DAMAGE EVOLUTION IN COMPRESSED CONCRETE

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ABSTRACT

This paper deals with different aspects of damage evolution in compressed concrete. Contributions are paid to mechanisms of damage evolution in compression, and supporting evidence from analytical approaches and experiments. This approach is basically at the mesoscopic level, although also microscopic aspects are briefly introduced. Fracture roughness and fractality are touched upon. Quantitative image analysis and stereological tools are presented for the study of damage evolution on engineering level. The stochastic concept of heterogeneity is introduced as sampling concept. A distinction is made between structure-insensitivity and structure-sensitivity of material properties. Sub-sampling (image analysis areas, or specimens in material testing) is demonstrated leading to biases when material properties are sensitive to structural details. This is the case for crack initiation strength, because governed by local tensile strength capacity. Experimental approaches to ultimate strength or post-ultimate properties are less liable to such size effects.

1 INTRODUCTION

Concrete is a particulate composite material on different levels of the microstructure. It is also referred to as a macroscopically heterogeneous quasi-brittle material. Already in the early 1960s research efforts demonstrated virgin concrete to contain myriads of tiny cracks resulting from stresses due to shrinkage and differential settlements [1]. They are not visible by naked eye, though. The underlying concept of a *continuous range of micro-structural dimensions* as well as the three discrete levels of aggregation, denoted by macro-, meso-, and micro-level in concrete technology, have been recognized for a long period of time in the physics and mechanics of deformable bodies [2]. They form reference for the stages of damage evolution described in this paper.

Composite material behaviour under forces reflects the properties of the composing parts of the material body and the material structure on the various aggregation levels. This behaviour can be defined in terms of macro or engineering properties, such as the mechanical ones. Properties are denoted as structure-insensitive when solely governed by material *composition*, *e.g.* mass, and to a lesser extent UCS. Contrary, structure-sensitive properties, such as the crack initiation strength, are affected by the so-called group pattern or *configuration* of particles. As a consequence, particle size, shape and spacing are involved. An engineering property can only be attributed to a material element of at least representative dimensions, the representative volume element (RVE). Similarly, the quantitative image analysis approach to damage evolution stages should be based on a representative area element (RAE) [2]. Sub-RVE/RAE elements are frequently employed in experimental studies. They reveal an increasing degree of stochastic heterogeneity at reducing volume dimensions. This is reflected by the increasing width of the probability density curve of the investigated material property or structural characteristic (like cracking). This is a complicated field, because each geometrical parameter or material property has its independent scale of heterogeneity, so that required sample sizes vary significantly. Implications are size effects.

2 GLOBAL MECHANICAL BEHAVIOUR IN COMPRESSION

On engineering level the material behaviour is generally expressed in terms of the stress-strain diagram. Strength, stiffness and toughness are derived from such diagrams. Secants value of

Young's modulus, E_s, is derived from the slope of the stress-strain diagram in direct compression (DC). Poisson's ratio, ν, is the ratio of transverse and axial strains. Significantly extended linear ranges up to about three-quarters of UCS demonstrate the quasi-elasticity of high strength concretes up to a moderate stress level. Constant values of E_s and ν are resulting, as a consequence. The specimen's volume will diminish also linearly. After passing the crack initiation strength or LOP, ν will increase and E_s will decline. Thereupon, the stress-strain curves in orthogonal directions will be curved, attaining maxim at UCS. Between LOP and UCS specimens will arrive at minimum volume, after which a reversed tendency can be observed. Beyond UCS, strength capacity will diminish gradually, while the specimen is disintegrating. This is established phenomenology of concrete in DC.

3 DAMAGE EVOLUTION MECHANISMS

The first study on damage evolution mechanisms on meso-level in direct compression is probably due to Vile, referred to in [1]. Stroeven completed this study by information from rock mechanics [1], generalized it to also include the micro-level, and provided supporting microscopic evidence. This has been extensively published, so only a brief description will be provided.

(1) Particles will debond in the equatorial area under rising DC levels. These isolated and small bond cracks will on average run in the loading direction and will hardly influence overall stiffness.

(2) Rising load level will cause such cracks to grow along the particle-matrix interface. Since the azimuth zones on the particles are tri-axially compressed, the tendency of the bond cracks to leave the interface and join with neighbours will increase. At LOP this process is accelerated.

(3) The phenomenon of crack coalescence has more dramatic effects when larger particles are involved. The smaller particles situated in the path of two such coalescing bond cracks of larger particles will stimulate this phenomenon because their interfaces are also pre-cracked in axial direction. The activated small particle interface cracks are roughly situated on the surface of a tri-axial compressed cone-like mortar element. They constitute en échelon crack arrays [1], which reduce the local shear resistance of the material.

