

# A FAST DESIGN-TOOL FOR CERAMICS UNDER STATIC AND CYCLIC CONTACT LOADING

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## ABSTRACT

Common strength and fatigue experiments for ceramic materials determine the failure behaviour at simple stress states with small stress gradients. Recently, ceramics are also used in applications like gears and pumps because of, compared to metals, their outstanding tribological properties. The resulting contact loading causes a multi axial inhomogeneous stress state with high stress gradients, which has to be taken into account in the experimental characterisation and failure prediction. A new method for prediction of the contact strength and contact fatigue behaviour of ceramic components under normal, sliding, and cyclic contact loading has been developed. The method is implemented in a fast Contact Design-Tool (CDT), that works independent from FEM methods and can be used to assess the lifetime and the failure probability of the loaded ceramic. Results obtained from the CDT for the friction contact of a continuous variable transmission gear show an exponentially decreasing dependence between the maximum allowed normal pressure and the friction coefficient of the system.

## 1 INTRODUCTION

A common method for the reliability assessment of ceramic components consists of measurement of the Weibull parameters of the material, the calculation of the stress field by finite element simulation and the numerical computation of the failure probability based on a weakest link model using e.g. the finite element post processor STAU [1]. This approach is convenient for most practical applications under thermal or mechanical loading of the ceramic component, causing simple stress states and small stress gradients.

Friction and sliding applications like continuous variable transmission (CVT) gears, clutches, and high pressure fuel injection pumps are currently under development [2], which use ceramic components because of their outstanding thermal and tribological properties. On the other hand, ceramics are sensitive to tensile stresses, which typically occur at the contact edge in a contact loading. An optimal design of such high performance tribological systems requires a method for the prediction of the failure behaviour under normal, sliding, and cyclic contact loading. This method needs to account for several experimental, modelling and numerical issues, which are specific to contact situations:

- Inhomogeneous stress state with high stress gradients
- Three-dimensional modelling necessary for bodies in sliding contact
- High resolution of the calculated stress field at the contact edge.

Two kinds of experimental set-ups for the measurement of contact strength and contact fatigue properties were developed by Fett et al. [3], which are based on cylindrical and spherical contact geometry, respectively, and test the materials under typical contact loading. Contact strength experiments with the two-roller test show same strength but a reduction of the Weibull exponent  $m$  by a factor of about two. Contact fatigue experiments show a different fatigue mechanism and a much smaller crack growth parameter  $n$  compared to conventional four point bending tests because of inhibited water diffusion in the region of the crack tip due to  $K_I < 0$  [4]. The prediction of the failure and fatigue behaviour in practical applications require a generalisation of these experimental results for arbitrary geometries and loading conditions. A corresponding method is proposed and described in this paper.

## 2 STRESS FIELD

Due to the requirements described in Sect. 1 finite element modelling of sliding contact situations is extremely time-consuming and should be avoided. Since tensile stresses develop in the vicinity of the contact edge, the computation of the stress field can be restricted to this area, based on the integration of Green's function  $s_{ij}$ , which is convenient for the solution of elastic contact problems [5] by

$$\sigma_{ij}(x, y, z) = \int_A p(x', y') s_{ij}(x - x', y - y', z) dx' dy'. \quad (1)$$

Here  $p$  is the pressure distribution in the contact and  $A$  is the contact area, which is bound by the ellipse  $e = \sqrt{1 - (b/a)^2}$  and the semi axes  $b < a$ . The effective radii of the two contacting bodies 1 and 2 are given by  $1/R' := 1/R_1' + 1/R_2'$  in  $x$ - and  $1/R'' := 1/R_1'' + 1/R_2''$  in  $y$ -direction, respectively. The semi axes are calculated from the implicit expression [5, p. 95]

$$\frac{R'}{R''} = \frac{(a/b)^2 E(e) - K(e)}{K(e) - E(e)}, \quad (2)$$

where  $K(e)$  and  $E(e)$  are elliptic integrals of first and second kind. The pressure distribution is then determined by the load  $P$  using

$$p(x, y) = \begin{cases} \frac{3P}{2\pi ab} \sqrt{1 - (x/a)^2 - (y/b)^2}, & \sqrt{(x/a)^2 + (y/b)^2} \leq 1 \\ 0, & \text{elsewhere} \end{cases}. \quad (3)$$

