MICRO-MACRO SCALING IN DISORDERED MATERIALS

J.G.M. VAN MIER
Institute for Building Materials, ETH, 8093 Zürich, Switzerland

ABSTRACT

Fracture of concrete, rock and other disordered materials is a complex process involving four stages, namely, an elastic stage (O), a (stable) microcrack stage (A), a (un-stable) macrocrack stage (B) and a bridging regime (C). Stages (O) and (A) define the pre-peak behaviour of the stress-deformation diagram in tension, whereas (B) and (C) are mechanisms active in the post-peak regime. Upon scaling different things may be changed. First of all, the material structure can stay constant while varying the structure size or one can envision a situation where the ratio of structure to material size is kept constant and the absolute sizes are scaled. The question is what would change in particular in the microcrack processes that determine the strength of the material. After a review of the fracture process according to stages (O) and (A), the effect of pre-critical micro cracking on macroscopic strength is considered. Knowledge on scaling of these microcrack processes as a function of material micro-structure is important for understanding what fracture strength actually means at all size-scales. Fracture strength appears to be a system property, which is not recognized in current scaling laws that are based on phenomenological considerations of the energy balance in structures only.

1 INTRODUCTION

Scaling of fracture properties in disordered materials like concrete, rock and ice has focused mainly on scaling of strength, [1]-[2], although in a few studies the scaling of fracture energy was considered as well [3]-[4]. In this paper the discussion is limited to scaling of strength in brittle disordered geo-materials. In Van Mier [5] it was shown that the tensile fracture process of many materials, ranging from glass, cement and concrete, to rock, and even metals, can conveniently be described by means of a 4-stage fracture model, which can be interpreted as a bridged crack model, where the bridging stress equals the stress carried by the tail of the softening diagram. This is a deviation from conventional cohesive softening models where the complete post-peak curve is considered as bridging stress. The macrocrack growth, which is part of the fracture process, is covered by the singular solution from classical fracture mechanics, whereas in most cohesive fracture models not the stress singularity, or stress intensity is used as a propagation criterion, but rather the tensile strength.

An essential feature of fracture processes in seemingly all types of materials is that macroscopic fracture is preceded by a process of stable microcrack growth. For glass, Célarié et al [6] made it plausible that nano-scale voids developed in front of the visible macrocrack. This means that even for materials like glass nano-scale fluctuations in material structural properties may lead to (stable) microcracking. This was hypothesized in the author's inaugural lecture [7], and it is interesting to see that such a process indeed takes place. Under the assumption that the bridged crack model is correct, and accurately describes the fracture process as presented in [5], the pre-critical microcracking is the most important stage since here the conditions are set for macroscopic failure to occur. The pre-critical cracks may be quite large compared to the size of the structure in which they grow, and their salient property is that they are stable, meaning they can be arrested by the meso- (or micro-, or nano-) structure of the material.

Pre-critical cracking is not just dependent on the material structure, but the global stress distribution has some effect as well. Under uniaxial tension the stress-state seems best defined, but from experiments it is quite well known that in tests on concrete and rock non-uniform stress- and deformation distributions occur, also before the maximum stress is reached, [8]. The situation may be better or worse depending on the rotational freedom of the loading platens. Irrespective of this, one can choose to apply the tensile load under some eccentricity, [9], and quite obviously the microcrack process will be affected accordingly. Thus, under practically uniform external stress (or deformation) the microcrack process is affected by the material only, whereas under stress-gradients, in addition to the material structure induced microcracks a new propagation and arrest mechanism is provided in the form of the gradients. Macroscopic strength, as well as scaling of strength, are thus defined by the pre-critical crack process, which needs to be studied in detail.

In this contribution several material-structure induced re-distribution mechanisms, leading to diffuse or localized cracking will be discussed, as well as the implications for scaling. Once the material structure dependant microcrack process are understood in detail, scaling laws for global strength will automatically evolve, and there is no need for curve fitting, which is the usual procedure to date, [1], [2]. Also the hitherto claimed universality of these fitted size effect curves can be checked, as can the validity of currently concave and convex extrapolations for very small and very large structures that are extremely difficult to test in a laboratory.

