STABLE EXTENSION UNDER NET SECTION YIELDING FOR 2024-T3 ALUMINUM: APPLICATION OF STRAIN ENERGY DENSITY

D.Y. Jeong[1]

[1] U.S. Department of Transportation, Volpe National Transportation Systems Center, Cambridge, Massachusetts 02142 USA

ABSTRACT

The term "widespread fatigue damage" refers to the occurrence of multiple fatigue cracking in structural elements in aging airplanes. Collinear cracks emanating from open holes are used to represent widespread fatigue damage in test specimens made from thin sheets of 2024-T3 aluminum. Residual strength tests conducted on such test specimens reveal that stable tearing occurred before failure. In this paper, the strain energy density criterion is used to predict stable crack extension and failure by plastic collapse. The nonlinear stress field in the test specimens is determined from finite element analyses assuming the incremental theory of plasticity. Reasonable agreement between the experimental data and predicted results was achieved using this approach.

1 INTRODUCTION

Fail-safe airframe designs are expected to arrest certain isolated large fractures that may initiate from fatigue and sudden damage (for example, penetration of a fuselage by a propeller blade). For the pressurized fuselage structure in commercial transport airplanes, this crack arrest capability must be demonstrated in order to comply with U.S. certification standards (14 CFR 25.271) [1].

Conventional design of a damage-tolerant fuselage structure is based on thin-skin fracture resistance, as characterized by the K_R curve [2]. For design purposes, fracture resistance for a given skin thickness, as expressed in terms of stress intensity K_R, is implicitly assumed to be a unique function of crack extension independent of initial crack length. Estimation of critical crack length for fracture initiation then requires only calculation of the applied stress intensity factor, K_A, and a graphical construction like the one shown in Figure 1. However, the in-flight failure of a jet transport fuselage in 1988 (Figure 2) dramatically demonstrated that the structure's crack arrest capability can be compromised by the formation, growth, and linking of multiple fatigue cracks along a fastener row [3]. Such multiple fatigue cracking is now referred to as widespread fatigue damage or multiple site damage.

The conditions in both the *R*-curve test and the airframe with a long isolated crack are such that the remaining ligaments along the crack line behave elastically during stable crack extension. Conversely, in the case of multiple site fatigue cracking, the remaining ligaments undergo plastic yielding. The *R*-curve method, however, is not valid when plastic collapse controls the stability limit, e.g., under multiple site damage conditions. Moreover, rough estimates for critical crack length under these conditions can be made by means of plastic collapse models, but such models do not account for stable crack extension.

Recent experiments on open-hole tensile coupons with hole diameter and spacing similar to typical aircraft construction details have shown that stable crack extension does indeed occur under multiple site damage conditions [4]. In this paper, a material mechanics model based on the strain energy density criterion is developed to account for stable crack extension when the remaining ligaments have already entered the plastic deformation regime.

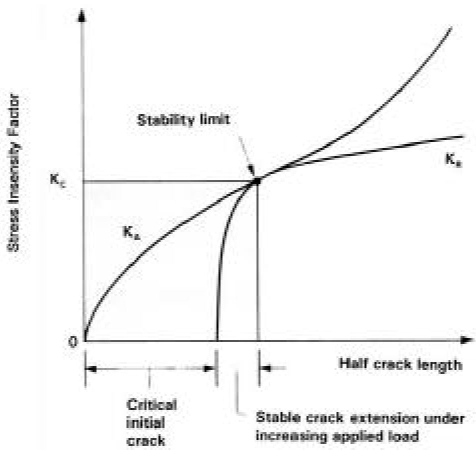

Figure 1. *R*-curve estimation of critical crack length.

Figure 2. N73711 fuselage failure.

2 STRAIN ENERGY DENSITY APPROACH

The strain energy density approach was originally developed as a rational approach to linear elastic fracture mechanics (LEFM) under mixed mode conditions [5], but was later found to be applicable to ductile fracture as well [6,7].