The shear stresses, caused by friction in a sliding contact, are assumed to be proportional to the pressure  $\tau(x, y) = \mu \cdot p(x, y)$  and oriented in direction of the transverse force, split in its  $x$ - and  $y$ -components. A verification of the calculated stress field of a plate in contact with a sliding pin is presented in Fig. 1. Only the relevant part of the finite element model is shown. The CPU time for the finite element solution was 34 hours on a IBM-Power-4 RS 6000 parallel clusters, while the Contact Design-Tool needed only 20 CPU minutes on a single processor PC. The different peak values can be regarded as inaccuracy of the finite element solution due to the

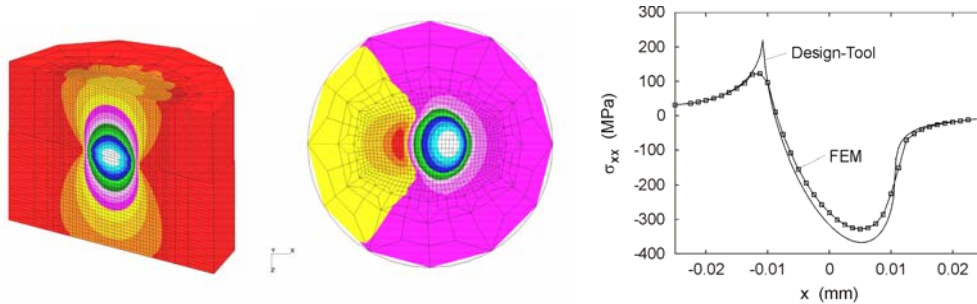


Fig. 1: Surface stress: Comparison of FEM and Contact Design-Tool  
Finite Element mesh.

## 3 COMPUTATION OF FAILURE RELIABILITY

The calculation of the failure probability from the stress field is described in detail in [6]. A typical distribution of the failure probability obtained from the Contact Design-Tool is given in Fig. 2. In a friction contact, high stresses occur in a small area behind the contact zone and the failure probability increases gradually with increasing normal load. The curves in Fig. 3(a) show

the resulting failure probability for the CVT gear test setup as a function of the normal force at  $K_{IC} = 125 \text{ MPa mm}^{1/2}$ ,  $\mu = 0.13$ ,  $R' = 100 \text{ mm}$  and  $R'' = 200 \text{ mm}$ . The sliding distance increases from  $s = 0 \text{ mm}$  to  $s = 471 \text{ mm}$ , which represents the circumference of a pair of cone pulleys at the smallest diameter of  $75 \text{ mm}$ . Since the CVT gear can be continuously adjusted, it can be assumed that during operation the whole cone surface will come in contact with the maximum local stress resulting in a jump of the failure probability at a critical load  $P_{krit}$ .

