LOCAL SECOND GRADIENT MODELS AND DAMAGE MECHANICS: APPLICATION TO CONCRETE

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ABSTRACT

The non linear behaviour of concrete is often simulated using local constitutive models based on the continuous damage mechanics theory. This approach however is not adequate for post-localisation studies with strain softening. It is well known that spurious mesh dependence appears in computations and cases of failure without energy dissipation. In order to improve computational performance second grade local models are chosen to include a meso scale in the continuous damage model. This approach differs from the nonlocal one in the sense that it is a local theory with higher order stresses depending only on the local cinematic history. 1D numerical computations with concrete specimens are presented. Using a random initialisation of the iterative solver of the equilibrium equation we search the existence of various solutions for the boundary value study and also to see if the second grade term regularise the problem giving results that are mesh insensitive and objective.

1 INTRODUCTION

Experimentally, concrete specimens exhibit a network of microscopic cracks that nucleate parallel to the axis of loading. Due to the presence of heterogeneities in the material (aggregates surrounded by a cement matrix), tensile transverse strains generate a self-equilibrated stress field orthogonal to the loading direction, a pure mode I (extension) is thus considered to describe the behaviour in compression. A classical local model based on continuous damage mechanics is used hereafter allowing accounting for the asymmetric behaviour of concrete under tension and compression. The influence of microcracking due to the external loads is introduced via a single scalar damage variable d ranging from 0 for the undamaged material to 1 for a completely damaged material. In order to introduce the non-symmetric behaviour of concrete, the failure criterion is expressed in terms of the principal extensions.

This approach however is not adequate for post-localisation studies where strain softening appears. Calculations performed with a local classical continuum model - which does not incorporate an internal length variable - are unable to model objectively intrinsic failure zones. It is now well known that a spurious mesh dependence appears in computations and cases of failure without energy dissipation. In order to improve computational performance the nonlocal damage approach is often used in the literature. A different solution is investigated within this work. Second grade local models are chosen to include a meso scale in the continuous damage model. This approach differs from the nonlocal one in the sense that it is a local theory with higher order stresses depending only on the local cinematic history.

Details on the damage mechanics constitutive law are given at the first part of the paper. The second gradient local approach is then introduced and different numerical computations with 1D concrete specimens in traction are presented. Using a random initialisation of the iterative solver of the equilibrium equation we search the existence of various solutions for the boundary value study and also to see if the second grade term regularise the problem giving results that are mesh insensitive and objective.

2 SCALAR LOCAL DAMAGE MODEL

Introduced in 1958 by Kachanov (e.g. Kachanov [14]) for creep-related problems, continuum damage mechanics has been applied in the 1980s for simulating the non linear behaviour of concrete (e.g. Krajcinovic [15], Lemaitre and Mazars [19], Ladeveze [17]). Thermodynamics of irreversible processes gave the framework to formulate the adapted constitutive laws (e.g. Lemaitre and Chaboche [18]). Considering the material as a system described by a set of variables and a thermodynamic potential, constitutive laws are systematically derived along with conditions on the kinematics of damage. However, an adequate choice of the potential and of the damage variable (scalar, tensor, etc.) remains to be made. Several anisotropic damage models have already been proposed (e.g. Dragon and Mroz [8], Mazars and Pijaudier-Cabot [23], Fichant et al. [9, 10]. Possible applications cover also dynamic problems (e.g. La Borderie [16], Ragueneau et al. [30]), porous materials (e.g. Pijaudier-Cabot and Burlion [29]) and chemical damage (e.g. Gérard et al. [12]). A recent literature review on damage mechanics can be found in Pijaudier-Cabot [27].

The outlines of a local scalar 3D damage mechanics law for concrete are presented hereafter (e.g. Mazars [21, 22]). In this model, the material is supposed to behave elastically and to remain isotropic. The loading surface takes the following form:

$$f(\varepsilon, d) = \varepsilon_{eq} - K(d) \; . \tag{1}$$

with ε_{eq} an equivalent strain defined as:

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3} < \varepsilon_i >_+^{\,2}} \text{ . where } \varepsilon_i \text{ are the principal strains} \tag{2}$$

The hardening-softening parameter $K(d)$ takes the largest value of the equivalent strain ever reached by the material at the considered point to retain the previous loading history. Initially $K(d)$ equals the threshold ε_{d0}. Evolution laws for damage are used to describe the response in tension or compression (index *i* refers either to tension (t) or compression (c)):

$$d_i = 1 - \frac{\varepsilon_{d0}(1 - A_i)}{\varepsilon_{eq}} - A_i \exp(-B_i(\varepsilon_{eq} - \varepsilon_{d0})) \; . \tag{3}$$

A_i and B_i and ε_{d0} are material parameters identified independently from compression tests on cylinders and bending tests on beams. The scalar damage variable d that has to be introduced in the constitutive equation is a weighted sum of d_t and d_c (variables that correspond respectively to damage measured in uniaxial tension and uniaxial compression).

