MULTI-LEVEL LINEAR DIMENSION IN FRACTURE. A FRACTAL COHESIVE CRACK REPRESENTATION

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Abstract

Several measures of the characteristic material length parameter have been suggested for solids weakened by presence of a fractal crack. The characteristic length parameter is discussed at various scale levels, beginning with macro-level and ending with a nano-level. Physical interpretation and the numerical estimates of these parameters are provided at all levels.

The fractal dimension of the crack D is allowed to vary from 1 (smooth crack limit) to 2, when the crack becomes a plane filling entity. Significant variations from the classic solution are demonstrated as the singularity exponent α, entering in the near-tip stress field, $r^{-\alpha}$, sweeps the range $(0, \frac{1}{2})$. The well-known concepts of the stress intensity factor and the Barenblatt cohesive modulus, which is a measure of material toughness, have been re-defined to accommodate the fractal view of fracture. Specifically, the cohesion modulus, in addition to its dependence on the distribution of the cohesion forces, is shown now to be a function of the "degree of fractality", reflected by the fractal dimension D, or by the fractal roughness parameter, H. For most fractal cracks, when D is not too close to 2, the characteristic length is chosen as the length of the cohesive zone, R. Above a certain threshold value of D, the root radius of the equivalent blunted crack ,ρ, is suggested as the primary characteristic length parameter. The equivalent blunted crack is selected by use of the Neuber stress magnification concept and the classic fracture mechanics equations for a crack with a finite root radius. The sequence of the meso-scale and the nano-scale material length parameters is then defined. It turns out that at each scale level there exists a different length constant, which determines the neighborhood "landscape", beginning with a critical crack opening displacement, quasi-static crack growth step, Wnuk's final stretch and ending with a lattice constant or an inter-atomic distance.

The fractal cohesive crack model used here is based on a simplifying assumption, according to which the original problem is approximated by considerations of a smooth crack embedded in the stress field generated by a fractal crack. As the degree of fractality increases, the characteristic material length constants are shown to rapidly grow to the levels around three orders of magnitude higher than those predicted for the classic case. Such phenomenon may be helpful in explaining an unusual size-sensitivity of fracture processes in materials with cementitious bonding such as concrete and certain types of ceramics, where fractal cracks are commonly observed. It is an experimentally confirmed fact that in these materials the size of the process zone, which frequently is identified with the primary characteristic length, and the size of the end zone as modeled by the cohesive crack representation, frequently approach the size of the entire specimen used in a typical laboratory test.

1. MATHEMATICAL PRELIMINARIES

In order to investigate the size effects associated with imperfect materials weakened by voids and crack-like defects, we need to establish a characteristic length defined by the material microstructure and geometry of the defect. A certain length parameter, R, is embedded in all cohesive crack models. Equilibrium between the restraining tractions S(X) =S_0G(X) applied over the end zone itself, is determined by the finiteness condition

$$K_{tot}(\sigma, S) = K_I(\sigma) + K_{coh}(S) = 0 \tag{1.1}$$

With the pressure σ applied to the crack surface, $|X| \le a$, and the pressure p = σ – S(X) applied over the end zones, $a \le |X| \le a + R$, equation (1.1) reads

$$\begin{aligned} &2\sqrt{\frac{a+R}{\pi}}\left\{\int_0^a \frac{\sigma\, dX}{\sqrt{a^2 - X^2}} + \int_a^{a+R} \frac{(\sigma - S(X))dX}{\sqrt{a^2 - X^2}}\right\} \\ &\qquad = 2\sqrt{\frac{a+R}{\pi}}\left\{\frac{\pi}{2}\sigma + \int_a^{a+R} \frac{-S(X)\, dX}{\sqrt{a^2 - X^2}}\right\} = 0 \end{aligned} \tag{1.2}$$

