Space–Time Finite Element Techniques for Computation of Fluid–Structure Interactions

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ABSTRACT

We describe the space–time finite element techniques we developed for computation of fluid–structure interaction (FSI) problems. Among these techniques are the Deforming-Spatial-Domain/Stabilized Space–Time (DSD/SST) formulation and its special version, and the mesh update methods, including the Solid-Extension Mesh Moving Technique (SEMMT). Also among these techniques are the block-iterative, quasi-direct and direct coupling methods for the solution of the fully-discretized, coupled fluid and structural mechanics equations. We present some test computations for the mesh moving techniques described. We also present numerical examples where the fluid is governed by the Navier–Stokes equations of incompressible flows and the structure is governed by the membrane and cable equations. Overall, we demonstrate that the techniques we have developed have increased the scope and accuracy of the methods used in computation of FSI problems.

1 INTRODUCTION

Fluid–structure interactions (FSI) is one of the most challenging classes of problems in computational engineering. Some of the computational challenges are encountered also in other classes of fluid mechanics problems involving moving boundaries and interfaces, such as free-surface, two-fluid interface, and fluid–object interaction problems. For example, the spatial domain occupied by the fluid changes in time, and the mathematical model to be used will need to be able to handle that. Also, the mesh needs to be updated as the spatial domain occupied by the fluid changes, and this could be complicated for 3D problems with complex geometries. In addition, we face a number of computational challenges that are more specific to FSI problems, such as the solution the coupled fluid and structural mechanics equations.

The Deforming-Spatial-Domain/Stabilized Space–Time (DSD/SST) formulation was introduced in early 1990's for computation of flow problems with moving boundaries and interfaces. The DSD/SST method is based on stabilized finite element formulations, which are written over the space–time domains of the fluid mechanics problems considered. The stabilized methods are the streamline-upwind/Petrov-Galerkin (SUPG) and pressure-stabilizing/Petrov-Galerkin (PSPG) formulations. These stabilized formulations prevent numerical instabilities that might be encountered in solving problems with high Reynolds or Mach numbers and shocks and strong boundary layers, as well as when using equal-order interpolation functions for velocity and pressure. Furthermore, this class of stabilized formulations substantially improve the convergence rate in iterative solution of the large, coupled nonlinear equation system that needs to be solved at every time step. The stabilized space–time formulations were used earlier by other researchers to solve problems with fixed spatial domains.

The space–time computations are carried out for one space–time "slab" at a time, where the "slab" is the slice of the space–time domain between the time levels n and n+1. This spares a 3D computational problem from becoming a 4D problem including the time dimension. Additionally, in most space–time computations, all the nodes of the space–time slab are at time level n or n+1, and the spatial mesh at level n+1 is simply a deformed version of the spatial mesh at level n.

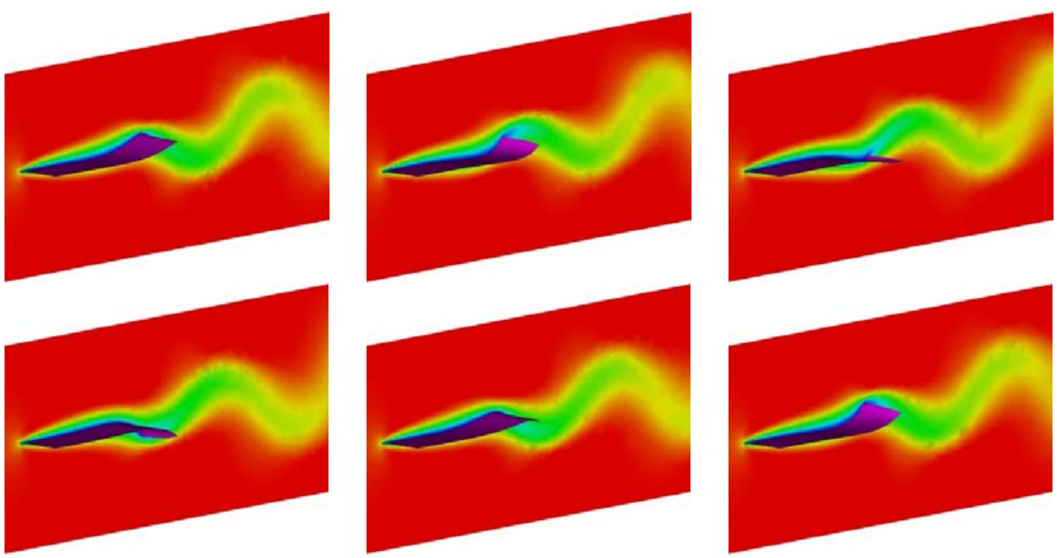
These additional special features are exploited in the Special DSD/SST (S-DSD/SST) formulation to make the calculation of the element-level vectors and matrices more efficient.