The above situations relate to scaling of the structural size while keeping the material structure constant, thus, using one type of rock or concrete for all structural sizes, see for example [9]. An interesting thought experiment is what would happen if the ratio of structure size to material characteristic size (for example the maximum aggregate grain) would be kept constant and everything is scaled down. The implications of this will be speculated in Section 4, but no answer will be provided. In concrete one could attempt to check these effects to some extent as particle sizes and specimen sizes can be scaled rather freely, although some limiting factors exist.

2 FOUR-STAGE FRACTURE MODEL

The four-stage fracture process alluded to in the introduction was proposed recently, [5]. The four stages refer to the fracture process under uniaxial tension, and are distinguished as follows: (O) elastic stage, (A) microcrack stage, (B) macrocrack growth, and (C) bridging. The stages are of course less distinct as indicated here, and a gradual changeover can be expected. The linear elastic stage is rather trivial, and is defined by the Young's modulus of the material. For a three-phase material like concrete the Young's modulus can be calculated from the rule of mixtures given that the moduli of the constituting phases as well as their relative volume concentrations are known. For a material like sandstone, the global Young's modulus depends on the particle stiffness and the modulus of the contact layers between the sand grains. The contact layers can be quite complicated, like the clay in Felser sandstone, which results in moisture dependant behaviour.

The next stage, (A) is probably most troublesome. Upon loading, microcracks will start to emerge inside the heterogeneous material structure at places where the ratio between local strength and local stress is most critical. In concrete, this is at the interface between aggregate and matrix, where, in case of a concrete with normal river gravel a devastating low tensile strength appears (in the order of 1 MPa). The interface strength is usually much smaller than the matrix strength (which derives more directly from the tensile strength of Portland cement (about 5MPa)) and the aggregate strength (about 10-15 MPa for good quality gravel). The appearance of pre-critical cracking in stage (A) can be simulated quite easily using lattice type models as mentioned briefly in Section 3. To detect the pre-critical cracks is tremendously difficult since either specimen manipulation (sawing, grinding, polishing, impregnating and drying) may cause additional microcracks, or in case of acoustic techniques (AE, ultrasonic wave propagation) the source of the defects is unknown. The salient property of the pre-critical cracks is that they can be arrested by the microstructure of the material itself. This means there is no size limitation, and a pre-critical crack may just as well traverse large part of a structures or specimens cross-section, provided the cracks are prevented to open un-stably, an no serious load-drop occurs. This can, for example, be achieved in fibre reinforced composites, like ECC [10] or Hybrid Fibre Concrete [11].

The peak of the stress-deformation diagram is like a phase transition where the initially distributed microcracks form an un-stable macro-crack that starts to grow through the specimen's cross-section. A macrocrack is a discrete displacement jump, and not a zone of distributed microcracking as erroneously assumed in models like the Fictitious Crack Model [12] and the Crack band Model [13], which have been frequented for analyzing crack propagation in (reinforced) concrete structures. The macrocrack in stage (B) is discontinuous in nature although locally (3-dimensional) crack overlaps occur that gradually fail in the long tail of the softening diagram. The bridging stress transmitted by the bridged macrocrack is not the stress profile generated by the complete post-peak softening curve, but rather the tail area only. This is stage (C), the bridging stage. Figure 1 shows the four-stage fracture model schematically. Ample experimental proof and numerical results support the suggested approach, [5].