When the nondimensional loading parameter $\pi\sigma/2S_0$ is denoted by Q, the condition (1.2) can be re-written as

$$Q = \int_a^{a+R} \frac{\widetilde{G}(X)\, dX}{\sqrt{a^2 - X^2}} \tag{1.3}$$

Here, the cohesive stresses are given by the Wnuk-Legat distribution law, cf. [2],

$$\widetilde{G}(X) = \left(\frac{X - a}{R}\right)^n \exp\left[\omega\left(\frac{a - X}{R}\right)\right] \tag{1.4}$$

The material parameters n and ω are subject to experimental determination at the mesomechanical level. If the pertinent variables are nondimensionalized as follows,

$$\frac{a}{a+R} = m,\ \lambda = \frac{x}{R} = \frac{X - a}{R} = \frac{x - m}{1 - m},\ x = \frac{X}{a + R} \tag{1.5}$$

Equation (1.5) reads

$$Q = \int_0^1 \frac{G(\lambda)(1 - m)\, d\lambda}{\sqrt{1 - [(1 - m)\lambda + m]^2}} \tag{1.6}$$

In what follows we shall restrict the considerations to the case of R<<a, or m$\rightarrow$1. This restriction is pertinent to the "small scale yielding" (ssy) condition frequently met in analyses of fracture in quasi-brittle solids. For this limiting case, when we consider (1-m) as a small quantity, the integral in Equation (1.6) can be simplified as follows

$$Q \cong \sqrt{\frac{1-m}{2}} \int_0^1 \frac{G(\lambda)}{\sqrt{1-\lambda}} d\lambda \tag{1.7}$$

Replacing (1-m) by R/a, we obtain

$$Q \cong \sqrt{\frac{R}{2a}} \int_0^1 \frac{G(\lambda)}{\sqrt{1-\lambda}} d\lambda \tag{1.8}$$

in which G(λ) results from G(X), when the appropriate transformation of variables, cf. Eq. (1.5), is completed. It reads

$$G(\lambda, n, \omega) = \lambda^n \exp\left[\alpha(1-\lambda)\right] \tag{1.9}$$

Multiplication of Q in (1.8) by the factor $2S_0\sqrt{a/\pi}$, where S_0 denotes the local value of the yield stress measured at the crack front, converts this expression to the so-called "cohesion modulus" (used in somewhat different form by Barenblatt [1]), namely

$$K_{coh} = \sqrt{\frac{2R}{\pi}} S_0 \int_0^1 \frac{G(\lambda, n, \omega)}{\sqrt{1-\lambda}} \, d\lambda \tag{1.10}$$

With the integral contained in (1.10) denoted by W_0(n,ω), it is customary to solve Equation (1.10) for the characteristic length

$$R = \frac{\pi}{2W_0^2}\left(\frac{K_{coh}}{S_0}\right)^2 \tag{1.11}$$

This value is considered to represent the characteristic length parameter reflecting the microstructural and mesomechanical properties of the material. The equations provided in this section are valid for a smooth crack.

2. A FRACTAL MODEL OF THE COHESIVE CRACK AND THE CHARACTERISTIC LENGTH

Recently proposed fractal model of Wnuk and Yavari [3] suggests a mathematical simplification based on associating a smooth crack to a stress field generated around a fractal crack. This is the so-called method of imaginary smooth crack. The approach provides a useful approximation to the problem at hand, including the cohesive aspects of a fractal crack. In the solutions now obtained a new variable enters: the fractal dimension of the crack. The fractal dimension, D, usually a non-integer, is a measure of how strongly a given entity diverges from its Euclidean counterpart. As a geometrical characteristic of the fracture surface, D enters as a new variable in most pertinent equations of the fractal fracture mechanics. Let us mention just one such relation – for a fractal version of the Griffith crack the familiar singularity of $r^{-1/2}$ is replaced by a somewhat weaker singularity for the near-tip stress, $r^{-\alpha}$, where for a self-similar crack α depends on the fractal dimension D, namely (see Yavari, et al. [4] and Yavari [5])

$$\alpha = \frac{2-D}{2}, \qquad 1 \le D \le 2 \tag{2.1}$$

The model allows one to generalize the formula for the cohesive modulus of a smooth crack (1.10), to the one representing a fractal crack, namely

$$K_{coh}^{f} = \frac{2}{\pi} S_0 \left(\frac{\pi R_f}{2} \right)^{\alpha} \int_0^1 \frac{G(\lambda, n, \omega)}{(1-\lambda)^{\alpha}} d\lambda \tag{2.2}$$