(4) The process of slip movement along the pre-weakened obliquely oriented shear zones leads to increasing lateral expansion of the specimens. In parallel, weak zones are eliminated, and redistribution of stresses takes place. The loading machine's stiffness prevents pre-mature destabilization of the discontinuous process of crack coalescence. This finally leads to UCS, after which the load capacity gradually declines.

(5) A stochastic process of gradual formation of series of fracture process zones starts at UCS, but continues during the gradual decline of the loading capacity. This ultimately leads to a structure of column-like elements of which the average cross-sectional dimensions equal those of the largest aggregate grains. This column-like structure finally collapses under load.

Solutions by elasticity theory for 2D cylindrical and 3D spherical inclusions have been available for a very long time [1,3]. Partially de-bonded situations are also covered. Reflective photo-elasticity allows assessing damage evolution characteristics at the surface of concrete specimens in which particle interference occurs [1]. These solutions present conclusive evidence for the afore-mentioned successive stages of damage evolution under direct compression. As a logic consequence of the gradual disintegration process, at final collapse of the column-like structure, small and larger aggregate grains can be found among the debris still provided with mortar cones. Such structural elements were found also by others [3]. The micro-cracks present in the virgin state are predominantly situated in the interface zone. The ones that are oriented in axial direction tend to grow and will appear at the meso-level. Their further grow is as indicated before.

4 DAMAGE EVOLUTION ON MACROLEVEL

Well-known *indirect* approaches (pulse velocity and resonance measurements) have been used for a long time for assessment of damage-induced stiffness reductions. Of course, the stress-strain diagram itself constitutes a basis for doing so. *Direct* measurements make use of quantitative image analysis procedures. Stereological tools additionally allow for 3D interpretation of the 2D crack measurements. Oldest studies are due to Stroeven [1], but more recent - scarce - applications can be found in the literature. Such approaches have revealed a slight increase in specific crack surface area over the quasi-brittle range, followed by an exponential growth beyond LOP with no special reference to ultimate loading [4], also supporting the sketched damage evolution stages.

The stereological framework for damage evolution assessment has been presented earlier [3]. The approach is based on the assumption of a partially-linear crack system in DC. This involves the development of an axis of axial symmetry (in stochastic sense). The actual specific surface area, S_V, is supposed to be composed of a 3D random portion, S_{V3}, and a 1D portion of which the crack planes are parallel to the symmetry axis, the linear one. In a vertical section (parallel to the orientation axis) the 3D portion is seen as a 2D random dispersion of crack traces and the 1D one as a completely oriented system of 1D traces in the direction of the orientation axis. Crack traces are ‘measured’ by covering the plane by a line grid and counting intersections, P. Normalizing by the grid length, L, yields $P/L=P_L$. For a grid successively in vertical and in horizontal direction:

$$P_L(vert) = \frac{1}{2}S_{V3} \quad \text{and} \quad P_L(hor) = \frac{1}{2}S_{V3} + S_{V1} \tag{1,2}$$

These two equations with two unknown parameters (S_{V3} and S_{V1}) allow determination of the specific crack surface area as function of the, say, axial deformation. The degree of 3D orientation can additionally be expressed by $w_3 = [P_L(vert) - P_L(hor)] / [P_L(vert) + P_L(hor)]$. This allows witnessing the successive stages of damage evolution, whereby strongly axially oriented cracking was found in the early stage, followed by increasing contributions due to oblique cracking (along the en échelon arrays). Thereupon, the axial orientation was getting more dominant, confirming the global effect of damage mechanisms [1,3].

The fracture surface’s roughness can also be determined by means of stereological tools [5]. The planar roughness index (real surface area divided by projected value), R_S, is given by

$$R_S = 1 + \frac{1}{2}V_V \tag{3}$$

in which V_V is the volume fraction of aggregate incorporated in the model underlying eqn (3). Experimental determination of surface roughness is equally governed by resolution. This phenomenon is reflecting the *fractal character* of cracking. In the derivation of eqn (3) the afore-mentioned damage evolution mechanisms are adopted. Surface roughness is a parameter correlated with fracture energy [5].