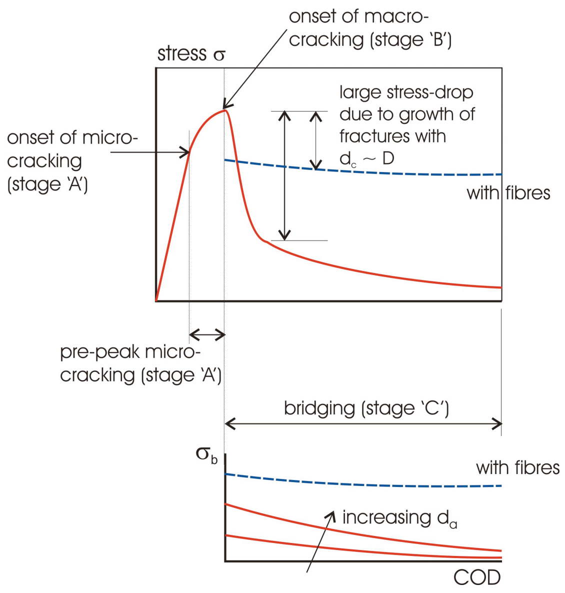

Figure 1. Four-stage fracture model for concrete and other brittle disordered materials, after [5].

The principal stages in the model of Figure 1 can be recognized in the mechanical behaviour of various types of materials. The difference is in the relative importance of each of the four phases, or actually three phases, (A), (B) and (C). For example in glass, stage (A) is limited to the formation of a few nano-sized voids, that would appear as in-significant if the specimen size becomes large in relation to the void size, say a factor 100. In sandstone, pre-critical cracking is more significant, and would lead to a larger (A) regime.

The other regime affected by material structure is the bridging stage (C). The size of crack face bridges in concrete is dependent on the aggregate size in the material, and the bridging stress depends directly on this size, [14]. When the localized crack has formed, bridging can be increased by adding fibres to concrete as shown schematically in Figure 1. In ECC [10], bridging is caused by polymer fibres, which require rather weak matrix strength in order to balance the stress-drop caused by macrocrack propagation through the matrix with the maximum fibre bridging stress provided by the total number of fibres bridging the macrocrack.

In summary, the four-stage crack model can be summarized by the following equations:

- for stage (O)-(A): $\sigma = f(\varepsilon)$
- for stage (B): $K_I = Y(a/W)\sqrt{\pi a} \geq K_{Ic}$
- for stage (C): $\sigma(\delta) = g(d_a, A_f, l_f, f_m, ...)$

where σ is the axial tensile stress, ε the axial strain in the pre-peak regime, K_I the stress intensity factor for the macrocrack growing through a microcracked material (due to the process in stage (A)), Y is the geometrical factor dependent on specimen size, geometry and relative crack length, W is the specimen width in the direction of crack growth, K_{Ic} is the critical stress intensity factor for the microcracked material (due to process (A)), and $\sigma(\delta)$ is the bridging stress at crack opening δ, which is caused by aggregate and/or fibre bridging. The parameters in the bridging stress equation refer to the aggregate size d_a, the fibre length l_f and fibre area A_f and the matrix strength f_m in which fibres and aggregates are embedded. There are other parameters of importance as well, which relate to the interface zone, the relative aggregate and fibre volumes, etc. The difficulty lies in describing the transition from stage (A) to (B) that requires knowledge on a critical microcrack length from which the macrocrack starts to grow.

3 PRE-CRITICAL CRACK GROWTH

Pre-critical cracking is the clue to scaling of strength, and in order to understand what happens both tensile and compressive stress should be considered. In tension the process is more difficult to trace since the deformations are small, in the micrometer range, whereas in compression larger deformations, and more stable microcrack processes take place because in addition to the material structure, the microcracks are arrested while they grow in a favourable direction in the compressive stress-field. Moreover, confinement may assist to stabilize the microcracks. In this paragraph briefly the most important issues are summarized.