The inverse relation that relates the characteristic length R to the material properties such as S_0, n, ω, K^{f}_{coh}, and the fractal geometry represented by the order of singularity α, is given now as

$$R_f = \left(\frac{\pi}{2} \right)^{\frac{1-\alpha}{\alpha}} \left[K_{coh}^{f} / S_0 \right]^{\frac{1}{\alpha}} \left\{ \int_0^1 \frac{G(\lambda, n, \omega)}{(1-\lambda)^{\alpha}} \; d\lambda \right\}^{-\frac{1}{\alpha}} \tag{2.3}$$

Index and superscript "f" have been added to designate quantities pertinent to a fractal crack. It is readily seen that for the limiting case of α = ½, the formulae (1.10) and (1.11), valid for a smooth crack, are recovered. With the notation

$$W(\alpha, n, \omega) = \int_0^1 \frac{G(\lambda, n, \omega)}{(1-\lambda)^{\alpha}} d\lambda, \qquad R_c^{f} = \left(K_{coh}^{f} / S_0 \right)^{\frac{1}{\alpha}} \tag{2.4}$$

equation (2.3) can be re-written in a nondimensional form

$$\frac{R_f}{R_c^f} = \frac{(\pi/2)^{\frac{1-\alpha}{\alpha}}}{W^{\frac{1}{\alpha}}(\alpha, n, \omega)} \tag{2.5}$$

Similarly, for a smooth crack, see Eqs, (1.10) and (1.11), we have

$$\frac{R}{R_c} = \frac{\pi}{2W_0^2}\ , \quad R_c = (\pi/8)(K_{coh}/S_0)^2 \tag{2.6}$$

Dividing equations (2.5) and (2.6) side by side, one obtains a measure of the material characteristic length associated with a fractal crack

$$\Theta(\alpha) = \frac{R_f R_c}{R R_c^f} = \left(\frac{\pi}{2}\right)^{\frac{1-2\alpha}{\alpha}} \frac{W_0^2}{W^{\frac{1}{\alpha}}} \tag{2.7}$$

It is easy to verify that for the limiting case of α = ½, when W $\rightarrow$ W_0, the function Θ(α) reduces to one, as expected. Fig. 1a shows the dependence of Θ on the fractal singularity exponent α, plotted for the range $\frac{1}{4} \le \alpha \le \frac{1}{2}$, while Fig. 1b shows a similar function, for which the fractal dimension D was chosen as an independent variable rather than α. As can be seen, the range ($\frac{1}{4}$, $\frac{1}{2}$) for α corresponds to the range (1, $\frac{3}{2}$) for D.

There is a problem with physical interpretation of a fractal crack behavior when D approaches 2. In this limiting case the crack resembles a 2D object spread over a plane (a plane filling curve). For this case, the stress intensity factor, cf. Wnuk and Yavari [3]

$$K_I^f = \frac{\sigma\sqrt{\pi a^{2\alpha}}}{\pi^{2\alpha}} \int_0^1 \frac{(1+s)^{2\alpha} + (1-s)^{2\alpha}}{(1-s^2)^{\alpha}} ds = \chi(\alpha)\sigma\sqrt{\pi a^{2\alpha}} \tag{2.8}$$

attains the value $2\sqrt{\pi}\sigma$. Note that the crack length "a" is suppressed entirely, and the entity $2\sqrt{\pi}$ should thus be interpreted as Nueber's stress magnification factor (rather than a stress intensity factor) corresponding to a certain 2D void. Assuming the void to be in the shape of a blunted crack with a finite root radius ρ, and the crack front identified at r = ρ/2, cf. Wnuk and Kriz [6], the stress at the crack front may be evaluated as follows

$$\sigma_{\max} = \frac{2K_I}{(\pi\rho)^{\frac{1}{2}}} = \frac{2\sigma\sqrt{\pi a}}{(\pi\rho)^{\frac{1}{2}}} \tag{2.9}$$

