The optimal design for fatigue life using biological method

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ABSTRACT

A methodology for structural shape optimization is presented. It can be applied to problems with fatigue life, as the design objective,. Initial cracks, of a user specified size, are automatically generated perpendicular to a (user) specified boundary. The software allows the rapid and accurate calculation of the fatigue life associated with each of the (user) specified cracks by using the new and simple method developed in [2] for estimating the stress intensity factor for cracks at a notch. These quantities are then used to determine the optimum shape. This method is ideal for use in structural optimisation as accurate results are obtained without the need to explicitly model a crack, i.e. it is only necessary to model the uncracked structure. This work confirms earlier findings that a stress optimized structure does not necessarily give the longest fatigue life. The importance of non destructive inspection (NDI) and the role it plays in determining optimum structural geometries is also revealed.

1. INTRODUCTION

Current optimum design tools do not readily lend themselves to treating fatigue life, as a design objective. This is because the majority of a component's life is frequently used up, as cracks grow from an initial non-detectable (part elliptical) flaw to a complex flaw size that is detectable. Thus, at each stage of the component's life, it is often necessary to analyse a complex 3-D flaw under arbitrary loading. The strain singularity along the crack (flaw) front requires a fine numerical mesh. As a result, the optimisation problem becomes extremely time consuming and requires large computer resources. The optimisation problem is further compounded by the need for every geometry under consideration, to allow for flaws at every potentially critical point.

To overcome this we need simple and accurate formulae for computing the stress intensity factors associated with cracks at a stress concentrator, or a notch. Unfortunately, there are few such solutions in the literature for this class of problems. The present paper uses a new and simple method for estimating the stress intensity factor for cracks at a notch [4], as well as an extension of the biological algorithms presented in [1], to study the problem of optimisation with fatigue life, as the design objective.

2. METHODOLOGY

2.1. Approximate formulae for 3D surface cracks

The simple technique presented in [2, 3] for determining the stress intensity factor of a through crack at a notch was recently extended [4] to three dimensions by noting that the solution for a semi-elliptical surface flaw, with a surface length of 2c and a depth of a, is expressed in the form.

$$K_I = \underline{K_I} M_f \tag{2.1-1}$$

where $\underline{K_I}$ is the solution to the equivalent embedded elliptical crack, in an infinite body acted upon by the same stress field as found at the surface. In equation (2.1-1) the boundary correction factor, M_f, accounts for the influence free surface, finite width of the specimen, crack length and the local radius of curvature. The factor M_f can be expressed as:

$$M_f = F_e + (F_s - F_e)e^{-\beta_\vartheta \frac{a}{\rho}} \tag{2.1-2}$$

where ρ is the local curvature, a is a depth length of the surface elliptical crack, as shown in Figure 2.1-1. The parameters β_ϑ, F_e and F_s were taken from [4].

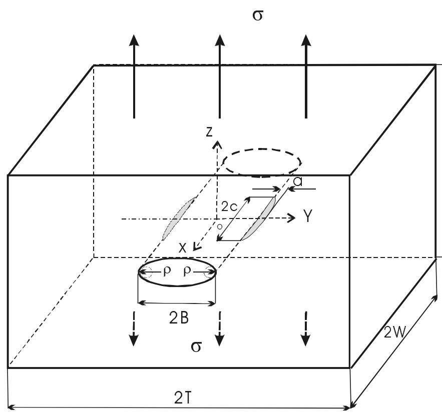

Figure 2.1-1 Two identical opposing semi-elliptical cracks at a internal notch

2.2. The Crack growth model

The crack growth analysis performed in this study used a modified version of the growth law used in FASTRAN [5], viz:

$$\frac{dc}{dN} = C_1(\Delta K_{eff})^{C_2} \frac{1-(\frac{\Delta K_0}{\Delta K_{eff}})^2}{1-(\frac{K_{max}}{C_5})^2} \tag{2.2-1}$$

Here K_0 and K_{max} are the maximum and the opening stress intensity factors respectively, and

$$\Delta K_0 = C_3(1 - C_4 \frac{K_0}{K_{max}}) \tag{2.2-2}$$

$$\Delta K_{eff} = K_{max} - K_0 \tag{2.2-3}$$

Here $C_1,...,C_5$ are materials constants respectively and the crack opening stress intensity factors K_0 was calculated using the analytical closure model described in [5].