5 SIZE EFFECTS

The parameters characterizing particular stages in the damage evolution process reveal a different degree of sensitivity to structural details. Crack initiation strength (CIS) is structure-sensitive, and depends on the group patter of particles involved (size, shape and spacing). UCS is rather structure-insensitive and depends primarily on quantities (volume fraction of aggregate). Data processing of such parameters for a sufficiently large sample yields average values and standard deviations. However, the shape of the probability density curve is symmetric (Gaussian) for a structure-insensitive property, but asymmetric (skew) in the case of structure-sensitivity. The width of the curve does reflect the stochastic heterogeneity of the sample, so is *not a material property*. Globalisation of data obtained on sub-RVE/RAE samples will yield biases in engineering characteristics when dealing with finite structure-sensitivity. Assessment of engineering properties on the basis of a sub-RVE sampling strategy requires therefore the availability of proper theoretical models, such as that of the weakest link on which Weibull's theory is based. With a lacking theoretical basis, correlations should be established experimentally between sub-RVE/RAE sampling results and those obtained on relevant RVEs/RAEs, conditions being similar, of course. And this has to be accomplished, basically, for each independent material property, and for different environmental conditions. The testing discrepancy between average engineering properties based on sub-RVE and on RVE samples is generally attributed as *size effect*. From the foregoing it must be obvious that size effects are different for different sample sizes, but also for different properties, because of inherent differences in structure-sensitivity. Although a recognized phenomenon in engineering testing, it is an inevitable source of discord and misinterpretation because of ignoring the fundamental background. Interpretation should not be in terms of the size of sub-RVE specimens with supposedly similar uniform (thus, homogeneous) structure, but based on the different heterogeneity levels involved as a logic consequence of investigating the materials on specific aggregation levels of particulate matter [6]. This type of approach would unify the engineering and materials science approaches. The relevance of the heterogeneity concept for investigating properties in direct compression is the impact this has on *sampling strategy* and on *data evaluation*, both pursuing the generation of unbiased engineering information.

The interest in the stochastic background of weakest link problems goes back about one century, and major statistical contributions to estimating fracture properties, recognizing their structure-sensitive background, appeared during the 1920th to 1940th. However, the underlying heterogeneity concept is still mostly ignored, and the implications missed. The situation is most serious in material science approaches in concrete technology, for which the heterogeneity concept equally holds, of course. It is commonly assumed in micro-mechanical models of concrete that flaws in the material body are at the basis of fracture properties. CIS in DC is selected as a property sensitive to material structure. Weibull has elaborated a theoretical concept for brittle fracture based on the weakest link concept for elementary units containing single cracks.

The probability density function of CIS of sub-volumes of the material body containing a single crack (elementary units) is denoted by $f(s)$, and the cumulative frequency curve by [6]

$$F(s) = \int_{-\infty}^{s} f(t)dt \tag{4}$$

When the material body encompasses a total number of i elementary units, the probability density function $g_i(s)$ of the *minimum value* of the brittle strength can be expressed in the form of $g_i(s) = i.f(s)[1-F(s)]^{i-1}$, with $G_i(s) = \int_{-\infty}^{s} g_i(t)dt = 1-[1-F(s)]^i$ as the cumulative distribution function. Determination of the median value of brittle strength, $\tilde{s}$, is straightforward. The mode $\hat{s}$ is obtained upon differentiation and equating to zero, resulting in

$$[1-F(s)]\frac{df(s)}{ds} - (i-1)f^2(s) = 0 \quad \text{with} \quad s = \hat{s} \tag{5}$$

This equation cannot be solved in an elementary way for *normally distributed brittle strength values* of the elementary units. A new variable z is therefore defined by $z = i.F(s)$. Substitution in eqn (5) and differentiation leads in the limiting case of $i \to \infty$ to the cumulative and probability density functions of z

$$G_{i\to\infty}(z) = 1 - e^{-z} \quad \text{and} \quad g_{i\to\infty}(z) = e^{-z} \tag{6}$$

An asymptotic development allows expressing s into z

$$s = m - s\left[\sqrt{2\ln i} - \frac{\ln\ln i + \ln 4p}{2\sqrt{2\ln i}} - \frac{\ln z}{\sqrt{2\ln i}}\right] \tag{7}$$

in which m and s are the mean and standard deviation of the brittle strength of the elementary units, governed by the normal distribution function $f(s)$.

The most probable value (mode) and variance of s are given respectively by

$$E(s) = m - s\left[\sqrt{2\ln i} - \frac{\ln\ln i + \ln 4p}{2\sqrt{2\ln i}}\right] \quad \text{and} \quad D^2(s) = \frac{p^2 s^2}{12\ln i} \tag{8}$$

In eqn (8) use was made of $[E(\ln z)] = 0$ and $E(\ln z)^2 = p^2/6$.