3.1 Fracture in uniaxial tension

The fracturing in tension is complicated in the sense that the microcracks are small in the pre-peak regime, and very difficult to detect. Deviation from linearity in the pre-peak regime suggests that some crack activity takes place, as was shown for example by Evans & Marathe [15]. Numerical simulation using micromechanics models also shows microcrack activity; see for example [16]-[19]. In these lattice simulations concrete is considered as a 3-phase particle composite, where the matrix phase is separated by and interfacial transition zone (ITZ) of constant thickness from the aggregates. The ITZ is chosen weaker than the matrix, which is in turn weaker than the aggregates. Depending on the aggregate density of the ITZ thickness, percolation of the weak ITZ may occur, leading to limited pre-critical cracking. ITZ percolation occurs at (i) high particle density or at (ii) thick ITZ at low particle density. The reverse cases lead to substantial pre-critical cracking, where distributed crack patterns develop in tensile specimens. It was estimated that the average crack width is in the order of 0.1 μm in the case of abundant microcracking, which makes these cracks rather difficult to detect. For example in [20] it was shown that by means of indentation experiments on cement a crack opening of a 1 μm could just be seen by means of synchrotron tomography. Viewing by means of electron microscopy has the advantage that smaller cracks can be observed, but the specimens must be scaled down tremendously (this applies to synchrotron experiments as well). Experimentally there are some hurdles to be taken in order to directly view pre-critical micro-cracks in tension, in large specimens, assuming of course that the lattice model predictions are correct and the fracture process goes as described. The interesting idea is actually that scaling will be dependent on pre-critical activity. When the ITZ is percolated the pre-peak microcracking is almost non-existent, which is actual the situation in classical concrete that contains very high particle volumes in order to decrease the costs. Therefore, it does not come as a surprise that for conventional concrete under uniaxial tension Weibull scaling applies, see [19].

3.2 Fracture under (confined) compression

As mentioned, in a compressive stress field vertical cracks propagate stably, and the external load must be increased in order to elongate the cracks. A popular model is the description of failure through the development of wing cracks, [21]-[22], which seems possible only when the material microstructure is not an obstacle for the development of (i) the inclined cracks, and also (ii) the wings. At large scales, when microstructural effects are minimal (in an earlier paper we proposed a transition scale of at least 8 times the maximum aggregate size; see also in [5]) the wing crack geometry can likely develop, assuming relatively long cracks. Note that it is common to discuss this particular wing-crack mode in PMMA or in glass, which at a specimen scale of 100 mm are rather homogeneous materials. Irrespective of whether the wing cracks develop or not, the detection of pre-critical cracks in compression, and the localized crack, which takes the form of a shear band (which may even be folded-up inside the specimen is the dimensions of the specimen do not allow for the growth of a single straight shear plane), are much easier than in the tensile case. In [23] a hollow cylinder was used, to profit from the stress-gradients near the hollow core to stabilize the crack growth even more. Curiously enough, for the experiments reported in [23] a Weibull law seems appropriate as well, which is hard to explain at the moment.

4 SCALING DOWN

As mentioned, detecting small microcracks requires scaling down the specimens. In view of restrictions regarding representative volumes, it is quite essential that the specimen is always larger than a multiple of the largest microstructural element. In synchrotron or in ESEM the specimen sizes are just a few millimeters large, which is just covering the cement phase in concrete. Cement grains can be as large as 100 μm, thus the smallest specimen would be 1 mm large. The same situation emerges in testing concrete samples in a structures lab, where the aggregate size is perhaps 10 mm and the specimen size 100 mm. Just based on geometry, one would expect the same behaviour since the ratio d_a/D is constant. But, is this so? For that question to answer, we have to look more carefully to the physical forces determining the strength of cement, which is indeed, as argued before, system strength as well. Due to lack of space not all arguments can be given here, but will be presented in a forthcoming publication.

REFERENCES

[1] Ba_ant, Z.P., Size effects in blunt fracture: concrete, rock, metal, *ASCE J. Engng. Mech.*, 110, 518-535, 1984.

[2] Carpinteri, A., Fractal nature of material microstructure and size effects on apparent mechanical properties, *Mech. Mater.*, 18, 89-101, 1994.

[3] Trunk, B. Einfluss der Bauteilgrosse auf die Bruchenergie von Beton, *Building Materials Reports 11*, ETH, Laboratory for Building Materials, Aedificatio Publishers, 2000.

[4] Van Vliet, M.R.A., Size effect of fracture in concrete and rock under uniaxial tension, *PhD-thesis*, Delft University of Technology, 2000.

[5] Van Mier J.G.M., Reality behind fictitious cracks? In: Li V.C., Leung C.K.Y., Willam K.J. Billington S.L., eds., *Proc. FraMCoS-V,* IA-FraMCoS, Evanston, (IL), 11-30, 2004.

