# COMPRESSIVE FRACTURE OF LAYERED HYPERELASTIC MATERIALS WITH INTER- AND INTRALAMINAR DEFECTS

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ABSTRACT

When a layered material is compressed along the layers, fracture due to interlaminar defects cannot be predicted using the classical Griffith-Irwin criterion or its generalisations, and therefore fracture due to mechanisms, specific to heterogeneous materials, needs to be considered. One of such mechanisms is internal instability, i.e. the loss of stability in the microstructure of the heterogeneous material. This paper investigates internal instability of layered hyperelastic materials with inter- and intralaminar defects undergoing large deformations under uniaxial or equi-biaxial loading. For interlaminar defects called “defects with connected edges”, the upper and the lower bounds for the critical load are established. The bounds are based on the analytical solutions for 3-D internal instability problem, considered within the model of piecewise-homogeneous medium. It is suggested that the Equivalent Constraint Model could be used to account for the presence of intralaminar defects in the material. Numerical results for hyperelastic layered materials, with layers described by the neo-Hookean potentials, are presented and discussed. They indicate that the bounds give a good estimation for considered modes of internal instability and material properties.

## 1 INTRODUCTION

Various types of inter- and intralaminar defects may occur in layered materials during the fabrication process or in-service. Interlaminar defects include cracks, zones of non-adhesion, reduced adhesion and slippage, and similar imperfections, while intralaminar defects could be cracks, voids, porosity etc.

When a layered material is compressed along the layers, fracture due to interlaminar defects cannot be predicted using the classical Griffith-Irwin criterion or its generalisations, since all stress intensity factors and crack opening displacements are equal to zero. This fact emphasises the importance and the necessity of investigation of fracture due to mechanisms, specific to heterogeneous materials. One of such mechanisms is the loss of stability in the microstructure of the heterogeneous material; the moment of stability loss in the microstructure of the material – internal instability according to Biot – is associated with the onset of fracture.

The most accurate approach to the analysis of the internal instability is based on the model of a piecewise-homogeneous medium, when the behaviour of each component of the material is described by the 3-D equations of solid mechanics provided certain boundary conditions are satisfied at the interfaces. It was used in numerous publications on the topic – see the reviews [1, 2]. Along with the exact approach, there are also approximate models proposed by Rosen and later by many other authors. Detailed comparative analysis of different approaches was given in [1, 2, 3]. It was concluded [1, 2, 4, 5] that the approximate methods are not accurate when compared to experimental measurements and observations. In the case of large pre-critical (applied) deformations, the approaches based on the Rosen model cannot be applied at all. The 3-D approach used in this paper allows us to take into account large deformations, geometrical and physical non-linearities and load biaxiality that the simplified methods cannot consider.

This paper investigates internal instability of layered hyperelastic materials with inter- and intralaminar defects undergoing large deformations under uniaxial or equi-biaxial loading. For interlaminar defects called “defects with connected edges”, the upper and the lower bounds for the critical load are established. The bounds are based on the analytical solutions for 3-D internal instability problem, considered within the model of piecewise-homogeneous medium. It is suggested that the Equivalent Constraint Model could be used to account for the presence of intralaminar defects in the material. Numerical results for hyperelastic layered materials, with layers described by the neo-Hookean potentials, are presented and discussed. They indicate that the bounds give a good estimation for considered modes of internal instability and material properties.

## 2 ANALYSIS

The material consists of alternating layers with thicknesses $2h_r$ and $2h_m$ (Fig. 1). Henceforth all values referred to these layers will be labelled by indices *r* (reinforcement) and *m* (matrix). Each layer is treated as an incompressible transversally isotropic solid with a general form of the constitutive equations.

It is assumed that an unspecified number of interlaminar defects called “defects with connected edges” [5, 6] or “perfectly lubricated interfaces” [7, 8] exist in the material. These defects refer to the zones of imperfect interlaminar adhesion, where the contact between the layers is implemented in such a way that infinitesimal sliding is allowed, but still there are no gaps between the layers (Fig. 2). In this case, the continuity at the interface is retained for normal components of stresses and displacements only.

For a layered material with these interfacial defects, the following estimation for the critical load can be suggested following [5]

$$P_{cr}^{sl} \le P_{cr}^{imp} \le P_{cr}^{pb} \qquad (1)$$

Here $P_{cr}^{imp}$ is the critical load for a material containing “defects with connected edges”, $P_{cr}^{pb}$ (upper bound) is the critical load for a material with the same internal structure with perfectly bonded layers, and $P_{cr}^{sl}$ (lower bound) is the critical load for a material with the same internal structure with sliding layers. The substantiation of the bounds is based on a general principle of mechanics, which states that the release from a part of connections inside of the mechanical system cannot increase the value of the critical load.

