MIXED MODE FATIGUE CRACK GROWTH USING THE STRAIN ENERGY DENSITY THEORY FOR WIDESPREAD FATIGUE DAMAGE

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ABSTRACT

"Widespread fatigue damage" is used to describe the occurrence of multiple cracking in structural elements in aging aircraft. These multiple fatigue cracks appear to propagate at an inclined angle, implying that the structural element is subjected to mixed mode loading. In this paper, a methodology to predict mixed mode fatigue crack growth is presented which uses the strain energy density criterion to determine the crack trajectory or angle of crack propagation. A quantity called an effective strain energy density factor is developed, and is applied in an empirical crack growth-rate equation to calculate incremental fatigue crack growth. Predictions are compared with data obtained from experiments conducted on test specimens made from thin sheets of 2024-T3 aluminum with inclined cracks emanating from an open hole.

1 INTRODUCTION

In 1988, an explosive decompression occurred in a commercial transport aircraft when approximately 5.5 meters (18 feet) of the upper crown skin and structure separated from the fuselage while in flight at about 7,320 meters (24,000 feet). The cause of this failure was formation, growth, and linkup of multiple cracks in the upper rivet row of the lap splice joint located near the fuselage crown. Figure 1 is a photograph of multiple site cracking discovered in a fuselage lap splice during an inspection of the fleet following the dramatic structural failure in 1988. This type of multiple site cracking in aging aircraft is now called "widespread fatigue damage." The fatigue cracks appear to propagate at an inclined angle rather than purely horizontal, implying that the fuselage lap joint is subjected to mixed mode loading. The source of the mixed mode loading is assumed to be biaxial stress due to cabin pressurization and transverse shear due to body bending.

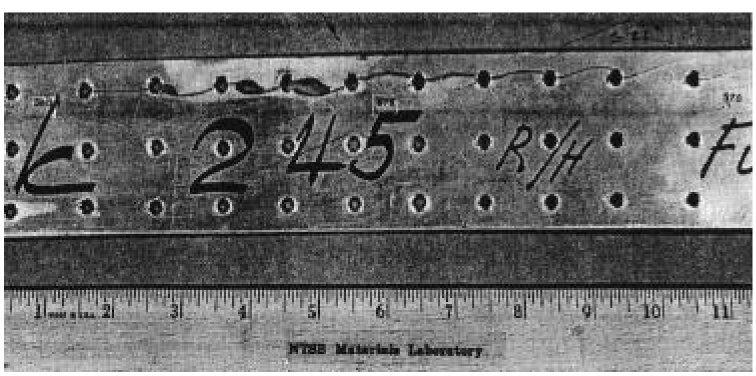


Figure 1. Inclined multiple fatigue cracks in the lap joint of an aircraft fuselage.

In this paper, a methodology to calculate the growth of inclined cracks emanating from open holes is presented. The methodology is based on an engineering approximation of a curved crack modeled as a straight crack of equal stress intensity [1]. The calculated results for crack trajectory and crack growth rate are compared to data obtained from test coupons made from 2024-T3 aluminum.

2 STRAIN ENERGY DENSITY APPROACH TO MIXED MODE FATIGUE

Sih and Barthelemy [1] proposed the following empirical crack growth-rate equation for mixed mode fatigue crack growth calculations:

$$\frac{da}{dN} = B(\Delta S)^m \tag{1}$$

where B and m are empirical constants. For linear elastic material behavior and mixed mode loading in two dimensions, the strain energy density factor range is defined as

$$\Delta S = a_{11}\left(K_{I_{max}}^2 - K_{I_{min}}^2\right) + 2a_{12}\left(K_{I_{max}}K_{II_{max}} - K_{I_{min}}K_{II_{min}}\right) + a_{22}\left(K_{II_{max}}^2 - K_{II_{min}}^2\right) \tag{2}$$

where $K_{I\max}$, $K_{I\min}$, $K_{II\max}$, and $K_{II\min}$ are the maximum and minimum stress intensity factors corresponding to tensile and in-plane shear loading. The auxiliary functions a_{ij} depend on elastic constants (modulus of elasticity, E and Poisson's ratio, n) and a polar angle measured from the crack tip:

$$\begin{aligned}
a_{11} &= \frac{(1+n)}{8p E}(3-4n - \cos q_o)(1+\cos q_o) \\
a_{12} &= \frac{(1+n)}{8p E}2\sin q_o\left[\cos q_o - (1-2n)\right] \\
a_{22} &= \frac{(1+n)}{8p E}\left[4(1-n)(1-\cos q_o) + (3\cos q_o - 1)(1+\cos q_o)\right]
\end{aligned} \tag{3}$$

