SIZE AND SCALE EFFECTS IN FRETTING FATIGUE THRESHOLDS

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ABSTRACT

Components subject to fretting experience a peculiar combination of loading conditions, where contact and classical fatigue interact intricately to produce failure. As a consequence, the prediction of fretting fatigue limit curves poses a challenge, in part due to the large number of parameters governing the phenomenon. This poses an obstacle to formulating efficient predictive approaches.

We demonstrate that these difficulties can be overcome successfully by means of a combination of experimental and computational approaches. Our analysis relies on various experimental data from Hertzian and 'flat and rounded' contact pad specimens and different calculation procedures developed previously, which resulted in fretting threshold curves for specific loading conditions. The derivation of such thresholds is however rather lengthy, so that for the purposes of formulating design rules a more efficient 'master curve' approach is proposed.

This paper presents comprehensive results on the application of an efficient and concise functional description of the fretting fatigue threshold curves based on the use of a 'multi-scaling power law'. The predictions encompass all of the results obtained for different loading conditions by the stress-based approach and by short crack arrest methodology.

1 INTRODUCTION

Fretting fatigue occurs when contacting components are subjected to cyclic loading, and is a significant cause of premature fatigue failure. Until recently it has been considered that surface damage due to microslip plays a dominant role in decreasing the fatigue strength of the material [1]. More recently, the severity of the local stress field has been recognised as important and the significance of the high stress gradients, and hence the presence of the "size effect" has also become clear. Multiaxial criteria [2] and short crack arrest methods [3] are examples of approaches used to understand the effects of contact stress on failure. Crack and notch analogies have been also proposed [4,5], since the contact stress field often gives rise to a finite stress concentration similar to that in notched components. Some of these approaches have also been used to generate fretting fatigue limit curves (or thresholds) for various geometries, materials, loading conditions and surface treatments.

In general, stress analyses of experimental results on crack initiation in fretting fatigue aim to define the fretting threshold as a function of governing loading and geometric parameters of the problem. In this paper two approaches are used: the short crack arrest method, based on the approach proposed by Nowell and Araújo [3], and a more conventional approach based on notch fatigue concepts.

In order to provide a compact and efficient formulation of design rules against fretting fatigue, thresholds derived using these methodologies are then used to calibrate a 'multi-scaling power law model' based on a particular functional representation of the limit curves.

2 SIZE EFFECTS AND SCALING PARAMETERS FOR FRETTING THRESHOLDS

It is appropriate to start by considering a generic fretting problem characterised by a punch (cylinder or flat and rounded) indenting a half-plane and subjected to oscillatory tangential loading. The loading variables that affect the material response are peak pressure, p_0, the load ratio, Q/P, and the underlying remote bulk tension, σ_b. The coefficient of friction, f, also plays an important role in determining the fretting fatigue strength, but is kept constant during the analyses[1].

The aim is to determine the fretting fatigue limit of the contact as a function of geometry and loading conditions, and hence to investigate the effect of loading parameters on the "size effect" induced by the steep stress gradient. As discussed by Nowell [8], if one considers a Hertzian contact depicted in the inset of Figure 1(b), contact width parameter a varies as $\sqrt{PR}$, whereas p_0 varies as $\sqrt{P/R}$. It is therefore possible to maintain p_o (hence preserving the stress concentration at the trailing edge of the contact) constant and vary a by varying the load and pad radius. We note in passing that similar reasoning may be applied to flat and rounded contacts. Nowell [8] undertook a number of series of experiments where the magnitude of p_0 was kept constant while the contact size was varied. Within each series, a critical size of contact above which short fatigue lives were observed was found, whereas smaller contacts specimen lasted for more than 10^7 cycles. Therefore, the fretting strength of the component decreases with the increase of the contact width as schematically described in Figure 1(a). Two different methodologies, which produce similar results, are now discussed. Space limitation does not allow a full description of the techniques to be presented, so these are only briefly summarised and the reader is referred to [9] for a complete explanation of the adopted procedures.

The critical distance procedure adopted to derive the threshold curves is based on the assumption that, in order for failure to occur, the average stress must exceed the fatigue limit over some critical volume surrounding the stress raiser (e.g. the concentration of stress at the notch root). Since the problem of averaging the stresses over a volume is generally computationally expensive, methods based on "single point" stress evaluation (critical point) or reduced averaging over critical lines and areas are used and these can be regarded as simplification of the volume method characterised by different levels of approximation [9]. An example plot of the variation of the fretting fatigue strength of Al4%Cu Hertzian contacts as a function of the peak pressure is illustrated in Figure 1(b).

