ANALYSIS OF INTERIOR STRESS FIELD OF THE SPECIMEN UNDER MIXED-MODE LOADING BY THREE-DIMENSIONAL LOCAL MODEL HYBRID METHOD

Kenji Machida

Department of Mechanical Engineering, Tokyo University of Science, Chiba, 278-8510, Japan

ABSTRACT

An experiment was conducted on the compact normal and shear (CNS) specimen made of acrylic resin subjected to various kinds of mixed-mode loading. The stress-intensity factors of mixed mode can be estimated by embedded speckle photography. However, the accuracy of the stress intensity factor inside the specimen was considerably large. To evaluate the 3-D stress field inside the specimen from displacement data on the free surface obtained from the 2-D intelligent hybrid method, we developed the 3-D local model hybrid method based on the inverse problem analysis. The accuracy of the 3-D local model hybrid method varies depending on the depth of the plane of error assessment, hybrid domain size and specimen thickness. Hence the optimal analysis condition was discussed for the acrylic homogeneous material and the acrylic-aluminum dissimilar material. The 3-D local model hybrid method can estimate the stress intensity factor with high accuracy, if error assessment is carried out between the displacement matrix at 1.5 mm depth of the 3-D full model FEM and the modified displacement data obtained by the 2-D intelligent hybrid method from the displacement data measured in the experiment. Accurate analyses can be conducted if hybrid domain size is taken more than two times of the thickness of the specimen with a straight crack of homogeneous and dissimilar materials. Although the accuracy of the specimen with a fatigue pre-crack is not so high as a straight crack, if hybrid domain size is taken by 2.5 times of the thickness of the specimen, the stress intensity factor at the central part of the specimen can be estimated with less than 1.0 % error. The 3-D local model hybrid method can evaluate the stress field inside the specimen of homogeneous material with high accuracy. On the other hand, in dissimilar material, accuracy goes down slightly.

1 INTRODUCTION

Real structures are three dimensions and it is very important to get to know the stress field inside an object. A stress-intensity factor inside a specimen was evaluated by embedded speckle photography [1]. However, the accuracy of the obtained result has not been enough yet. The displacement obtained from the experiment is including large error and it is impossible to evaluate the stress and the strain with high accuracy using raw displacement data. Therefore, we applied the 2-D intelligent hybrid method proposed by Nishioka et al. [2][3] in order to evaluate the 2-D displacement field. It is elucidated that although the 2-D intelligent hybrid method does not evaluate the surface stress-strain field of the dissimilar material strictly, it can evaluate the internal (a depth of about 0.5-1.0 mm) stress-strain field [4]. Then, to evaluate the 3-D stress field inside the specimen from displacement data obtained from the 2-D intelligent hybrid method, the 3-D local hybrid method based on the inverse problem was developed. Accuracy evaluation was performed from comparison with the 3-D direct full model FEM.

2 THEORETICAL BACKGROUND

The 3-D local hybrid method was built according to the approach of Murakami and Yoshimura [5]. Now let Γ denote the measurement area of the displacement at the surface of the 3-D body as shown in Fig. 1. The nodal displacement of 8-nodes isoparametric elements in Γ region is supposed to be measured by the experiment. Let M and u denote the number of nodes and displacement vector respectively. It is considered that the analysis of the 3-D stress field is possible, if the nodal forces on boundary Γ are determined from the information of u. Let us consider the 3-D mesh which extended the 2-D mesh to the thickness direction as shown in Fig. 2. Γ is the boundary at the surface, several points or a narrow region is constrained, and unit uniform

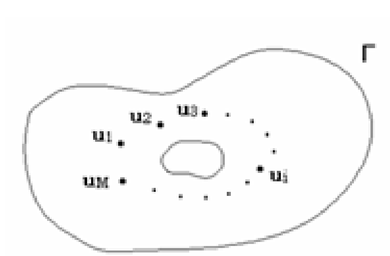


Figure 1: Measurement of $\boldsymbol{u}_{\mathrm{i}}$

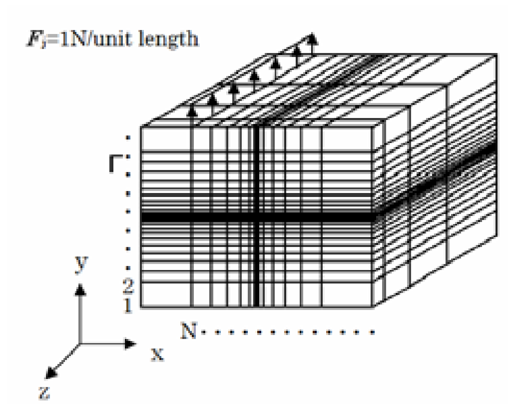


Figure 2: Application of unit uniform load in thickness direction

load is applied to one line in thickness direction (the number of lines j=1~N) including a principal node on Γ. Let $\boldsymbol{u}_{ij}$* denote the displacement vector of point i due to this unit uniform load. Let F_j denote the nodal force applied to the point j. If F_j is the appropriate value, the error $\boldsymbol{e}_i$ shown by the following equation should become zero.