Moreover, q_0 is the angle of fracture. The elastic constants assumed for 2024-T3 aluminum are listed in Table 1.

Table 1. Mechanical Properties for 2024-T3 Aluminum.

E GPa	n
72.4	0.33

According to the strain energy density criterion [2], the angle of fracture, q_o is assumed to coincide with the value of q that yields the minimum strain energy density factor. This angle also yields maximum dilatation or volume change. The physical interpretation is that dilatation promotes void formation, growth, and coalescence. Mathematically, the fracture angle is determined by differentiating DS with respect to q, and setting the resulting equation equal to zero.

In this paper, a modified version of equation (1) was developed based on the concept of a so-called effective strain energy density factor [3]:

$$\frac{da}{dN} = B\left(\Delta S_{eff}\right)^m \tag{4}$$

The effective strain energy density factor range is defined in this paper as

$$\Delta S_{eff} = \left(\frac{1-R}{1+R}\right)^n \Delta S \tag{5}$$

where R is the stress ratio and n is an empirical constant. The empirical constants in equations (4) and (5) for 2024-T3 aluminum were determined using a least squares regression analysis, and are listed in Table 2.

Table 2. Empirical Constants in Equations (4) and (5) for 2024-T3 Aluminum.

B m-(N-m^{-1})$^{-m}$-cycle^{-1}	m	n
2.430×10^{-6}	1.733	0.604

Figure 2 shows a comparison between the fatigue crack growth-rate data for 2024-T3 aluminum under Mode I loading and equations (4) and (5). An interesting feature of this comparison is that the data points for different stress ratios collapse onto a single curve.

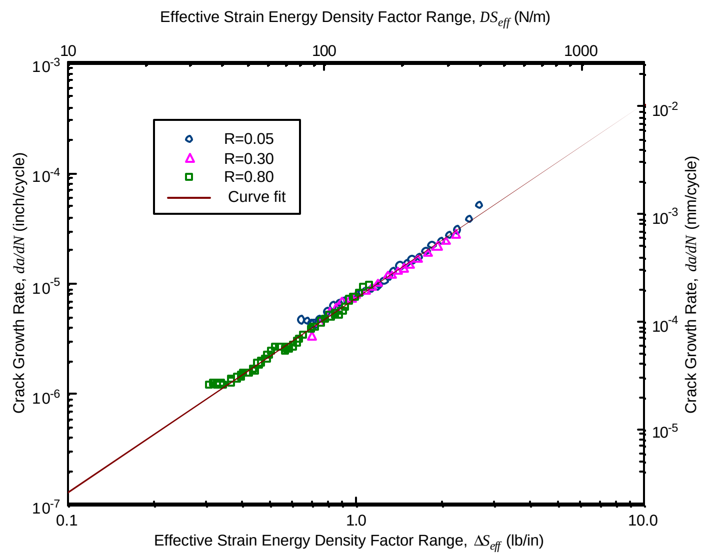


Figure 2. Crack growth-rate data for 2024-T3 aluminum correlated with Equations (4) and (5).

3 APPROXIMATION OF INCREMENTAL CRACK GROWTH

An approximate method, originally developed by Sih and Barthelemy [1], was modified in the present work to estimate incremental growth of inclined cracks emanating from an open hole. In this method, a bent or zigzag crack is modeled as a straight-line crack of equal stress intensity for each increment of growth. This straight-line equivalence for two radial cracks emanating from an open circular hole is shown in Figure 3.