Herewith, the mean and standard deviation of the material containing a very large number of cracks can be formulated by $\hat{s} = m - a_i s$ and $s_i = b_i s$ in which the coefficients are

$$a_i = \sqrt{2\ln i} - \frac{\ln\ln i + \ln 4p}{2\sqrt{2\ln i}} \quad \text{and} \quad b_i = \frac{p}{2\sqrt{3\ln i}} \tag{9}$$

Tables are available in the literature for a_i and b_i. The decline rate in scatter with increasing sample volume is exceeding the one in strength. So, also the coefficient of variation is declining with increasing sample volume. In other words, sampling sub-volumes of the RVE (defined at an acceptable level of scatter) will lead to improved strength values accompanied by disproportionately increased scatter (or, *heterogeneity*).

The strength ratio of an arbitrary material volume and the RVE is obtained by eqn (9)

$$\frac{\bar{s}}{\bar{s}_{RVE}} = \frac{m - a_i s}{m - a_{RVE} s} \approx 1 - \frac{s}{m}\left(a_i - a_{RVE}\right) \quad \text{with} \quad a_i - a_{RVE} = \left(\frac{b_{RVE}}{b_i} - 1\right)a_{RVE} \tag{10,11}$$

Hence, the strength increase due to sub-RVE sampling strategy is reflected by

$$\frac{\bar{s}}{s_{RVE}} \approx 1 + \frac{s}{m}\left(1 - \frac{b_{RVE}}{b_i}\right)a_{RVE} \tag{12}$$

Eqn (12) depicts the *strength increase by heterogeneity* on sub-RVE sampling level. The value of b_{RVE} indicates the acceptable scatter limit ($= s_i/s$) for declaring the sample volume representative for brittle strength, and thereby defining it as homogeneous. Heterogeneity is expressed by the ratio $b_i / b_{RVE} = s_i / s_{RVE}$. The microscopic material parameter $(s/m)^2 = v$ has been referred to in the international literature as the unit coefficient of variation. When b_{RVE} is selected, a_{RVE} is given by the relationship $a_{RVE} = p/(\sqrt{6}\,b_{RVE})$.

Upon combination of eqns (9) and (12), the ratio of linear dimension of sample and the RVE (proportional to $\sqrt[3]{i/i_{RVE}}$) would be introduced as the running parameter. Hence

$$\frac{\bar{s}}{s_{RVE}} \approx 1 + \frac{s}{m}a_{RVE}\left(1 - \frac{\sqrt{\ln i}}{\sqrt{\ln i_{RVE}}}\right) \tag{13}$$

This defines the so-called *size effect* [6], in the present case supposedly for CIS. The unit variation coefficient in eqns (11) and (12) is available for adjusting to experimental data. Epstein

[7] presents a survey of this type of approaches starting from different assumptions as to $f(s)$ including the so-called Weibull distribution. Generalizing, the author concludes “that the most probable value of the smallest item in samples of size i must decrease as $(\log i)^{1/p}$ with p a positive constant” (p=2 in the elaborated concept). Identical conclusions can be drawn for this example as formulated for the spatial homogeneity case.

6 SUMMARY AND CONCLUSIONS

Early stages in damage evolution of concrete in direct compression reveal crack initiation and propagation in particle matrix interfaces as a result of tensile stresses. Slip as a result of shear stresses takes place as a secondary phenomenon along pre-weakened en échelon zones along cone-like mortar elements at azimuth sides of grains. This forms the introduction to near neighbour crack coalescence predominantly in the loading direction, which characterizes the engineering property of crack initiation strength or LOP. Crack density at UCS is of the same order as obtained by crack coalescence of all near neighbouring cracks. This process continues in the post-ultimate range. Series of fracture process zones are gradually formed in this way, leading to a column-like structure, with cross-sectional dimensions of the order of maximum grain size. The roughness of the fracture surface is proportional to volume fraction of aggregate, and not to details of the sieve curve. Changing sensitivity or resolution in experimental or modelling approaches changes the roughness of the fracture surface as a reflection of its fractal nature.

Heterogeneity is not a material characteristic. Instead, it is a stochastic concept governing the design of experiments. LOP is an example of structure-sensitive properties, whereas UCS is a relatively structure-insensitive property. Scatter - as a reflection of heterogeneity - among observations on similar parameters of samples of equal size is larger the more extreme structure-sensitivity of the property at issue. This points to significantly larger dimensions of the RVE/RAE in the latter case. Commonly, sub-RVE/RAE sampling is pursued, however. In the case of CIS the probability density function as a weakest link phenomenon will be skew, and sample averages will be biased. This is the so-called size effect, which will be different for sub-RVE/RAE samples of different size and for properties with different degrees of structure-sensitivity.

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