Failure was by brittle cleavage: plane strain conditions prevailed throughout. Of course the K_c values measured here are only equal to the actual plane-strain fracture toughness when the notch root radius approaches zero: in practice K_c remains almost constant up to a critical root radius of about ρ=0.05mm, thereafter increasing almost linearly with $(\rho)^{1/2}$. For relatively small root radii one can use the analytical solution of Creager and Paris [6] for $\sigma(r,\theta,\rho)$ which, combined with the PM (eqn.1) gives a relationship between the measured toughness K_m and the sharp-crack value K_c:

$$K_m = K_c \frac{(1+\rho/L)^{3/2}}{(1+2\rho/L)} \tag{4}$$

However the Creager and Paris relationship is only valid for sharp notches so it becomes unusable for large ρ (above 0.2mm in the present case); for larger root radii the predictions were made using stress fields obtained from finite element analysis. The optimum values of the material constants were found by trial and error but, given eqn 3, there is in fact only one choice to be made: for example if one chooses a value of L then, since K_c is already known for the material, σ_o can be found from eqn 3. In this case the values were: K_c=25.7MPa$(m)^{1/2}$; L=0.03mm; σ_o=2447MPa. The predictions are clearly very accurate (maximum error 10.1%); we can conclude that, in cases

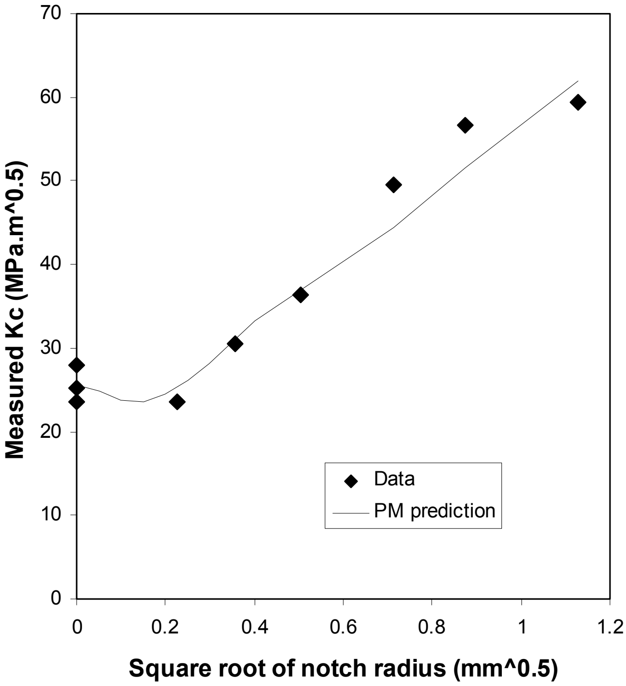

Fig.1: Experimental data and predictions of notch radius effect in plane strain conditions.

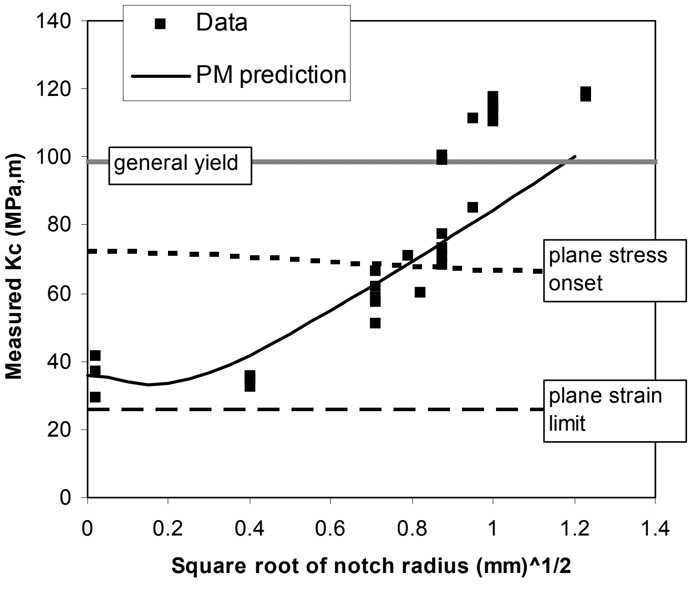

Fig.2: Data and predictions under conditions of mixed constraint.

where plane strain conditions prevail, TCD is a suitable method for predicting notch effects. Similar accuracy was achieved in predicting data on an aluminium alloy [7], in which the brittle-fracture mechanism was ductile void growth rather than cleavage. Both the PM and LM gave reasonable predictions but LM was slightly better in this case.

3 THE EFFECT OF CONSTRAINT

Fig.2 shows some data from Yokobori and Konosu [8] whose material and test conditions were similar to those of Wilshaw et al, except that thinner specimens were used. It can be seen that the TCD (PM) prediction also works well in this case (and with similar values of the constants: L=0.05mm) except for the data at the largest root radii, for which the measured K_c was considerably higher than predicted. We suspected that this was caused by a lack of constraint: higher stresses are needed to cause failure in the blunt notches, leading to larger plastic zones and thus the possibility of loss of plane strain conditions. To investigate this we applied some simple rules to estimate constraint, as follows. Plane strain conditions can be assured by applying the criteria used in several national standards for toughness testing, which can be written so as to define a limiting value of K_c, below which plane strain conditions certainly apply:

$$K_c \text{ [plane strain limit]} = \sigma_y (B/2.5)^{1/2} \qquad (5)$$

Here B is the thickness of the specimen and σ_y is the yield strength. This criterion is known to be somewhat conservative, therefore it forms a lower bound for the plane strain limit. A higher value of K_c, at which plane stress conditions begin to appear, can be set (using the conclusions of Irwin [9]) at the point where the plane stress plastic zone size is equal to B. To estimate the size of the plastic zone ahead of the notch we used a variation of the PM in which the critical stress is σ_y and the critical distance is B/2.