Stress intensity factors for cracks emanating from an open circular hole in an infinite sheet subjected to constant amplitude loading at an arbitrary angle were derived by Hsu [4,5] using a complex variable method.

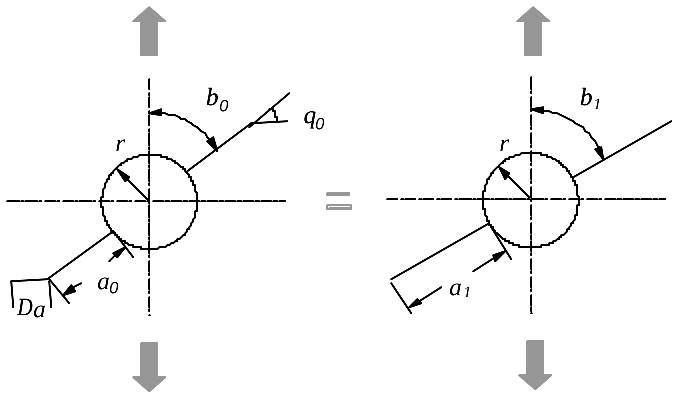


Figure 3. Straight crack approximation of two radial cracks emanating from a circular hole subjected to uniform tension at an arbitrary angle, β.

The following expressions were derived from geometrical considerations, and were used in the analyses to update the crack configuration after each increment of crack growth:

$$b_1 = b_0 + \frac{\Delta a \sin q_0}{\left(a_0 + r\right) + \Delta a \cos q_0} \tag{6}$$

$$a_1 = a_0 + \left[\frac{(a_0 + r)\cos q_0 + \Delta a}{(a_0 + r) + \Delta a \cos q_0}\right]\Delta a \tag{7}$$

In this paper, the increment of crack growth *Da* is calculated from

$$\Delta a = B\left(\Delta S_{eff}\right)^m \Delta N \tag{8}$$

The choice of cycle increment in equation (8) affects the crack-growth calculation. This effect is examined by comparing the approximate incremental method with direct integration of the crack growth-rate equation for the Mode I case. Figure 4 compares the growth rate of two cracks emanating horizontally from an open hole (i.e. Mode I), calculated by the incremental method with different cycle increments and by direct integration of equation (4). Three different values for the cycle increment *DN* were assumed: 500, 1000, and 2500 cycles. The figure shows that the crack growth rate calculated by the approximate method and by direct integration is practically the same up to about 10,000 cycles. The growth rates calculated from the approximate method and from direct integration start to deviate from each other after 10,000 cycles, and the difference becomes larger as the cycle increment increases. Based on these comparisons, a cycle increment of 500 cycles was assumed in subsequent calculations of crack growth for mixed mode loading.

4 COUPON TESTS

Fatigue tests were performed on coupons made from sheets of clad 2024-T3 aluminum, 101.6 mm (4 inches) in width by 1.02 mm (0.04 inch) in thickness. A hole was drilled in the center of each coupon with a diameter of 4.67 mm (3/16 inch). The hole was notched with one or two cracks emanating from its periphery at an inclined angle. Three coupon tests were conducted for this study: (1) a singe crack emanating from an open hole at an inclined angle of about 70 degrees from the vertical and an initial crack length of 4.2 mm (0.16 in.), (2) two radial cracks emanating at an angle of about 45 degrees and an initial crack length of 2.4 mm (0.09 in.), and (3) two radial cracks at an inclined angle of about 70 degrees and an initial crack length of 2.4 mm (0.09 in.).

The fatigue tests were conducted at constant amplitude with a maximum stress of 2.32 MPa (16 ksi) and a stress ratio of 0.1. The frequency of loading was 3 Hertz (or 180 cycles per minute). The grain direction in the test coupons was perpendicular to the direction of applied load. Crack growth was monitored, and measured visually at periodic intervals using a 20x traveling microscope.