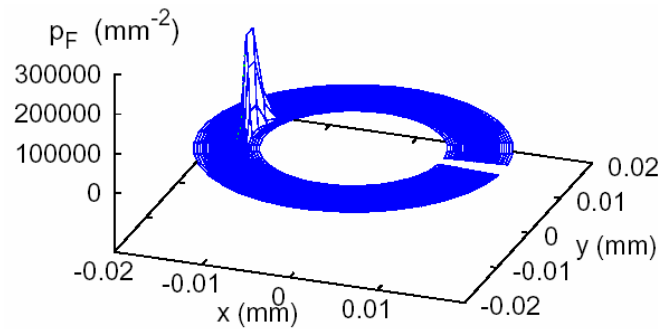


Fig. 2: Typical distribution of local failure probability in sliding contact

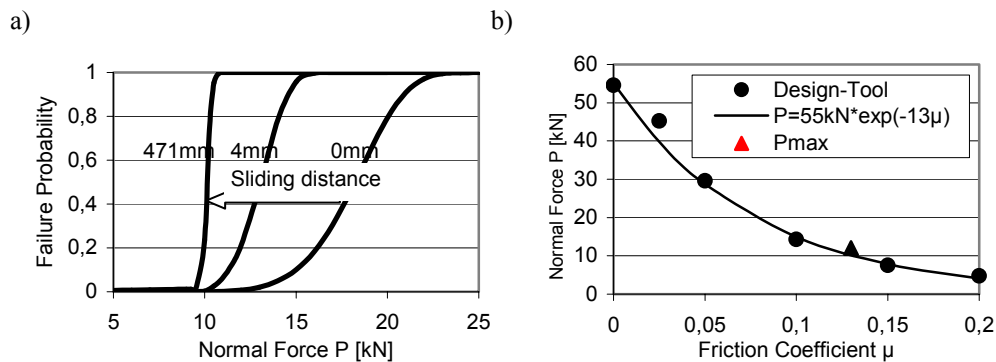


Fig. 3: a) Failure probability of the CVT cone pulley; b) Limit load to failure

Figure 3(b) shows the calculated critical normal load as a function of the friction coefficient  $\mu$ .

The determined relation can be fitted with an exponential function of the form

$$P_{krit} = 55 \text{ kN} \cdot \exp(-13\mu). \quad (4)$$

Under extreme conditions of maximum hydraulic pressure and a minimum number of pins in contact, a maximum load of  $P = 12 \text{ kN}$  can be obtained in the experimental test setup [2, pp.57-64]. This value slightly exceeds the critical load in Fig. 3(b) at the friction coefficient of  $\mu = 0.13$ , which has been measured for the material combination of the steel chain and the Al2O3-cone pulley (triangle marker).

#### 4 CONTACT FATIGUE

In the vicinity of the contact edge the mode-I stress intensity factor is typically negative causing a different fatigue mechanism under cyclic loading. The contact fatigue model [4] determines the number of cycles to failure for cyclic contact loading by

$$N_f = N_0 \log \left[ \frac{\mu_0 K_I}{\sqrt{\frac{2}{3}} K_{IC} - |K_{II}|} \right], \quad (5)$$

where  $N_0$  and  $\mu_0$  are contact fatigue parameters, and  $K_{IC}$  is the fracture toughness. This model is implemented in the Contact Design-Tool, which computes the stress intensity factors  $K_I, K_{II}$  at each point of the surface mesh (cf. Fig. 2). Running in the fatigue option, the Contact Design-Tool determines the effective stress intensity factor taking the high stress gradients into account as described in [6]. Three cases can occur: no damage, fatigue, and spontaneous failure. The parameters  $N_0 = 4 \cdot 10^6$  and  $\mu_0 = 0.1$  are obtained from fitting the cyclic fatigue data from the two-roller test device [4] (see Fig. 4).

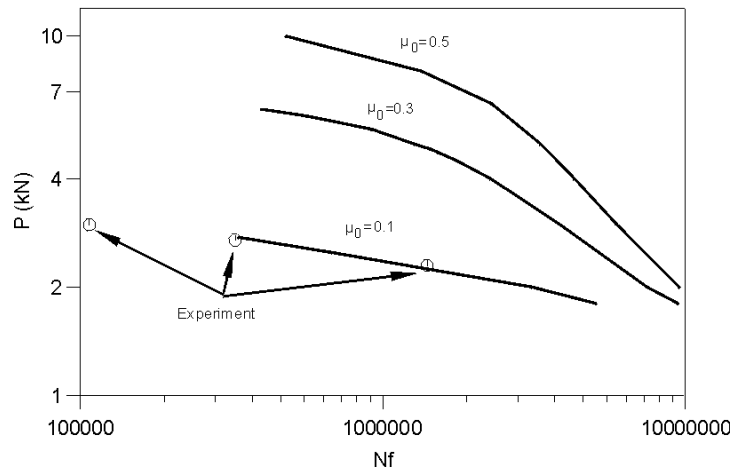


Fig. 4: S-N diagram of the contact fatigue simulations and fit to fatigue data

A spherical contact with cyclic loading, first causes a circumferential crack starting from a natural crack, which can be described with the same theory. When the circumferential crack has formed, a cone crack develops immediately. During further cycling the cone crack can also grow further till the part breaks. Using the Contact Design-Tool and the material parameters obtained from the two-roller test, the fatigue behaviour with regard to the first circumferential crack has been determined as a function of the radius of the sphere in contact with a ceramic plate. The results presented in Fig. 5 show, that only a small gap exists in between when fatigue takes place. Underneath, no damage occurs, while above a cone crack is formed during the first half cycle. For spherical radii above 15 mm, no contact fatigue takes place. Here, the load at the crack tip changes from compression to tension so that no shielding by a friction zone exists. As a consequence for a CVT gear with pin radii of 200 mm, no contact fatigue is expected and the reliability assessment of the cone pulleys can be carried out with focus only on the failure probability.