$$d = \alpha_t d_t + \alpha_c d_c \; . \tag{4}$$

We call σ_+ and σ_- ($\sigma = \sigma_+ + \sigma_-$) the tensors in which appear only the positive and negative principal stress, respectively, and ε_t, ε_c the strain tensors defined as:

$$\varepsilon_t = \Lambda^{-1} : \sigma_+ \text{ and } \varepsilon_c = \Lambda^{-1} : \sigma_- \; . \tag{5}$$

$\Lambda(d)$ is a fourth-order symmetric tensor interpreted as the secant stiffness matrix and it is a function of damage. The weights α_t and α_c are defined by the following expressions:

$$\alpha_t = \sum_1^3 H_i \frac{\varepsilon_{ti}(\varepsilon_{ti} + \varepsilon_{ci})}{\varepsilon_{eq}^2} , \alpha_c = \sum_1^3 H_i \frac{\varepsilon_{ci}(\varepsilon_{ti} + \varepsilon_{ci})}{\varepsilon_{eq}^2} \; . \tag{6}$$

$H_i = 1$ if $\varepsilon_i = \varepsilon_{ci} + \varepsilon_{ti} \geq 0$, otherwise $H_i = 0$. α_t and α_c are the coefficients defining the contribution of each type of damage for general loading. From eqn (6) it can be verified that for uniaxial tension $\alpha_t = 1,\ \alpha_c = 0,\ d = d_t$ and vice versa for compression.

3 LOCAL SECOND GRADIENT MODEL

It is today well established that strain softening induces bifurcation, strain localisation and failure without energy dissipation (e.g. Bazant [2]). One of the possible remedies is to use nonlocal constitutive models (e.g. Pijaudier-Cabot and Bazant [28]). A different approach is investigated within this work using second grade local models to introduce a meso scale in the continuous damage model.

Since the work of Aifantis (e.g. Aifantis [1]) second grade models are often used, especially within the flow theory of plasticity. Peerlings (e.g. Peerlings et al. [25, 26]) and Fremond (Fremond and Nedjar [11]) have also studied second grade damage models. The study presented in this paper is about uniqueness of solution involving damage mechanics and the local second gradient model proposed in Chambon et al. [4, 5, 6]. Here the word local means that the constitutive equation is a relation only between local quantities. The model is a direct extension of microstructured or micromorphic continua (Germain [13] and Mindlin [24]). A 2D large strain finite element formulation and the corresponding constitutive equations have been developed using a mathematical constraint between the micro kinematics description and the usual macro deformation gradient field. This constraint is enforced in a weak sense by using Lagrange multipliers in order to avoid difficulties with the C^1 continuity (second grade models involving the first and the second derivatives of the displacement field - Matsushima et al. [20]). The implementation of the method in the finite element code LAGAMINE (Université de Liège) has recently been completed (e.g. Besuelle [3]).

4 1D NUMERICAL SIMULATIONS

One-dimensional traction in plane deformations is studied hereafter and the results are compared with the analytical solutions (else than the homogeneous one that is always possible) calculated for small strains (eg. Chambon et al. [4, 5]). Figure 1 shows the boundary conditions used in the 2D version of LAGAMINE. In order to avoid a 2D effect, the condition $u_2 = 0$, is applied at the upper and lower boundaries along the bar (u for displacement). The section of the bar is 0.1x1 m^2 and its length 1m. The right end of the bar is fixed ($u_i = 0$) and the external traction force is applied at the left end. The additional external forces are assumed to be zero at both ends.

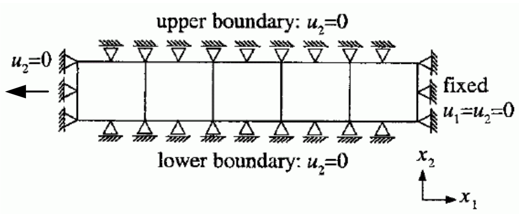

Figure 1. Boundary conditions for simulating 1D traction in a 2D FE code.

It is assumed that there is no coupling between the first and the second gradient part of the model. In order to get localization, the first gradient part has to exhibit softening. The constitutive relation is shown in Figure 2. The parameters chosen for the damage law (first gradient part) correspond to a typical concrete specimen ($A_1 = 30.E{+}09Pa, \varepsilon_{d0} = 1.E{-}04, A_t = 0.5, B_t = 2.E{+}04$, parameters that provide $A_2 = -16.7E{+}09Pa$). The second gradient 1D model implicitly defines two internal lengths, one (namely $\sqrt{B/A_1}$) corresponding to the unloading regime of the first gradient part of the model and the other (namely $\sqrt{B/(-A_2)}$) corresponding to the softening loading regime just after the peak. The parameter B is chosen in order to have possible analytical solutions and to avoid snack back phenomena (B=0.37E+09N , Chambon et al. [4, 5]). Assuming a two-part solution (built with a patch of a hard part and a soft one) is possible one finds analytically that the length of the soft part is equal to 0.37m. For the case of a three-part solution (hard – soft – hard) the length of the soft part equals 0.78m.

