FRACTURE OF RUBBERS UNDER MIXED MODE CONDITIONS

M. NAÏT ABDELAZIZ[1] , N. AÏT HOCINE[2] and F. BECHET[1]
[1]LML, UMR CNRS 8107, Polytech'Lille, Villeneuve d'Ascq, France.
[2]Laboratoire de Rhéologie, Brest, France.

ABSTRACT

This work deals with fracture of rubbers under mixed mode. A filled Styrene Butadiene Rubber (SBR) was tested using specimens with initial inclined cracks. A numerical analysis has allowed the determination of the J-integral and of the strain energy density factor S. S is numerically evaluated around the crack tip. As suggested by Sih in the framework of linear elastic fracture mechanics (LEFM) (Sih [1]), the strain energy density factor seems to be a good indicator of the crack propagation direction of the studied rubber. The numerical predictions are in good agreement with experimental data. The fracture criterion concept has also been discussed. Everything suggests that the critical J and S values combined with the initial inclined angle of the cracks govern the fracture of the SBR.

1 INTRODUCTION

Generally speaking, there are two methods to study the fracture of rubbers : the global and the local energetic approaches. The main advantage of the first method is the fact that the knowledge of the stress and strain distribution through the structure is not required. It is based on the energy balance, written for the whole specimen and leads to the definition of a parameter, noted J and represents the energy needed to extend the pre-existing defect of an unit area A :

$$J = -\left.\frac{dU}{dA}\right|_{u} \tag{1}$$

In the relation (1), U is the potential strain energy (equivalent to the area under the load-displacement curve issued from the uniaxial tensile test). The suffix u indicates that derivation is taken under a constant displacement. The parameter J given by the equation (1) is equivalent to the popular J-integral introduced by Rice[2] and Cherepanov [3]**.** Hence, crack growth will start at some critical value J_c of J.

The global energetic approaches are, however, only used to qualitatively characterise the fracture of materials under simple loadings. Under complex loadings, the local approaches represent a good alternative method that can define the initiation of the crack, the direction of the crack growth and the kinetic of the propagation. Dealing with linear elastic mechanical behaviour, in a combined mode I and mode II loading, Sih has theoretically analysed the strain energy density field near the crack tip (Sih [1]). He has postulated that the direction of crack growth and the fracture toughness are governed by the critical value of the minimum of the strain energy density factor S expressed as :

$$S = W.r \tag{2}$$

where W is the strain energy density and r is the radial distance measured from the crack front (figure 1).

In a recent work (Aït Hocine [4]), we have found that the critical J-integral and the critical S parameter values seem to be a fracture criterions of rubber under simple loading (mode I). Moreover, the strain energy density factor S clearly suggests the crack propagation direction.

Double edge notch in tension (DENT) specimens with several cracks were experimentally and numerically analysed.

In this work, we will verify if these results can be extended to such highly deformable material under a mixed mode (mode I+II). To simulate such a loading, inclined center cracked sheets with varying angles have been chosen.

2 EXPERIMENTAL STUDY

2.1 Materials, specimens, tests

The experiments were carried out using a styrene butadiene rubber (SBR) filled with a carbon black. Such synthetic elastomers show substantially non-linear reversible elasticity and display large deformation.

The constitutive law of the material is required for the numerical analysis. Therefore, the measure of our material's stress-strain relationship is provided by tensile tests on three rectangular unnotched sheetsof 2 mm thickness (length=30mm, width=6mm). A camera that follows the changes in the distance l_0 between two circle marks printed on the surface of the specimen provides the elongation measurements. This camera is connected to a computer in order to record the instantaneous stress-strain data.

The fracture tests were performed using inclined centre cracked specimens of 70 mm width (w), 116 mm length (h) and 2 mm thickness (B) (Figure 1). Four orientation values defined by the angle φ={0°, 15°, 30°, 45°} of the crack (the length of which is a=38.5mm) were analysed. Razor blades were used to create initial cracks.