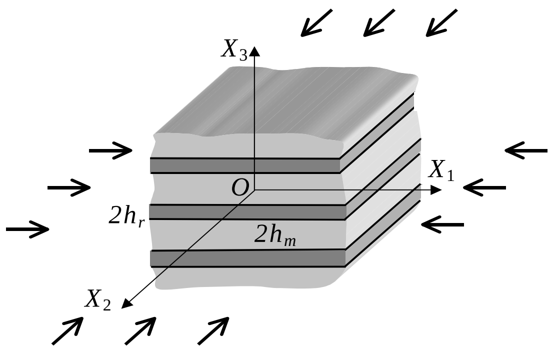


Figure 1: The co-ordinate system and applied loads.

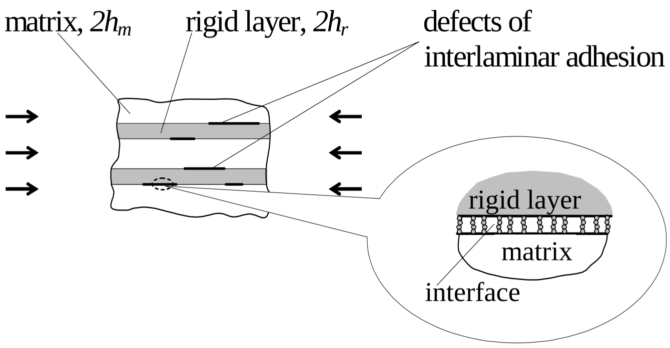


Figure 2: An interface with defects of interlaminar adhesion.

The same estimation can be written in terms of shortening factors as

$$\lambda_{cr}^{pb} \le \lambda_{cr}^{imp} \le \lambda_{cr}^{sl} \tag{2}$$

where $u_i = (\lambda_i - 1)x_i$. Inequalities, eqns (1) and (2), are true for an arbitrary number, size and disposition of the defects with connected edges.

In order to calculate the lower and upper bounds, the non-axisymmetrical problem of the internal instability is considered within the model of a piecewise-homogeneous medium using the equations of the 3-D stability theory [1]. This allows us to eliminate the restrictions imposed by using the approximate theories as well as the inaccuracies they involve. In both cases (perfectly bonded layers and sliding without friction layers), the characteristic determinants are derived analytically for the modes, which are more commonly observed. The proposed method can also give the solutions for modes with periods, which are equal to 3, 4, 5, …. periods of the internal structure. Other modes with periods, which are not multiples of the period of the internal structure, can also be examined. The solution for them would be based either on the Floquet theorem for ordinary differential equations with periodic coefficients [9], or on reducing the problem to an infinite set of equations with the consequent solution by a numerical method [10].

In many cases, layered materials contain not only interlaminar, but also various sorts of intralaminar defects such as cracks, voids, pores etc. One of the strategies to account for the presence of these intralaminar defects is to replace the layers with defects with equivalent homogeneous ones with appropriate effective properties. It is now increasingly accepted that the layer with defects behaves within the layered material in a different manner compared to an infinite medium containing many defects. In particular, elastic properties of the cracked layer are strongly influenced by its neighbouring layers. To take account of the in-situ constraint of the neighbouring layers on the effective properties of a particular layer with defects, it is suggested to use the Equivalent Constraint Model (ECM) of the damaged layer (Fig. 3). The model was successfully used to predict effective properties of cracked layers in fibre-reinforced composite laminates [11, 12].

To calculate the effective properties of the particular $k^{th}$ layer with intralaminar defects using ECM, all the layers above and below it (except the immediate ones), are replaced with homogeneous layers having the equivalent constraint effect (Fig. 3). The effective properties of all


Figure 3: Equivalent Constraint Model for the layered material with intralaminar defects.

layers with defects are then calculated from a set of inter-related problems for ECMs and incorporated into the fracture analysis described above.

## 3 RESULTS AND DISCUSSION

The effect of the different types of inter- and intralaminar defects, layer thickness and stiffness on the lower and the upper bounds is examined for a number of particular non-linear models of materials under various kinds of loading. The obtained results show that the bounds present a good estimation. One of the examples is given below.

Let the composite (Fig. 1) consist of hyperelastic layers described by the simplified version of Mooney's potential, namely neo-Hookean potential, with the strain energy density function $\Phi = 2C_{10} I_1(\varepsilon_{ij}^0)$, where $C_{10}$ is a material constant, and $I_1(\varepsilon)$ is the first algebraic invariant of Cauchy-Green strain tensor. This potential is also called Treloar's potential, after the author who obtained it from an analysis of model of rubber regarded as a system of long molecular interlinking chains [13].

The upper bound for critical shortening factors is found as a result of the following procedure. Solving the characteristic equations derived for the case of sliding layers for different modes of stability loss (see, for example [14]), the shortening factors are obtained as

$$\lambda_1^N = \lambda_1^{(N)}(\alpha_r) \tag{3}$$

where $\alpha$ is the normalised wavelength, and $N$ is the number of the mode ($N = 1, 2, \dots$). The critical value for the particular mode, $\lambda_{cr}^{(N)}$, can be found as a maximum of the corresponding function. The maximum of these $N$ values will be the critical shortening factor of the internal instability for the considered layered material with sliding layers, $\lambda_{cr}^{sl}$,

$$\lambda_{cr}^{sl} = \max_N \lambda_{cr}^{(N)} = \max_N \left( \max_{\alpha_r} \lambda_1^{(N)} \right) \tag{4}$$

which is also the upper bound for the critical shortening factor for composites with interlaminar defects with connected edges.