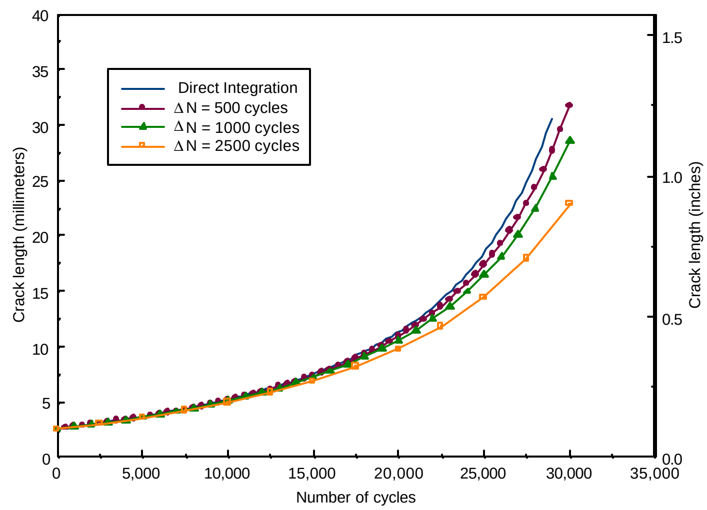
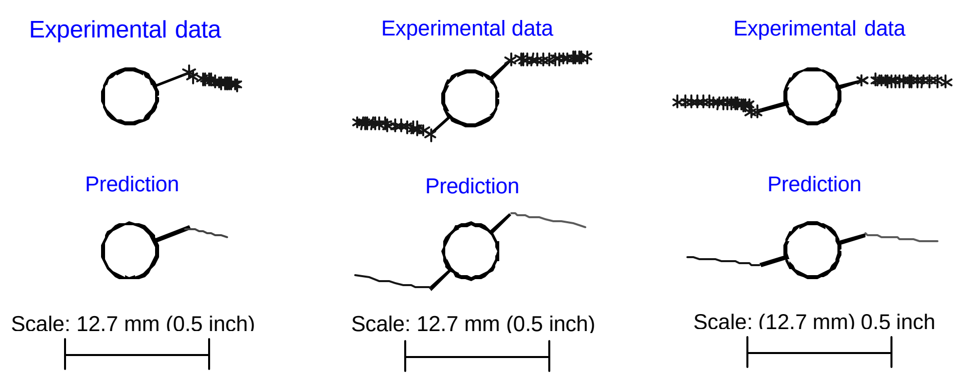


Figure 4. Effect of cycle increment on Mode I crack growth using equation (4).

5 COMPARISON BETWEEN TEST RESULTS AND ANALYSES

Figure 5 compares the measured and calculated crack trajectories for the different coupon tests after a certain number of cycles have been applied. In each case, the calculated trajectories resemble those measured in the coupon tests.



(a) Coupon Test 1 after 25,000 cycles.

(b) Coupon Test 2 after 30,000 cycles

(c) Coupon Test 3 after 20,000 cycles.

Figure 5. Comparison of crack trajectories.

6 CONCLUDING REMARKS

In this paper, a methodology based on the strain energy density approach was developed and applied to calculate mixed mode fatigue crack growth in test coupons made from 2024-T3 aluminum. The predicted crack trajectories were shown to be nearly identical to those observed in the tests.

Test coupons with a single, open hole were analyzed in this paper. In an actual aircraft fuselage, rivets apply bearing loads to the periphery of the hole. The calculation of crack growth in this case becomes more complicated, not only because of the load on the hole, but also because the magnitude of the load may vary as the cracks propagate. In addition, the effect of adjacent holes and cracks was not examined in this paper. Examining the effect of holes carrying rivet loads and the effect of adjacent holes and cracks on mixed mode fatigue crack growth may be considered in future research.

An effective strain energy density factor was developed in this paper, which was then applied to calculate crack propagation using an empirical fatigue crack-growth equation. This effective quantity was assumed to be a function of the Mode I and Mode II stress intensity factors. Since stress intensity factors are based on linear elasticity, their application to the irreversible effects of damage accumulation is inherently limited. Ultimately, however, more accurate calculations of fatigue crack propagation may be achieved if the strain energy density factor can be determined from a theory that properly accounts for damage accumulation and irreversibility [6,7].

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