The loads vs. displacements were recorded up to total breaking. The critical loads and displacements corresponding to crack initiation were recorded for each test using a LCD camera that screens on a computer the moving picture of the crack tip zone. The recorded images clearly show that a pre-existing notch always propagates perpendicularly to the loading direction (figure 2), whatever is the initial crack orientation.

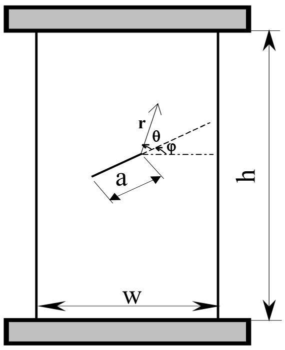


Figure 1: sheet with an inclined centre crack (r and θ are the polar co-ordinates).

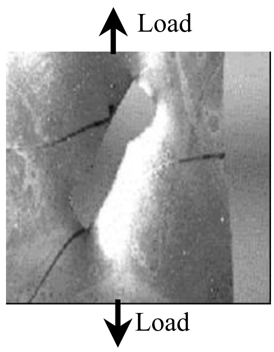


Figure 2: crack propagation on the sheets with an inclined centre crack .

2.2 Constitutive law

The mechanical behaviour of rubbers is commonly described using hyperelastic formalism derived from a strain energy density function W (Oden [5]).

The constitutive-law of the studied material follow the form of the Ogden model developed for nearly incompressible rubber materials [6] :

$$W = \sum_{k=1}^{n} \frac{u_k}{\alpha_k} \left(\lambda_1^{-\alpha_k} + \lambda_2^{-\alpha_k} + \lambda_3^{-\alpha_k} - 3 \right) \tag{3}$$

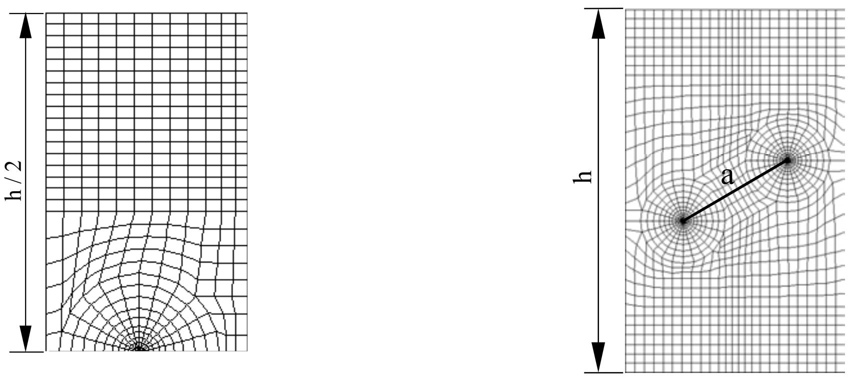
where μ_k and α_k are the material constants obtained from the curve fitting of the experimental data; λ_1, λ_2 and λ_3 are the principal stretches. The constant α_k and the ratio $\frac{\mu_k}{\alpha_k}$, experimentally identified for our material, are reported in the table 1.

K	α_k	$\frac{\mu_k}{\alpha_k}$ (N/m²)
1	-1.7815	238.273
2	1.4787	-11.190
3	1.7427	-250.257

Table 1: Material constants

3 FINITE ELEMENTS ANALYSIS

All the experimental fracture tests above-mentioned were numerically simulated using the MARC finite elements (FE) program.



a) φ = 0 b) φ = 30°

Figure 3 : Grid-works used in the FE analysis

A whole sheet was modelled except when the crack orientation is horizontal (φ=0) where, according to the symmetries, only a quarter of the specimen was considered. Figures 3a and 3b show examples of selected meshes for φ=0 and for φ=30°. These meshes contain triangles with six nodes in the vicinity of the crack tip and quadrilaterals with eight nodes far from the crack. Plane stress and large strains were assumed in the analysis. The Ogden constitutive law (eqn 3) is available in the FE program that only requires the materials constants to be introduced.