The stress-based critical distance approach is largely empirical and there is some evidence that the critical distance may not be geometry-independent. It is also unclear how to incorporate residual stresses, frequently used as a fretting fatigue palliative, into the approach. In order to overcome these limitations the short crack arrest methodology is employed. This makes use of either the Kitagawa-Takahashi [10] or the El Haddad threshold [11] on the K-T diagrams in order to predict the propagation and arrest of cracks emanating from stress concentrations. For any test conditions, the growing crack may be thought of as describing a curve on the diagrams, although it should be noted that the two forms of the plot are equivalent only in the case in which the crack grows under a uniform far-field stress. For cracks growing in a varying stress field it is most convenient to use the ΔK form of the diagrams, where the stress intensity factor range is plotted as a function of the growing crack. The diagrams may be used to establish the fatigue limit by finding the applied stress amplitude which raises the curve above the threshold [9].

The semi-analytical method employed in the short crack arrest approach may be summarised as follows:

1. The contact stress field is calculated using Muskhelishvili's potentials applied to pressure and traction distributions derived by solving the singular integral equation governing the contact of the two elastically similar bodies, initially within the assumption of the half-plane approximation.

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[1] The values of the coefficient of friction used for the computation of the stresses can be derived using the method described in [6, 7] respectively for the Hertzian and 'flat and rounded' configurations

2. Crack tip stress intensity factors are calculated, using the distributed dislocation method described in detail by Nowell and Hills [12] and Hills et al. [13], at the trailing edge of the contact, where cracks are found to nucleate.

3. ΔK (Mode I stress intensity range) is calculated as a function of the crack length and compared to the threshold defined by the K-T diagram. The fatigue limit is then determined by establishing the bulk stress required to raise the curve above the threshold.

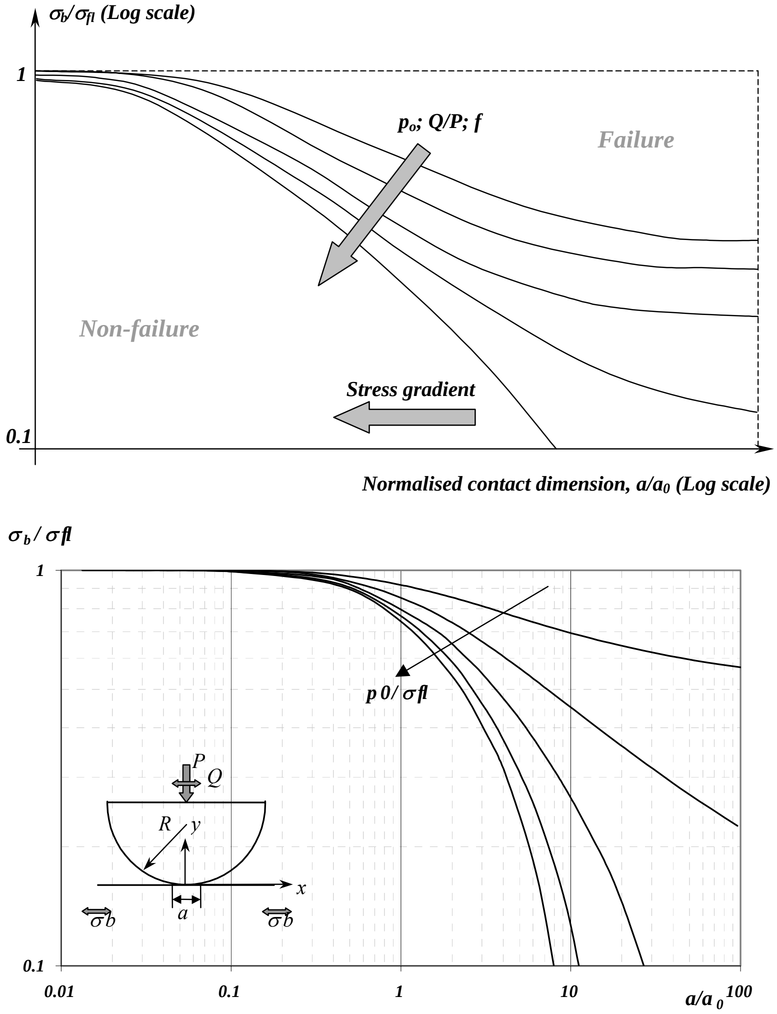

Figure 1: (a) Schematic of the predicted "size effect" in fretting fatigue as a function of the loading parameters; (b) example of critical distance predictions for Hertzian contacts as a function of p_0.