It can be seen from fig.2 that the TCD predictions begin to break down at the point corresponding to the onset of plane stress conditions, at a value of $(\rho)^{1/2}$ of approximately 0.8mm$^{1/2}$. At slightly higher radii (0.9 and 1.0mm$^{1/2}$) the experimental data show a lot of scatter, with some results still conforming to the TCD predictions whilst others fail at higher stresses. This may imply a ‘bimodal’ distribution of failure, the value of K_c alternating between a plane stress value and a plane strain value. Fig.3 shows brittle-fracture data for a high-strength steel tested at room temperature (Irwin [9]). As with the results on fig.2, no single set of constants could be found to predict all the data using TCD, but it seems that the results can again be understood in terms of the plane-strain/plane-stress transition. Here two different predictions have been made, for plane strain and stress respectively, showing that the data move from one to the other with increasing root radius. Finally, as fig.4 shows, data known to be entirely in plane stress [10] can also be accurately predicted using the TCD approach.

4. DISCUSSION AND CONCLUSIONS

TCD is a linear-elastic theory, and in this respect it is similar to LEFM. For the case of a long, sharp crack the two theories coincide. However, TCD is more generally applicable: LEFM only applies to long, sharp cracks because it relies on the $1/(r)^{1/2}$ singularity as a description of the stress field. In other respects, however, LEFM and TCD have the same limitations. In particular, since they rely on an assumption of linear elastic behaviour they can only be used in situations where the local elastic stress field provides a unique description of the stress concentration. Another way to express this concept is that there should be a one-to-one relationship between the elastic stress field and the true, elastic-plastic field. This implies the condition of contained plasticity (i.e. a

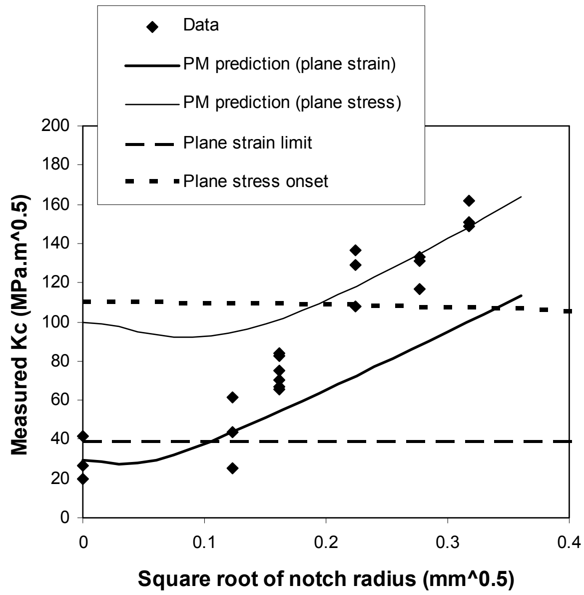

Fig.3: Experimental data and predictions for plane strain and plane stress conditions.

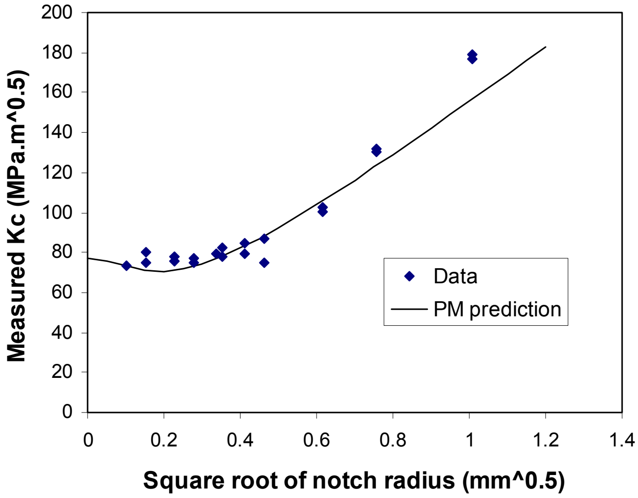

Fig.4: Experimental data and predictions for conditions of plane stress.

small plastic zone). Another major limitation of LEFM, which is also a limitation of TCD, is that the relevant material constants will change with the degree of constraint. This reflects a real variation in material properties: crack propagation is easier under plane strain conditions than under plane stress, and this difference is generally accompanied by a change in the mechanism of fracture. What the present paper has demonstrated is that this effect, which in LEFM is accommodated by a change in the value of K_c, can be included in the TCD prediction also by changing the values of the relevant constants. In the case of TCD there will be two constants to change but, provided sufficient experimental data is available, it is in principle quite possible to predict fracture under any constraint condition. We have considered the relatively simple, two-dimensional situation of a through-thickness notch in a plate, for which the extreme conditions of plane stress and plane strain can be defined. For a more general, three-dimensional case, the essentially 2D concepts of plane stress and plane strain no longer apply and must be replaced with a general definition of the level of constraint, such as the ratio between hydrostatic and deviatoric stresses. Future work on TCD will be directed towards finding values of the two material constants as a function of this constraint factor.

In conclusion, the Theory of Critical Distances provides a relatively simple method of predicting failure. From a theoretical point of view it belongs to the same family of methods as LEFM; from a practical point of view the information needed to make predictions can easily be obtained using simple test data, and applied to components through the use of elastic FEA. The present paper has demonstrated that, in addition to its uses to predict high-cycle fatigue limits and fracture in composites, the method is applicable in the field of brittle fracture in metallic materials provided constraint effects are taken into account.

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