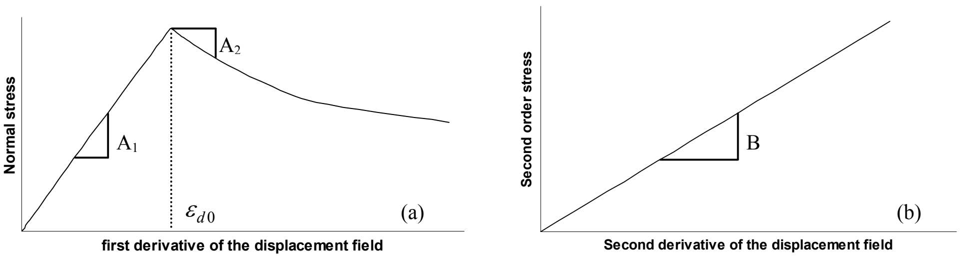

Figure 2. Constitutive model: (a) first gradient part, (b) second gradient part.

A soon as the peak is reached the problem exhibits a loss of uniqueness. In order to determine numerically bifurcation thresholds, an algorithm of random initialisation of the iterative solver of the equilibrium equation is used just after the peak (at $\varepsilon = 1.2E-04$, e.g. Chambon et al. [7]). For every step, a full Newton-Raphson involving a numerical consistent tangent stiffness operator for the complete model (i.e. the second gradient terms as well as the classical ones) is used. The results of two meshes with 14 and 50 elements are presented hereafter.

Figure 3 shows the global force displacement curve for both meshes. Figure 4 presents the distribution of the damage variable d just after the peak ($\varepsilon = 1.2E-04$) and Figure 5 at the end of the loading ($\varepsilon = 2.9E-04$). The differences in the global curves just after the peak are due to the different corresponding localisation patterns. The mesh with 14 elements converges to a solution with two patches (a hard part and a soft one with a length equal approximately to the length calculated analytically). The mesh with 50 elements converges to a three-part solution (hard – soft – hard) with the length of the soft part again very similar to the analytical value. The 50-element mesh switches after to the two-part solution, thus the localisation pattern and the global curves become identical. It is obvious that the use of local second gradient models with local damage mechanics laws provide internal lengths and consequently regularization of the solutions but it does not restore uniqueness properties for the corresponding boundary value problem.

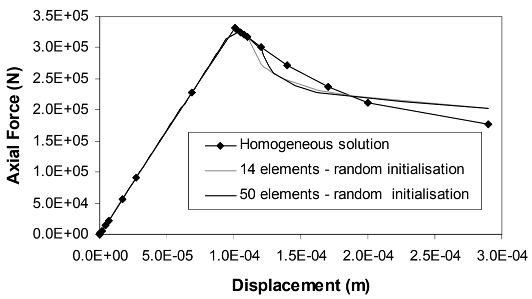

Figure 3. 1D traction: Force - displacement curves for the two meshes.

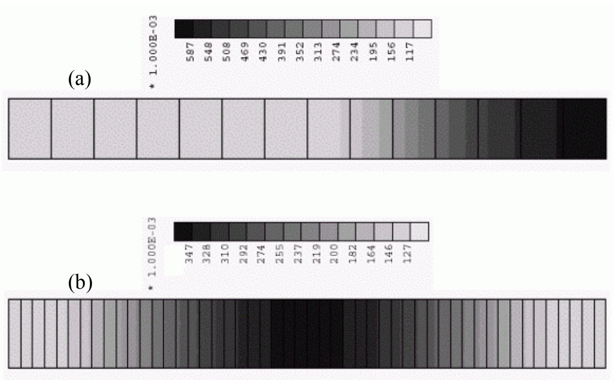

Figure 4. Localisation patterns (distribution of damage variable) just after the peak ($\varepsilon = 1.2E - 04$): (a) 14-element mesh, (b) 50-element mesh.

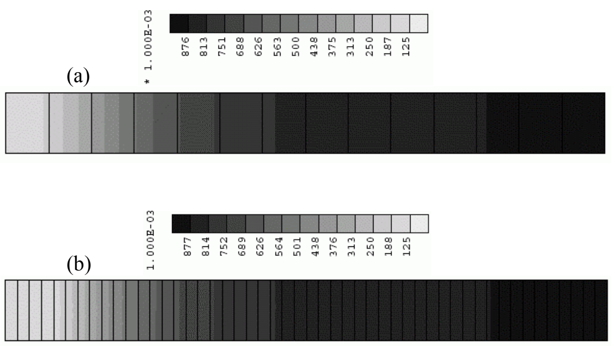

Figure 5. Localisation patterns (distribution of damage variable) at the end of the loading ($\varepsilon = 2.9E - 04$): (a) 14-element mesh, (b) 50-element mesh.

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