In general, we address the mesh update challenge with an automatic mesh moving method. In this method, the motion of the nodes is governed by the equations of elasticity. The mesh deformation is dealt with selectively based on the sizes of the elements. When the mesh becomes too distorted, a full or partial remeshing (i.e., generating a new set of elements, and sometimes also a new set of nodes) takes place. We introduced a number of enhancements to this general mesh update technique, including the Solid-Extension Mesh Moving Technique (SEMMT). The SEMMT was introduced to address the challenge involved in moving the very thin fluid elements typically seen near the solid surfaces.

The fully-discretized equations of fluid and structural mechanics need to be solved in their coupled form, and we propose a number of ways to accomplish that. They are: block-iterative coupling, which we have widely used in our computations; quasi-direct coupling; and direct coupling. The direct coupling approach is based on the mixed analytical/numerical element-vector-based (AEVB/NEVB) computation technique.

2 FLOW PAST A "FLAG"

In this test problem, we simulate the FSI involved in the flapping of a "flag". We use the quasi-direct coupling approach. The dimensions of the flag are 1.5 m in the flow direction and 1.0 m in the span-wise direction. The fluid velocity, density and kinematic viscosity are 2 m/s, 1 kg/m^3 and 0.008 m^2/s, respectively. The flag is modeled as a membrane with density 1000 kg/m^3, thickness 0.2 mm, and Young's modulus 40,000 N/m^2. The leading edge of the flag is held fixed and the lateral edges of the flag are constrained to move only in a normal plane. The FSI computations are carried out until a nearly cyclic pattern of flapping is reached. Figure 1 shows a sequence of snapshots of the flag and the horizontal velocity on a normal plane. Figure 2 shows the displacement and velocity for the midpoint of the free edge of the flag.



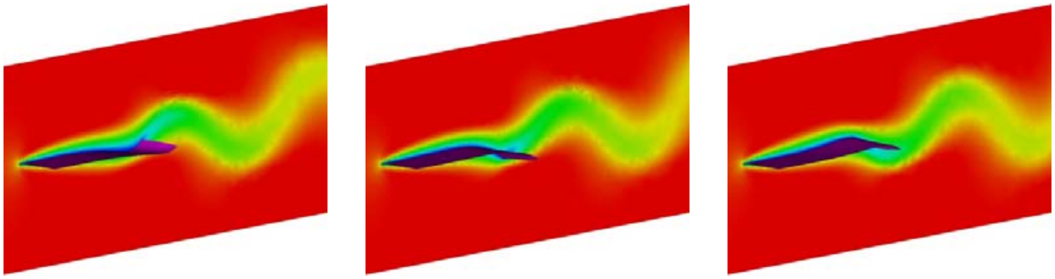


Figure 1: Time history (left to right and top to bottom) of the shape of the flag and the horizontal velocity on a normal plane.

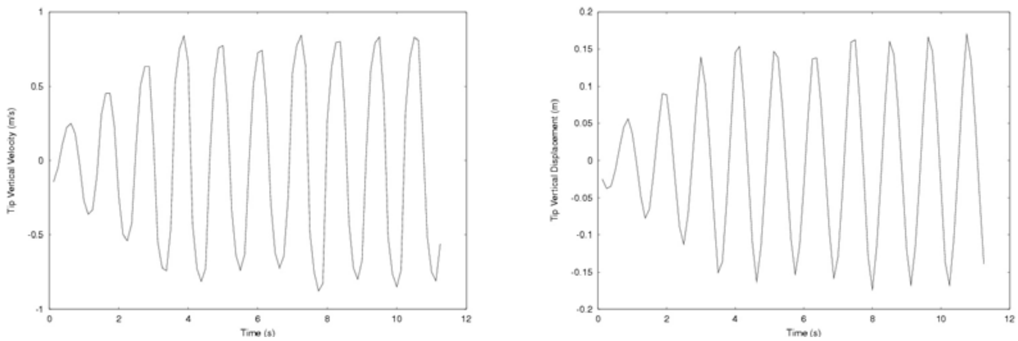


Figure 2: Vertical displacement (left) and velocity (right) of the midpoint of the free edge of the flag.

3 CONCLUDING REMARKS

We described a set of techniques for computation of fluid–structure interaction (FSI) problems. The core method is the Deforming-Spatial-Domain/Stabilized Space–Time (DSD/SST) formulation. We developed a number of techniques to support and enhance this core method. The Special DSD/SST (S-DSD/SST) formulation helps us make the element-level vector and matrix computations more efficient. The mesh update methods, which include the Solid-Extension Mesh Moving Technique (SEMMT), help us update the mesh effectively as the spatial domain occupied by the fluid changes in time. A number of techniques have been developed for the solution of the fully-discretized, coupled fluid and structural mechanics equations. These are the block-iterative, quasi-direct, and direct coupling techniques. The quasi-direct and and direct coupling techniques are particularly suited for FSI computations where the structure is light. We presented test computations for the mesh moving techniques we developed. We also presented results from test computations for FSI applications with light structures. We believe we demonstrated in this article that the techniques we are developing are substantially increasing the scope and accuracy of the simulations for FSI problems.

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