Setting it equal to the value predicted by the fractal model

$$\sigma^{f}_{\max} = \left[\frac{\chi(\alpha)\sigma\sqrt{\pi\, a^{2\alpha}}}{(2\pi\, r)^{\alpha}} \right]_{r=\rho/2} = \frac{\chi(\alpha)\sigma\sqrt{\pi}\, a^{\alpha}}{(\pi\,\rho)^{\alpha}} \tag{2.10}$$

leads to an expression defining the root radius of the fractal crack when $D\to 2$, or $\alpha\to 0$, namely

$$\frac{\chi(\alpha)\, a^{\alpha-\frac{1}{2}}}{(\pi\,\rho)^{\alpha}} = \frac{2}{(\pi\,\rho)^{\frac{1}{2}}} \tag{2.11}$$

or

$$\rho = \lim_{\alpha\to 0} \frac{a}{\pi} \left[\frac{\chi(\alpha)}{2} \right]^{\frac{2}{2\alpha-1}} \tag{2.12}$$

3. NUMERICAL ASSESSMENT OF THE LENGTH CONSTANTS GOVERNING FRACTURE PROCESS AT MULTI-SCALE LEVELS

If all the various fracture mechanics parameters, used to measure an enhancement of fracture toughness during the early stages of fracture, such as δ_5 or J_R, are denoted by a common symbol R, then the rate of toughness increase associated with growth of the subcritical crack, can be predicted as follows

$$\frac{dR}{da} = M - \frac{1}{2} - \frac{1}{2}\log(\frac{4R}{\Delta}) \tag{3.1}$$

This equation was first proposed by Wnuk (1972, 1974) on the basis of his theory of quasistatic crack and assuming a structured nature of the end-zone adjacent to the crack front, and several years later it was derived independently by Rice and Sorensen (1978) and Rice et al. (1980) from considerations of the Prandtl slip-line field in the near-tip region. Eq. (1.1) defines material resistance R-curve for the small scale yielding range. However, studies have shown the equation remains valid and produces correct results for loads σ raised to 70% of the yield stress σ_Y, cf. Wnuk (1990, 2001). Symbol M in (3.1) denotes the tearing modulus, while Δ is the characteristic microstructural length parameter identified with the size of the process zone, i.e., the zone of intensive necking occurring just prior to the final act of fracture.

For the range of crack tip plasticity considered here the resistance parameter R and the J_R variable are directly related, namely $R = (\pi E/8\sigma_Y^2)J_R$, while the nondimensional tearing modulus M is related to Paris' tearing modulus $T_J=(E/\sigma_Y^2)(dJ_R/da)_{ini}$ and to Shih's crack tip opening angle CTOA=δ/Δ, with δ denoting Wnuk's constant of final stretch, cf. Wnuk (1974,1979), in the following way: $M = (\pi/8)T_J$ and $M = (\pi E/8\sigma_Y)$CTOA. Here, E denotes the Young modulus and σ_Y is the effective yield stress at the crack front, while σ_0 is the uni-axial yield stress. For a pressure-vessel steel such as A 533 B, the approximate values of the pertinent material constants are, $\sigma_0/E = 3*10^{-3}$, $T_J = 50$, CTOA = 0.15, CTOD = 0.1 mm, and the characteristic material length parameter is defined as $R_c = (\pi/8)K_{Ic}^2/\sigma_Y^2$.