The FE calculation was achieved by gradually incrementing the displacements applied to the nodes situated at the top of the specimen, with equilibrium iteration at each step (full Newton-Raphson method).

4. RESULTS – DISCUSSION

4.1 J-integral

The FE J-integral was evaluated using the modified Parks method (Hellen [6]). More details about this procedure can be found in a previous paper (Aït Hocine [4]).

The path independence of the *J*-integral has been pointed out for all studied configurations even when the first ring of elements immediately surrounding the crack tip is concerned. These results confirm the appropriateness of the meshing.

The values of the J-integral corresponding to crack initiation are reported on the figure 4 as a function of the initial crack orientation φ. This graph clearly shows that the critical J_c values increase with φ. Are also reported on this figure the values of the multiplicative parameter "J_c.cos(φ)" that seems to be quite independent on the angle φ. These results mean that, unlike J_c, the parameter "J_c.cos(φ)" could represents a fracture criterion of rubbers under mixed mode.

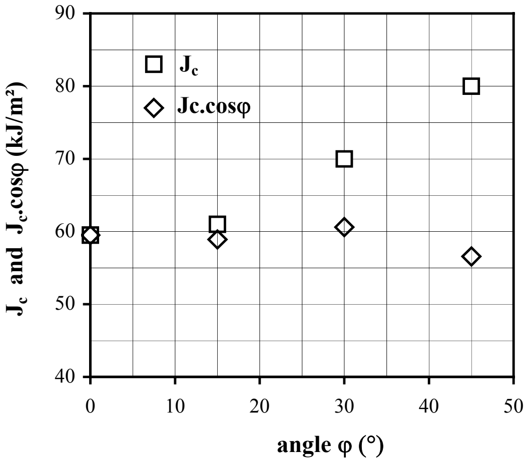


Figure 4 : Critical values J_c and J_c.cosφ against initial orientation angle φ (°) of the cracks.

4.2 Strain energy density factor

According to the work of Sih [1], the strain energy density field was analysed in the vicinity of the notch at different steeps of the loading and more precisely at the onset of crack growth.

Firstly, the influence of the meshing was examined. So, the strain energy density was calculated using four kinds of meshing refined in the vicinity of the crack. It's found that the elements size has a significant influence on the strain energy density values. But, this influence becomes negligible when considering the parameter S (eqn. 2) whatever the crack orientation. Hence, one may conclude that it is better to denote the magnitude of the strain energy density W by the strain energy density factor S as it has been done by Sih in LEFM area (Sih [1]).

The strain energy density W was estiamted around the crack tip using a meshing with element size of 0.5mm in the vicinity of this crack. Then, the factor S has been deduced and its evolution as function of the angle θ is shown in the figure 8 for a given radius r=2.12mm and for the critical displacement values. It is clearly pointed out that, whichever the initial orientation of the crack, the minimum value of S coincides with the axis perpendicular to the loading direction. Although the results are only illustrated in this graph for a particular case, it must be noted that the same trends have been observed when considering other loading levels and other rows of the integration points situated far from the disturbances induced by the singularity. The same trends have been also observed when considering the others values of the initial crack angle. That suggest, according to the assumption of Sih [1] in LEFM, that for highly deformable materials under mixed mode I+II, the crack always propagates in the same direction, independently of the initial crack angle. Such a result is in good agreement with experimental observations (§ 2.1., figure 2).

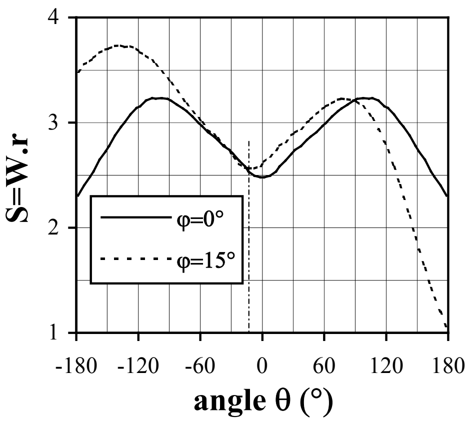


Figure 8 : The factor S as function of the angle θ for a given radius r=2.12mm and for an imposed displacement.