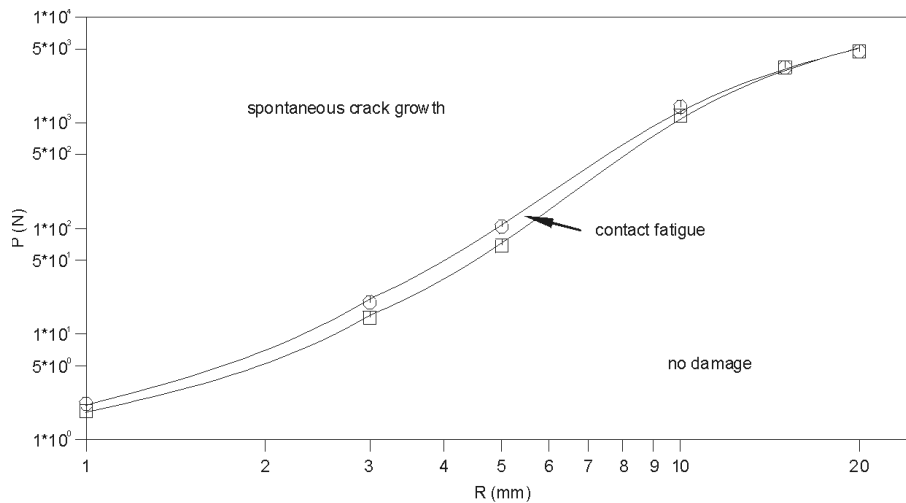


Fig. 5: Load range for contact fatigue for spherical contact loading

## 5 CONCLUSIONS

When the reliability of ceramics under contact loading is determined, new methods have to be applied that take into account the high stress gradients along a crack near the contact edge and that include a proper model for different fatigue mechanisms. At the same time, finite element simulations are very time consuming due to the necessary complex three-dimensional meshing with high resolution at the contact edge and high number of elements on the one hand and the highly nonlinear contact problem on the other hand.

The new Contact Design-Tool presented in this paper is able to perform automatically the meshing of the contact problem and then a quick computation of the stress field and the distribution of the failure probability. In addition, a fatigue option has been implemented, that allows the prediction of stable crack growth under typical conditions of cyclic contact loading.

Using this method, the critical load of a CVT gear has been determined in dependence of the friction coefficient. Contact fatigue is not relevant for this geometries due to the big radii of the contacting bodies. The next step is a verification experiment using the CVT test setup.

## 5 REFERENCES

- [1] K. Heiermann, et al.: STAU – A Postprocessor for a Finite Element Program to Calculate the Failure Probability of Multiaxially loaded Ceramic Components, Users Manual Version 4, 2002, Forschungszentrum Karlsruhe, 2002.
- [2] K. H. zum Gahr, J. Schneier (Eds): 2. Statuskolloquium des Sonderforschungsbereichs 483, Karlsruhe (in German), 27. Jan. 2004.
- [3] T. Fett, D. Munz, G. Thun: Test devices for strength measurement of bars under contact loading, Journal of Testing and Evaluation, Vol. 29, pp. 1-10, 2001.
- [4] T. Fett: Interpretation of failure under cyclic contact loading, J. Mater. Sci. Letters, Vol. 39, pp.1795-1797, 2004.
- [5] K. L. Johnson, Contact Mechanics, Cambridge University Press, 1985.
- [6] V. Licht, P. Hülsmeyer, T. Fett: Probability of cone crack initiation due to spherical contact loading, Journal of the European Ceramic Society, Vol. 24, pp.2907-2915, 2004.