As already discussed in [9], thresholds derived using the short crack arrest methodology for Hertzian contacts are very similar to those derived using the critical distance methods. The thresholds obtained using the critical distance and the short crack arrest methodologies are validated by comparison with the experimental results obtained by Nowell [8]. Further validation is carried out using the flat and rounded contact geometry using the experimental results of a number of fretting fatigue experiments carried out with Ti6Al4V Titanium alloy [9].

3 THE NEW FUNCTIONAL FORM FOR FRETTING FATIGUE THRESHOLDS

As discussed in the previous section, fretting thresholds depend on key parameters concerning problem geometry, loading conditions and material properties combined into dimensionless power groups. The challenge for the designer is then to identify a suitable form of the fatigue threshold 'master curve' that would allow limits to be readily calculated for any combination of parameters. One approach is to construct a series of look-up tables, but instead we propose to identify a functional form of the relationship between the limiting bulk stress, σ_b, on the one hand, and the contact size, a, on the other.

The functional form of this relationship was originally proposed by Korsunsky et al [14] in relation to the scaling of the hardness of coated systems with coating thickness. The approach was further developed and applied to a wide range of scaling and size effect problems in the context of materials failure [15]. The functional form of the multi-scaling power law described there can be shown to arise from the analysis of different power law asymptotic regimes of strength.

The functional form for the dependence of fretting strength of a contact, σ_b, on its size, a, proposed here is as follows:

$$\frac{\sigma_b}{\sigma_{fl}}\left(a/a_0,\mathbf{\Lambda}\right)=C_0(\mathbf{\Lambda})+\frac{C_1(\mathbf{\Lambda})}{\left(1+(a/a_0)^{C_2(\mathbf{\Lambda})C_3(\mathbf{\Lambda})}\right)^{\frac{1}{C_3(\mathbf{\Lambda})}}} \qquad (1)$$

where a_0 is the combined size-material parameter defined by El Haddad [11]. Symbol $\mathbf{\Lambda}$ stands for the set of parameters (R_P, R_Q, f, G), where $R_P = p_0/\sigma_{fl}$ and $R_Q = Q/fP$ are respectively normal and tangential load dimensionless parameters, f is the coefficient of friction, and G is a geometrical factor that accounts for the shape of the couplings.

An example application of the derivation of threshold curves for fretting of Hertzian contacts is now discussed. The formulation relies on the optimisation of the functional form using a series of curves previously derived adopting one of the techniques described above. In particular, making use of the thresholds derived by Dini [9] for Al-4%Cu alloys (metal on metal contact was only considered, so that the coefficient of friction was 0.55), a general expression for the dependency of the unknown functions C_0, C_1, C_2 and C_3 in (1) on the loading parameters R_P and R_Q was found to be:

$$\begin{aligned}C_0 &= 1-k_0R_P\\ C_1 &= k_0R_P\\ C_2 &= \left(k_1{R_Q}^2+k_2R_Q+k_3\right)\\ C_3 &= \left(k_4{R_Q}^3+k_5{R_Q}^2+k_6R_Q+k_7\right)\end{aligned} \qquad (2)$$

where

$$k_0=1.4, k_1=0.52, k_2=0.43, k_3=0.031, k_4=-133, k_5=233, k_6=-142, k_7=33 \qquad (3)$$

Comparison between the prediction curves obtained using the functional form (1) incorporating expressions from (2) (continuous curves), together with the results from Dini [9] (markers) are shown in Figure 2. In order to appreciate the significance of the quality of agreement the reader is invited to note the logarithmic scales used for both axes, and the fact that the single formula (1) with parameters from (2) captures the entire set of predicted threshold curves depending on not less than three independent parameters.