The lower bound for critical shortening factors is found in a similar way following the approach [15]. Solving the characteristic equations derived for the case of perfectly bonded layers for different modes of stability loss, the shortening factors are obtained as functions of the normalized wavelength. The critical value for the particular mode can be found as a maximum of the corresponding function. The maximum of these $N$ values will be the critical shortening factor of the internal instability for the considered layered material with perfectly bonded layers, $\lambda_{cr}^{pb}$, which is also the lower bound for the critical shortening factor for composites with interlaminar defects with connected edges.

The examples of the upper bound were given in [14]. The examples of the lower bounds for equi-biaxial compressive loading are shown in Fig. 4 and 5. They were calculated for the typical ratios of the material constants, $C_{10}^{r} / C_{10}^{m}$. In both cases, the value of the critical shortening factor increases with increasing relative stiffness of the layers. The increase rate is much higher for smaller values of the relative stiffness of the layers, i.e. for $C_{10}^{r} / C_{10}^{m} < 50$. At that, the increase is very sharp for $C_{10}^{r} / C_{10}^{m} < 20$.

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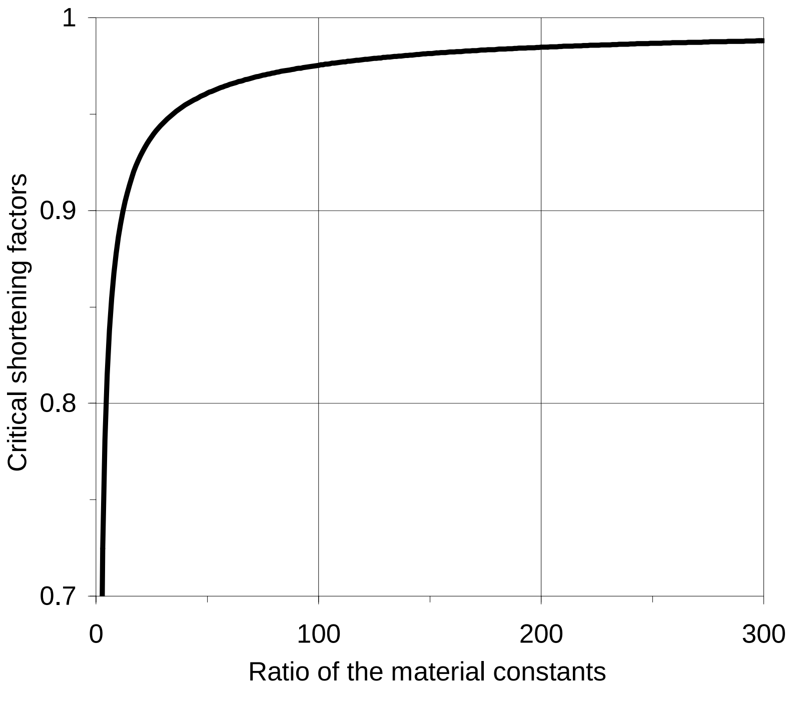


Figure 4: The lower bound for $\lambda_{cr}^{imp}$; $h_r / h_m = 0.06$.

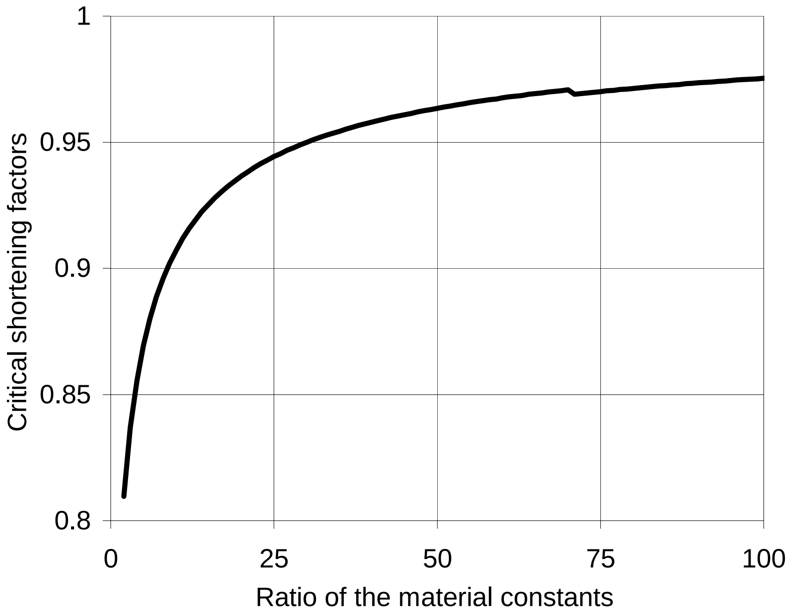


Figure 5: The lower bound for $\lambda_{cr}^{imp}$ ; $h_r/h_m = 0.09$ .

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