The length Δ represents the constant crack growth step, which can be estimated as follows

- for brittle materials

$$\Delta = \frac{(2\gamma)E}{\sigma_{mol}^2} \tag{3.2}$$

- for quasi-brittle or ductile materials

$$\Delta = \frac{EG_f^{true}}{S_{\max}^2} \tag{3.3}$$

Here, the symbol γ is used to denote surface tension, σ_{mol} is the molecular strength ($\approx E/30$), G_f (equivalent to 2γ for the brittle fracture case) is the true work of fracture, while S_{max} is the maximum stress in the Wnuk-Legat cohesive-stress distribution law proposed for a quasistatic crack, cf. Wnuk and Legat (2002).

To get a numerical estimate of these length parameters, we replace γ by $\sigma_{mol}b$ and then substitute E/30 for σ_{mol}, in where b denotes the average interatomic distance (b =0.2 nm), arriving at

$$\Delta = (E/\sigma_{mol})b = 30\ b = 6*10^{-9}\ \text{m} = 6\ \text{nm} \tag{3.4}$$

Let us compare this estimate with the one obtained for quasi-brittle fracture, as would be expected in a pressure vessel steel, such as A 533 B. Using the data provided by Smith (1981), we have δ_{Ic} as .1 mm = 10^{-4} m. With G_f^{true} estimated as 9/100 of the total energy dissipated within the end zone, $G_f^{true}=.09\Phi_Y*_{Ic}$, the equation (3.3) can be re-written as follows

$$\Delta = (1/100)(E/\Phi_Y)\ \delta_{Ic} = .01*111*10^{-4}\ \text{m} = 111\ \mu\text{m} = 555{,}000\ \text{b} \tag{3.5}$$

Note that we assumed that $S_{max} = 3\sigma_Y$ while $\sigma_Y/E = 3\sigma_0/E = 0.9*10^{-2}$ yielding the value of the ratio E/Φ_Y as 111, and the length Δ in (3.5) was thus estimated as $1.11*10^{-4}$ m, which is equivalent to 555,000 b.

Now, let us proceed from the other end of the scale, beginning with the macroscopic range of length parameters characteristic of fracture in a pressure vessel steel. Let us start with an evaluation of the characteristic length $R_c = (\pi/8)K_{Ic}^2/\sigma_Y^2$, which when $\pi/8$ is approximated by 1/3 reads as follows

$$R_c = (1/3)\ EG_f/\sigma_Y^2 = (1/3)(E/\sigma_Y)\delta_{Ic} = (1/3)(100)(10^{-4}\ \text{m}) = 3*10^{-3}\ \text{m} = 3\ \text{mm} \tag{3.6}$$

The crack tip opening displacement δ_{Ic} is evaluated as

$$\delta_{Ic} = (8/\pi)(\sigma_Y/E)R_c = (3)(.9*10^{-2})(3*10^{-3}) = 0.9*10^{-4} \sim 10^{-4}\ \text{m} = 0.1\ \text{mm} \tag{3.7}$$

This agrees with the value assumed previously for the CTOD. Now, the crack growth step Δ and the final stretch δ, both pertinent to the slow stable crack growth, can be estimated as follows:

$$\delta = (\sigma_Y/E)^2 T_J R_c = (0.9*10^{-2})^2(50)(3*10^{-3}) = 121*10^{-7} = 12\mu m = 60{,}750b \qquad (3.8)$$

$$\Delta = \delta/CTOA = 121*10^{-7}\ m/0.15 = 81*10^{-6}\ m = 81\ \mu m = 405{,}000\ b \qquad (3.9)$$

The latter value of the estimated crack growth step is somewhat less than the one given in (3.5), and based on the data pertaining to the specific fracture energy G_f, but the order of magnitude of both deltas is the same. The material constants such as Paris tearing modulus $T_J = 50$, and the crack tip opening angle CTOA = 0.15, were taken from Smith, (1981). It is noteworthy that the crack growth step Δ equals about 4/5 of the CTOD and about 1/30 of the R_c, while the final stretch δ is about one-tenth of the CTOD and 1/250 of the R_c. We note that for a fractal crack all these numbers increase up to two orders of magnitude as the fractal dimension D approaches 2, and crack degenerates to a two-dimensional void.