[6] Célarié, F., Prades, S., Bonamy, D., Ferrero, L., Bouchaud, E., Guillot, C. and Marlière, C., Glass breaks like metal, but at the nanometer scale, *Physical Review Letters*, 90(7), 2003.

[7] Van Mier J.G.M., *"De Kunst van Breken en Scheuren"* (The art of breaking and cracking), Inaugural Lecture, Delft University of Technology, 40 p., 2000.

[8] Van Mier, J.G.M. *Fracture Processes of Concrete - Assessment of Material Parameters for Fracture Models,* CRC Press, Boca Raton, Florida, USA, 448 p., 1997.

[9] Van Vliet, M.R.A. and Van Mier, J.G.M., Experimental investigation of size effect in concrete and sandstone under uniaxial tension, *Engng. Fract.Mech.*, 65(2/3), 165-188, 2000.

[10] Li, V.C. 2003. On engineered cementitious composites (ECC) – A review of the material and its applications, *J. of Advanced Concrete Technology*, 1(3), 215-230, 2003.

[11] Markovic, I., Van Mier J.G.M. and Walraven, J.C., Development of high performance hybrid fibre concrete, In: Naaman, A.E. and Reinhardt, H.W., eds., *Proc.4th Int'l. RILEM Workshop on 'High Performance Hybrid Fibre Concrete'*, RILEM Publications, Bagneux, 277-300, 2003.

[12] Hillerborg, A., Modeer, M. and Petersson, P.-E., Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements, *Cem.Conc.Res.*, 6. 773-782, 1976.

[13] Ba_ant, Z.P. and Oh, B.-H., Crack band theory for fracture of concrete, *Mater. Struct. (RILEM)*, 16, 155-177, 1983.

[14] Van Mier, J.G.M., Crack face bridging in normal, high strength and lytag concrete, In: Van Mier, J.G.M.. Rots J.G. and Bakker, A. eds., *Fracture Processes in Concrete, Rock and Ceramics*, Chapman & Hall, London/New York, 27-40 1991.

[15] Evans, P.H. and Marathe, M.S., Microcracking and stress-strain curves for concrete in tension, *Mater. Struct. (RILEM),* 1, 61-64, 1968.

[16] Prado E.P. and Van Mier, J.G.M., Effect of particle structure on mode I fracture process in concrete, *Eng. Fract. Mech*., 70, 1793-1807, 2003.

[17] Lilliu, G. and Van Mier, J.G.M., Effect of ITZ percolation on tensile fracture properties of 3-phase particle composites, In: Vermeer, P.A., Ehlers, W., Herrmann, H.J. and Ramm, E., eds., *Proc. 2nd Int'l. Symp.'Continuous and Discontinuous Modelling of Cohesive Frictional Materials'*, Balkema Publishers, Leiden, 101-108, 2004.

[18] Lilliu, G. and Van Mier, J.G.M. 3D lattice type fracture model for concrete, *Engng. Fract. Mech.*, 70(7/8), 927-942, 2003.

[19] Van Mier J.G.M. Van Vliet M.R.A., Influence of microstructure of concrete on size/scale effects in tensile fracture, *Eng. Fract. Mech*., 70(16), 2281-2306, 2003.

[20] Trtik, P., Van Mier, J.G.M. and Stampanoni, M., Three dimensional crack detection in hardened cement pastes using synchrotron-based computer microtomography (SR□CT)", *ICF-11*, Torino, March 20-25, 2005 (these proceedings).

[21] Horii, H. and Nemat-Nasser, S., Brittle failure in compression: splitting, faulting and brittle-ductile transition, *Phil. Trans. Royal Soc. London: Series A.*, 319(1549), 337-374. 1986.

[22] Schulson, E.M., Compressive shear faults within artic sea ice: fracture on scales large and small, *J. Geophys. Res.*, 109(C07016), 1-23, 2004.

[23] Elkadi, A.S., Van Mier J.G.M. and Sluys, L.J., Multiaxial failure of low-cohesive frictional materials: fracture behaviour and size-dependency, *Trans. Royal Soc.*, (in press), 2005.