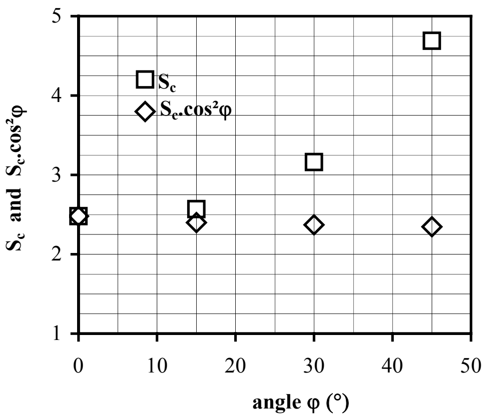


Figure 9 : Critical values S_c and "S_c.cosφ" vs initial orientation angle φ (°) of the cracks.

Then, the minimum S^{mini} of the factor S corresponding to onset of crack growth has been plotted on the figure 9 as a function of the initial orientation φ of the crack. This graph shows that the critical values S_c^{mini} of S^{mini} increase with the angle φ. However, the term "$S_c^{mini}.\cos^2\varphi$" is quite independent on the angle φ. Thus, we can postulate that the crack initiation in the studied material is also governed by the critical parameter "$S_c^{mini}.\cos^2\varphi$". If such a criterion is combined with a crack direction criterion (the minimum of the S factor), then a prediction of crack propagation in more complex loading could be expected.

5. CONCLUSION

In this study, a Styrene Butadiene Rubber (SBR) under mixed mode was analysed. The inclined centre crack specimens were experimentally tested and numerically investigated.

The Fracture tests highlights that pre-existing cracks in sheets of rubber under mixed mode loading seem to always propagate perpendicularly to the direction of the loading.

The factor S (eqn 2) has been numerically evaluated on the integration points around the crack tip, for several loading levels. The minimal value of this factor, that correspond to the probable direction of crack propagation according to Sih [1], seems to be independent of the initial crack orientation. The crack propagation direction is always perpendicular to the loading direction, which is in agreement with the experimental observations.

Finally, it appears that neither the critical value J_c of J nor the critical value $S_c^{min\,i}$ of the strain energy density factor S^{mini} is a fracture criterion of the studied material under mixed mode (I+II). However, the modified terms "$J_c.\cos\varphi$" and "$S_c.\cos^2\varphi$" seem to be independent on the initial crack orientation angle φ. Thus, these last parameters could govern the fracture of rubber under such complex loading.

As an outlook, it will be interesting to extend this study to other kinds of rubber. But, we must previously theoretically the modified terms "$J_c.\cos\varphi$" and "$S_c.\cos^2\varphi$" are able to represent fracture criterions for such materials.

6 REFERENCES

[1] Sih G. C., *Mechanics of Fracture Initiation and Propagation*, Kluwer Academic Publ., Dordrecht, p 428, 1991.

[2] Rice J. R., *A path independent integral and the approximate analysis of strain concentration by notches and cracks*, J. of Applied Mech., 35, 379–386, 1968.

[3] Cherepanov G.P., *The propagation of cracks in a continuous media*, J. Appl. Math. Mech., 31, pp 503-512, 1967.

[4] Aït Hocine, N., Naït Abdelaziz, M. & Imad, A., *Fracture problems of rubbers: J-integral estimation based upon η factors and an investigation on the strain energy density distribution as a local criterion,* International Journal of Fracture, 117, pp. 1–23, 2002.

[5] Oden J. T., *Finite elements of nonlinear continua*, McGraw-Hill Book Company, 1972.

[6] Finite Element Marc program, version k7, Volume A : *Theory and User Information*.

[7] Hellen, T.K., *Virtual crack extension method for non-linear materials,* International Journal for Numerical Methods in Engineering, 28, pp. 929–942, 1989.