To put these numbers into the context of nano-mechanics and quantum physics, we look at the two characteristic length parameters governing physical events at the nano-scale, namely the Compton wavelength λ = (1/500)nm or λ = (1/100)b, in where the other constant b = 0.2 nm = $2*10^{-10}$ m is the interatomic distance. We have represented all the length constants pertinent to non-elastic fracture process in terms the constant b. A consistent atomistic model of fracture would certainly incorporate all these constants in a natural way.

4. CONCLUSIONS

It has been demonstrated that a solid weakened by a fractal crack possesses a characteristic length, which is determined by the mesomechanical properties of the material, such as S_0, n, ω, K_{coh}^f and also by the fractal dimension of the crack, D. Two measures of this length have been suggested here for two intervals of the fractal dimension D. For the fractal dimension ranging from 1 (a smooth crack limit) to 1.846, the characteristic length R_c^f is used, and it varies between 1 for D = 1, and 201.8 for D = 1.846. It should be noted that the value of 201.8 is three to four orders of magnitude greater than the values of the characteristic length observed in ductile metals. This result is of significance when the size effects related to fracture in cementitious materials is interpreted. The value of D = 1.846 is used as a cut-off value for a fractal dimension. Beyond this limit a root radius of a hypothetical blunted crack (equivalent to a fractal when D approaches 2) is suggested as a measure of the characteristic material length. In the limit of D = 2, this root radius equals a/π, where "a" denotes the nominal crack length.

In Section 3 we provided an assessment of the numerical values of the length parameters pertinent to a process of non-elastic fracture. The constants were compared to the constants governing the physical events at the nano-scale, as is common in quantum physics. An example of such work is provided by the research of Ortiz et al., cf. Ortiz and

Phillips (1999), Ortiz, Cuitino et al. (2001), Nguyen, Repetto and Ortiz (2001) and Nguyen and Ortiz (2002).

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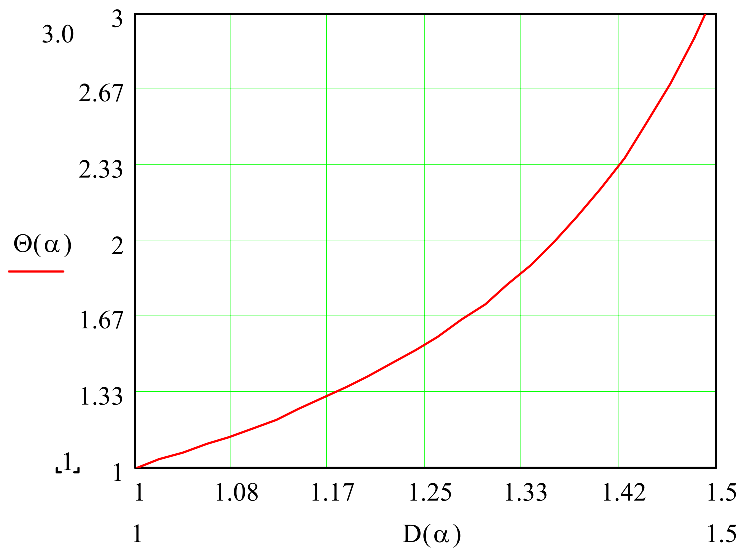