2.3. Shape optimisation

2.3.1 The biological optimization algorithm

The "biological" optimisation procedure used in this work was an extension of that used by Kaye and Heller [6], which was first proposed by Mattheck and Burkhardt [7]. The "biological" optimisation procedure outlined in [7] states that the amount of material to add, or remov, is directly proportional to the difference between the local tangential (hoop) boundary stress σ_i and a suitable reference threshold stress σ_{th}. In this approach the i'th boundary node is moved normal to the local boundary by an amount d_i, as shown in equation 2.3.1-1.

$$d_i = \left(\frac{\sigma_i - \sigma_{th}}{\sigma_{th}} \right) s_f \qquad (2.3.1\text{-}1)$$

Here s_f is a step size parameter. It is clear from equation (2.3.1-1) that if σ_i is greater than σ_{th} material will be added and if σ_i is less than σ_{th} then material will be removed.

In this paper the stress based optimisation algorithm expressed in equation (2.3.1-1) was extended by replacing the terms, such as σ_i by $N^i f$ (the fatigue life for a crack initiating at the i'th point). As a result equation (2.3.1-1) is used to determine d_i for fracture strength based optimisation and fatigue life based optimisation respectively, viz:

$$d_i = \left(\frac{N^i f_i - N f_{\min}}{N f_{\min}} \right) s_f \qquad (2.3.1\text{-}2)$$

Here $N f_{\min}$ is minimum value of $N^i f$.

2.3.2 Shape optimisation procedure

The shape optimisation methodology presented in this report is an iterative procedure that was based on the "biological" optimisation procedure presented in [1], and used the semi-analytical procedure [4] to determine the stress intensity factors associated with surface cracks emanating from a notch. The basic steps in this approach is given in below:

1. In this (iterative) optimisation approach, an IGES (Initial Graphics Exchange Specification) file was used to describe the three-dimensional geometry.
2. CUBIT, an automated mesh generation program developed at the Sandia National Laboratories, was used to create the FEM mesh (uncracked structure). The finite element model was then analysed using NE-NASTRAN to evaluate the stress field.
3. An approximate formulae for 3D surface cracks was used to determine the stress intensity factors for a complex 3-D flaws under arbitrary loading. In this instance, the stress intensity factors around the crack front need to be determined at each stage of the fatigue life calculation.
4. The change in the design boundary depends on the structural response.
5. The convergence of the optimisation process was monitored. Convergence was assumed to be achieved when the relative/absolute change in the design variables and/or objective function between successive iterations was less than a pre-defined value.
6. The steps 2-5 were repeated until convergence was achieved.

3. APPLICATION TO SHAPE OPTIMIZATION

Let us illustrate further the versatility of this technique by considering the problem of flaws in a rib stiffened structure that contains a cutout. The geometry of the structure, and the applied loading are as shown in Figures 3.1 and 3.2. This structural components is found in an aircraft wing and the cutouts are sometimes referred to as “mouseholes”.

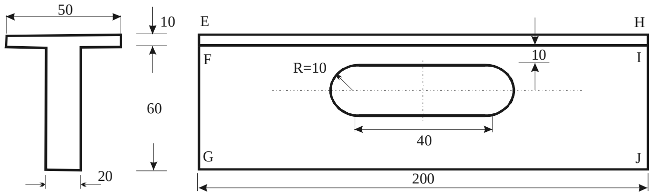

Figure 3.1 The geometry of stiffener and plate

The loading, as illustrated in Figure 3.2 was assumed to be a remote uniform tensile stress of σ =100 MPa applied to the surface HIJ, and a concentrated force of 6800 N, applied half way through the thickness of the stiffener, at point J. The left surface EF (i.e. the upper plate) was fixed. The thickness of the (upper) plate and the stiffener (rib) were taken to be 10.0 mm and 20.0 mm respectively.

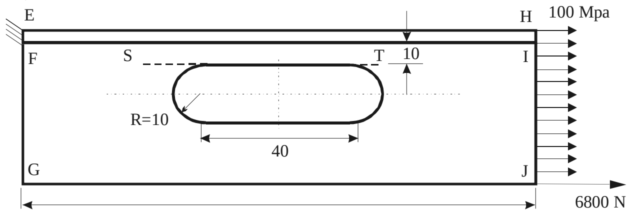

Figure 3.2 Loading and boundary conditions

The upper segment of the hole was taken to be fixed. This is because the upper plate is the primary load bearing member. Thus any damage accidentally induced in the upper plate as a result of manufacturing defect arising from reworking the mousehole could compromise the load carrying capacity of the component. Therefore, care must be taken to avoid inducing any defects near the upper plate. One way to ensure that this region is defect free, is to avoid reworking this region of the mousehole. Consequently, the line ST, as shown in Figure 3.2. was taken to be a constraint on the position of the upper boundary of the mousehole.