$$\boldsymbol{e}_i = \boldsymbol{u}_i - \sum_{j=1}^{N} \boldsymbol{u}_{ij}^{*} \boldsymbol{F}_j \ , \ \ i=1\sim M, \ \ j=1\sim N. \tag{1}$$

A least squares method is applied to determine the unknown value, F_j. Here, S is defined by the following equation as the sum square error.

$$S = \sum_{i=1}^{M} (\boldsymbol{e}_i)^2 \ . \tag{2}$$

It is considered that F_j considered appropriate satisfies the following equation.

$$\frac{\partial S}{\partial \boldsymbol{F}_j} = 0 \ . \tag{3}$$

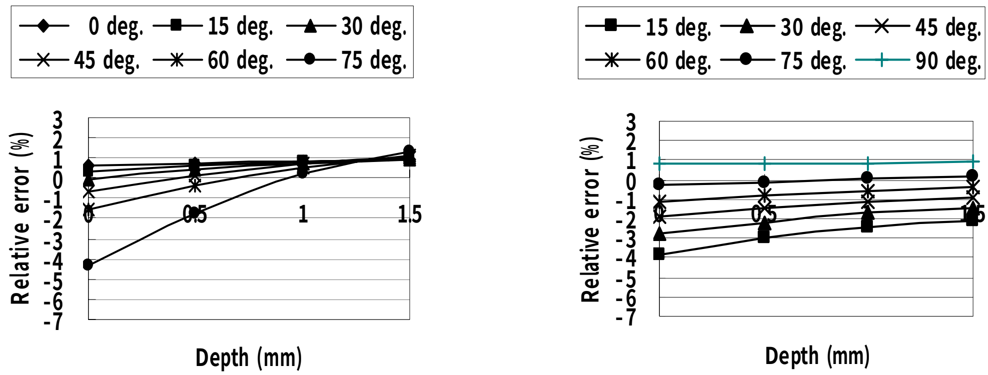
Equation 3 becomes 2N simultaneous equations since a displacement and a nodal force vector have two components of the x and y directions, respectively. F_{jx} and F_{jy} (j=1~N) can be determined by solving eqn (3). If F_j is obtained, we can determine the stress and strain of the arbitrary points inside the 3-D model shown in Fig. 2 by FEM with the boundary condition of F_j.

3 MODEL FOR ANALYSIS

Two kinds of specimens that were made of acrylic homogeneous and aluminum acrylic dissimilar materials were analyzed. Young's modulus and Poisson's ratio of aluminum and acrylic resin were 67.3 GPa, 0.33 and 3.06 GPa, 0.38, respectively. Width is 60 mm, height is 100 mm, and thickness is 3, 6, 15, 30 and 50mm. The crack length was 30 mm. The number of layer division of full model was 4, and the numbers of elements and nodes were 1440, and 7801, respectively. We carried out seven kinds of mixed-mode loadings by changing a load application angle (α). The mixed-mode stress-intensity factors were evaluated by the virtual crack extension method [6] and the superposition of an asymptotic solution [7]. Virtual experimental data were extracted from the surface displacement data of 3 D full model analyses.

4 RESULTS AND DISCUSSIONS

4.1 Depth of error assessment plane



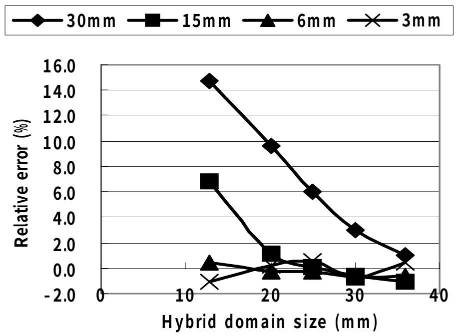
(a) K_1 (b) K_2

Figure 3: Relative error of the stress intensity factor between the full model FEM and the local model hybrid method of dissimilar material

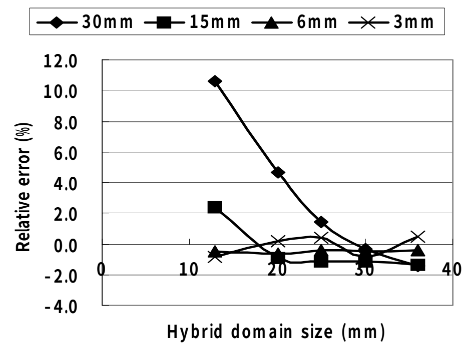
Appropriate displacement is evaluated by the 2-D intelligent hybrid method from the displacement data obtained from the experiment by assuming the plane stress condition. However, the obtained displacement data is not strictly in agreement with the result of the surface obtained by the 3-D full model FEM. It is considered by error assessment on the 3-D local model surface that high accuracy is not acquired. Then, the depth of error assessment plane was examined.