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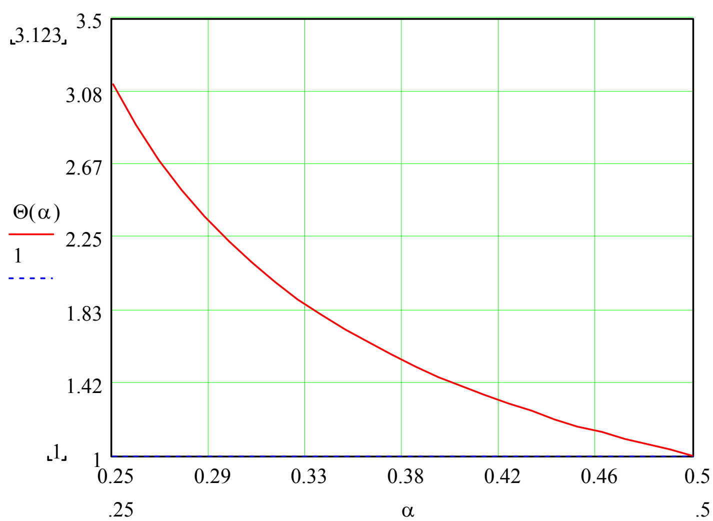
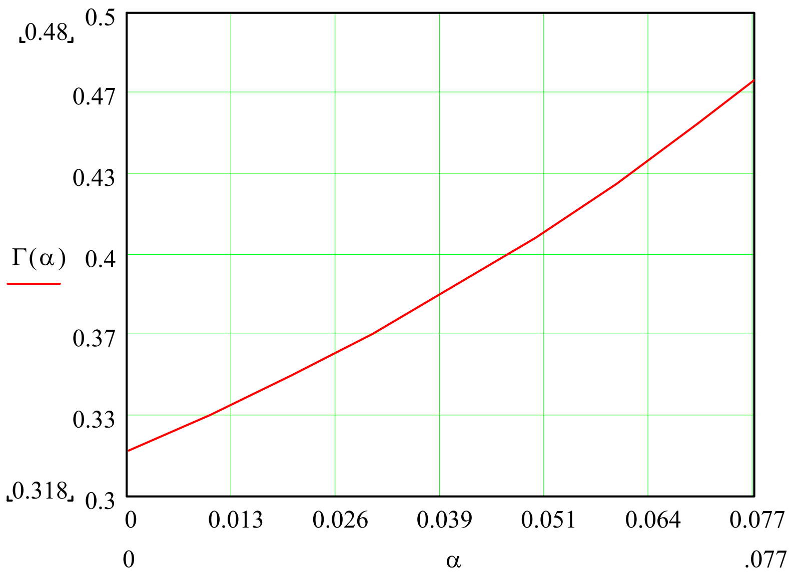


(b)

Fig. 1 (a) Nonondimenisonal measure of the characteristic length for a material weakened by a fractal crack shown as a function of the singularity exponent, α, (b) Function Θ shown with the fractal dimension D featured as the independent variable.