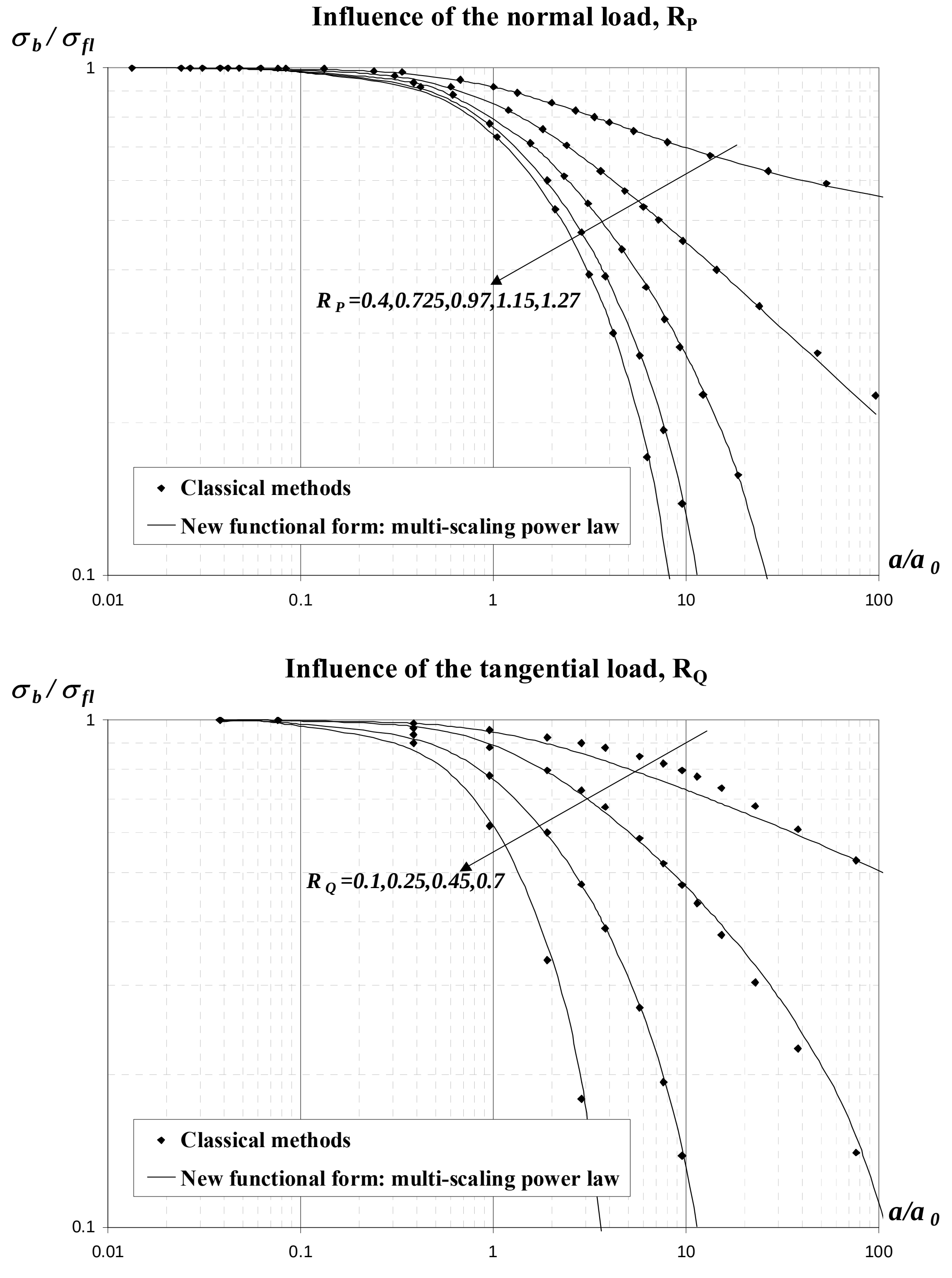

Figure 2: Comparison between threshold predictions obtained using classical methods and the new 'multi-scaling power law' formulation.

4 DISCUSSION

In the present paper we introduced the application of the multi-scaling power law functional description to fretting fatigue strength thresholds. The approach demonstrates that the choice of a suitable functional form allows a vast range of experimental and theoretical threshold curves to be conveniently 'condensed' into the form of a single function, making it extremely convenient for use by a practical designer.

The multi-scaling power law approach introduced in this paper possesses remarkable flexibility and range of application, and can be successfully used to describe scaling behaviour of strength under conditions of quasi-brittle or elasto-plastic fracture, fatigue crack initiation and propagation, short crack effects, hardness scaling, etc. Fretting fatigue thresholds considered here present a particularly complex case, and thereby serve as a good illustration of the utility of this approach.

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