Figure 3 shows the relative error of the stress intensity factor between the 3-D full model FEM and the 3-D local model hybrid method. In the case of homogeneous material, the relative error of K_1 increases monotonically, and becomes -0.2 % in a depth of 1.5 mm as the depth of plane to evaluate an error with virtual experimental displacement data changes from the surface to a depth of 1.5 mm. On the other hand, although the relative error of K_2 varies considerably from the surface to a depth of 1.0 mm, and becomes almost uniform 0.8 % at 1.5 mm depth. The above-mentioned tendency hardly depends on the load application angle. In the case of dissimilar material, the relative error of K_1 increases monotonically as the depth of plane to evaluate an error with virtual experimental displacement data changes from the surface to a depth of 1.5 mm as shown in Fig.3 (a). As the load application angle increases, a variation becomes large. However, in the case of 1.5 mm depth, the relative error becomes constant about 1.0 % except α=75 degrees. On the other hand, the relative error of K_2 increases with the increase in depth, but it becomes almost constant with less than about 2% at 1.5 mm depth as shown in Fig.3 (b).

4.2 Optimal hybrid domain size



(a) K_1



(b) K_2

Figure:4 Variation of relative error between stress intensity factors of full model FEM and local model hybrid method of dissimilar material with hybrid domain size (α=45 degrees)

The accuracy of the 3-D local model hybrid method depends on the hybrid domain size [4]. From the above-mentioned result, the depth was fixed 1.5 mm and error assessment was performed. Four kinds of thickness 3, 6, 15, and 30 mm were used. The domain size was changed 7.2 mm × 7.2 mm, 12.8 mm × 12.8 mm, 20 mm × 20 mm and 25 mm × 25 mm for the homogeneous material. Since the error was large in the dissimilar material, 30 mm x 30 mm and 36 mm x 36 mm which were bigger than those of the homogeneous material were also examined.

Figure 4 shows variation of the relative error of the stress intensity factor of the full model FEM and the local model hybrid method with the hybrid domain size in α=45 degrees as a typical example in mixed mode. In the case of a homogeneous material, a relative error decreases as hybrid domain size becomes large. In the case of 6 mm thick specimen, the relative error of K_1 and K_2 becomes less than 0.5 % at domain size 12.8 mm which are bigger than the thickness of the specimen. In the case of 15 mm thick specimen, the relative error becomes less than 1 % at domain size 20 mm. Similar tendency can be seen in other load application angles. Therefore, it is considered that accurate analyses can be conducted if hybrid domain size is taken more than two times of the thickness of the specimen. The relative error decreases as hybrid domain size becomes large also in the case of a dissimilar material. In the case of 3 mm thick specimen, the relative error vibrates between -1 % and 1.5 % rather than approaches to 0 % even if hybrid domain size is taken with 6 or more times of the thickness of the specimen. However, the relative error of 6 mm thick specimen is less than that of 3 mm thick specimen and vibrates between -1 % and 0 %. Unlike the case of homogeneous material, in the case of dissimilar material, the relative error from α= 0 degree to 90 degrees is not stabilized. The relative error of K_1 becomes large as α increases, while that of K_2 changes irregularly according to α. At domain size of two times of the thickness, the relative error becomes less than ±3%. However, even if domain size becomes larger, the relative error does not decrease. Therefore, it is considered that accurate analyses can be conducted if hybrid domain size is taken more than two times of the thickness of the specimen as well as the homogeneous material.

Although the above result is about a straight crack, if hybrid domain size is taken by 2.5 times of the thickness of the specimen, the stress intensity factor at the central part of the specimen with a fatigue pre-crack can be estimated with less than 1.0 % error.

6 CONCLUSIONS

(1) The 3-D local model hybrid method can estimate the stress intensity factor with high accuracy, if error assessment is carried out between the displacement matrix at 1.5 mm depth of the 3-D full model FEM and the modified displacement data obtained by the 2-D intelligent hybrid method from the displacement data measured in the experiment.

(2) Accurate stress analyses can be conducted if hybrid domain size is taken more than 2.5 times of the thickness of the specimen of homogeneous and dissimilar materials.

(3) The 3-D local model hybrid method is a practically very useful approach which the stress field inside the specimen can be analyzed by measuring only the displacement field near a crack or a hole without analyzing entire structure by FEM or BEM.

REFERENCES

[1] Machida, K., Geni, M. and Okamura, H. Eds. Geni, M. and Kikuchi, M., Key Engineering Materials, Vols. 243-244, pp. 153-158, (2003).

[2] Nishioka, T., Ikekita, H. and Tamai, K., Computational Mechanics, Vol. 20, pp. 101-108, (1997).

[3] Nishioka, T., Kurio, K. and Nakabayashi, H., Exp. Mech., Vol. 40, pp. 170-179, (2000).

[4] Machida, K., Ed. Brust, F.W., PVP-Vol. 438, pp. 221-228, (2002).

[5] Murakami, Y. and Yoshimura, M., M., Trans. Jpn. Soc. Mech. Eng., Vol. 61, pp. 2482-2488, (1995).

[6] Parks, D.M., International Journal of Fracture, Vol.10, pp.487-502, (1974).

[7] Sun, C.T. and Jih, C.J., Vol. 28, No. 1, pp. 13-20, (1987).