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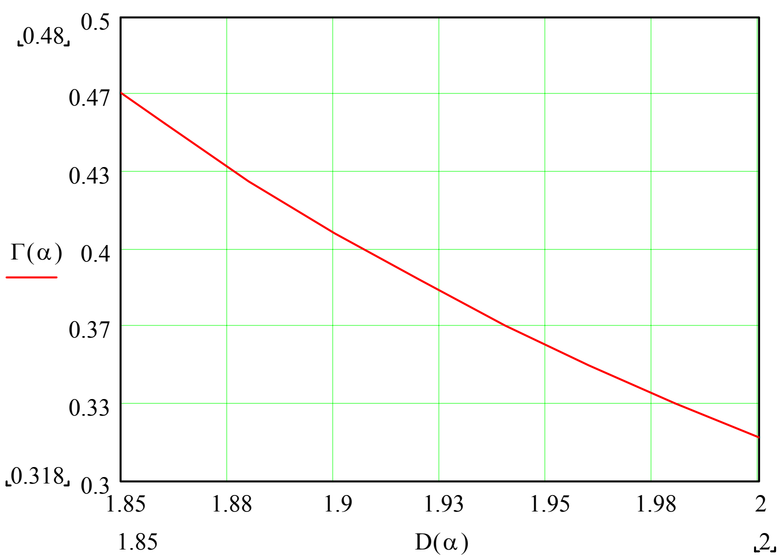
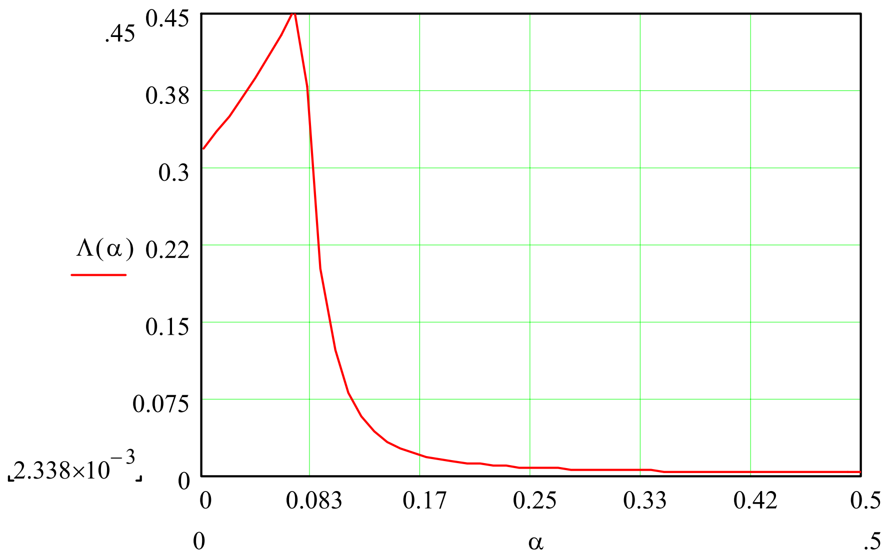


(b)

Fig. 2 (a) Function Γ shown against the singularity exponent of the fractal crack, (b) variations of the same function as in Fig.1a, but shown against the fractal dimension of the crack, D.



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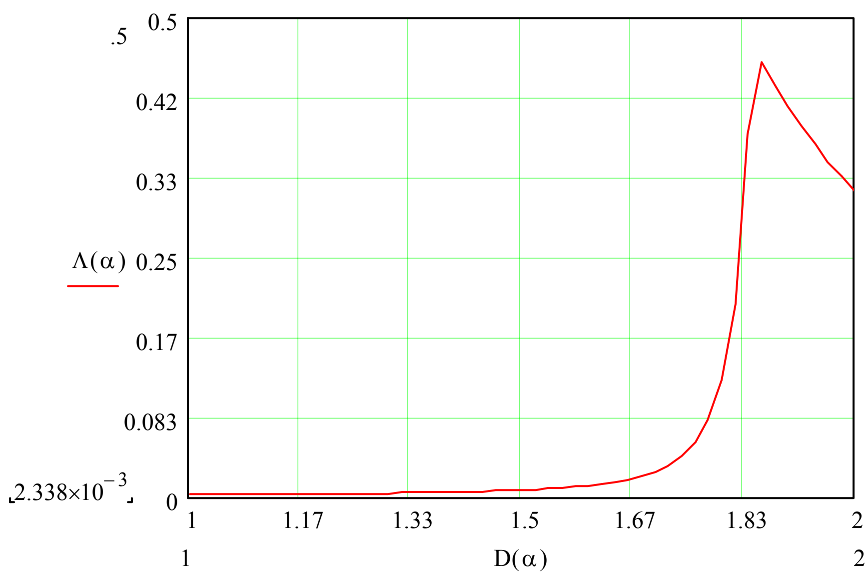

(b)

Fig. 3 (a) dependence of the function Λ on the singularity exponent α,
(b) dependence of Λ on the fractal dimension of the crack, D.