As the (uncracked) problem was symmetric, only half of the uncracked structure was analysed. In this study both the plate and the stiffener was assumed to have a Young's Modulus (E) of 72000 MPa, and a Poisson's ratio (ν) of 0.3, a yield stress of 365 MPa, an ultimate tensile strength of 448 MPa, and a fracture toughness of 36.27 MPa√m. The stiffener mousehole problem described above was used to study optimisation for the case when the (initial) flaws around the periphery were uniformly long around boundary. In this work, the flaws were considered to be surface flaws emanating from the boundary of the mousehole. In this optimisation study, 61 control points (nodes) were used around the boundary of the mousehole.

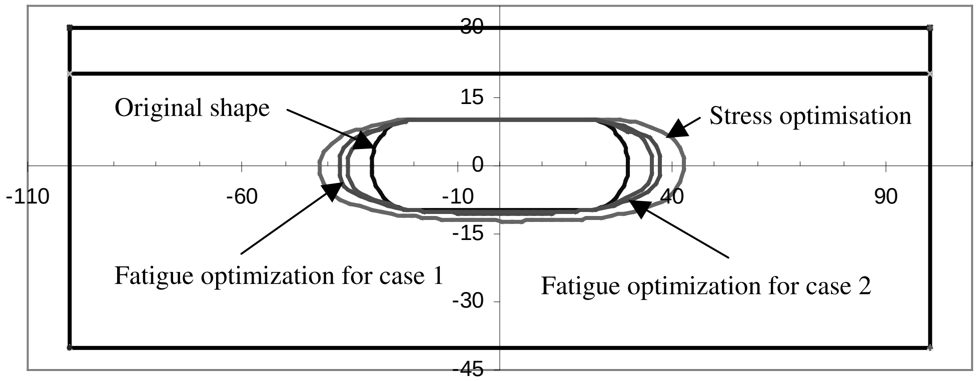

Figure 3.3. Comparison of stress optimised, fatigue life optimised and initial shape.

In this work we will determine the shape of the mousehole needed to optimize the fatigue life of the structure for two difference cases. sizes (a, c) , viz: The initial crack size (a_i, c_i) and final crack size (a_f , c_f) were chosen, viz:

1). a_i = 1.0 mm, a_f = 2.0 mm and c_i = 3.0 mm, c_f = 5.0 mm;
2). a_i = 2.0 mm, a_f = 4.0 mm and c_i = 5.0 mm, c_f = 8.0 mm.

The crack paths, for cracks growing at a number of points around the boundary and the "optimized" geometry are shown in Figure 3.3. In both cases, the "near optimal" shape was significantly different from the initial shape, and the fatigue life of the mousehole was remarkably improved, see Table 3.1. For case 1, we obtained a maximum fatigue life of 852 cycles at the 40th iteration and for case 2, the maximum fatigue life of 602 cycles at the 41th iteration. The fatigue lives for the stress and the fatigue (N_f) optimized shapes are presented in Table 3.2. This table shows that the fatigue optimized shape has a fatigue life up to ~40% greater than the stress optimized shapes.

Table 3.1 Summary of Fatigue Lives for Mousehole Problem:

Descriptor (mm)	Initial geometry N_f (cycles)	Optimised geometry N_f (cycles)	Difference (%)
a_i = 1.0, a_f = 2.0 c_i = 3.0, c_f = 5.0	602	852	41.53
a_i = 2.0, a_f = 4.0 c_i = 5.0, c_f = 8.0	363	601	65.56

Table 3.2: Fatigue life (cycles) for various hole shapes

Case	Stress opt. shape	Fatigue (N_f) opt. shape	Difference (%)
1	763	853	11.79
2	428	601	40.42

4. CONCLUSIONS

This paper has presented a range of tools for use when studying the problem of structural optimization with fatigue life, or fracture strength, as the objective function.
This finding reveals the importance of non-destructive inspection (NDI) and the role it plays in determining optimum structural rework geometries. Consequently, when designing a major structural item, such as a rail bogey or sideframe, or a wing carry through box, it is important to note that structures designed to minimise the peak stresses are not necessarily the most durable. In this context, it is clear that unlike stress based optimisation, the optimal design process should account for the nature of the initial flaw(s) at the time of manufacture and the available NDI tools as well as the loading spectra.

5. REFERENCES

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