A property of the strain energy density function is that, for the two-dimensional crack problems, it may be expressed in terms of polar coordinates (r, q) with the origin at the crack tip:

$$U = \frac{S(q)}{r} \tag{1}$$

where S is defined as the strain energy density factor, a quantity independent of r.

According to the strain energy density criterion, the minimum S or U corresponds to the expected angle trajectory for crack extension. This angle also yields maximum dilatation or volume change. The physical argument in support of this interpretation is that the dilatation fosters void formation, and coalescence along that trajectory. Although the original criterion was developed for linearly elastic isotropic material behavior, the criterion can be extended to ductile fracture since the $1/r$ singularity of the strain energy density function is independent of the constitutive equation.

The strain energy density criterion includes hypotheses for crack extension and a stability limit based on two physical parameters: the critical strain energy density U_c and the critical strain energy density factor S_c. The first parameter is obtained from basic material properties, while the second depends on structure geometry, crack size, and load at the point of fracture instability.

The critical strain energy density is defined as the maximum energy density that a material can absorb before the onset of local plastic failure (micro-ligament necking and void coalescence). For elastic bodies, the hypothesis is that a macro-crack will extend radially, along the expected trajectory, as far as there exist material points at which the applied-load strain energy density exceeds U_c. The value of U_c is estimated as the area under the tensile stress-strain curve, based on the conventional hypothesis that this curve also represents the relation between equivalent plastic stress and strain. For elastic-plastic bodies, the consumption of energy at points inside the plastic zone, before crack extension, must be accounted for. Figure 3 illustrates an example, in which the state at a given material point in the plastic zone is represented by the point P on the equivalent stress-strain curve. The energy *U(P)* has already been consumed, therefore, the energy available to extend the crack is equal to $U_c - U(P)$. Moreover, the amount of stable crack extension is determined by the intersection of the strain energy density curve (with the $1/r$ singularity) and the available energy curve.

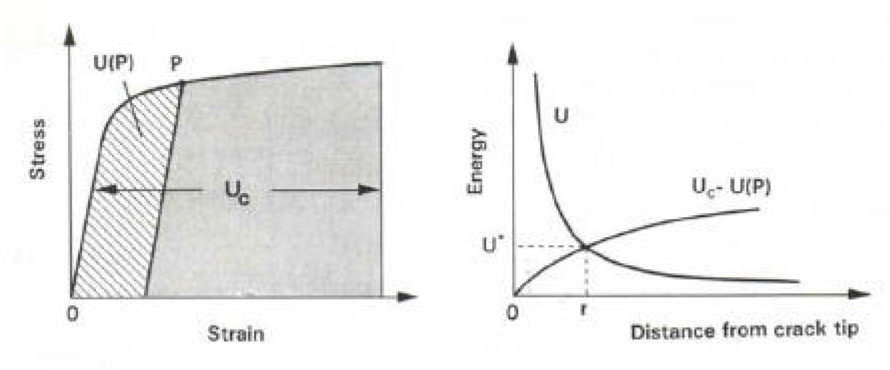

Figure 3. Strain energy density criterion for stable crack extension.

3 APPLICATION

The strain energy density approach was applied to examine the tensile coupon experiments mentioned in the introduction [4].

The coupons consisted of clad 2024-T3 aluminum sheet, 4 in. (101.6 mm) wide by 0.04 in. (1.02 mm) thick, with one row of three drilled holes 3/16 in. (4.76 mm) in diameter and 1.0 in. (25.4 mm) on center. The central hole was symmetrically notched, and fatigue loads were applied to produce sharpened cracks. The inner edges of the outside holes were then notched to produce a final configuration for the experiment, such that the cracks and notches all had approximately the same initial length a_0 in a given coupon (Figure 4). Coupons with a_0 = 0.15, 0.22 and 0.26 in. (3.81, 5.59, and 6.60 mm) were tested.

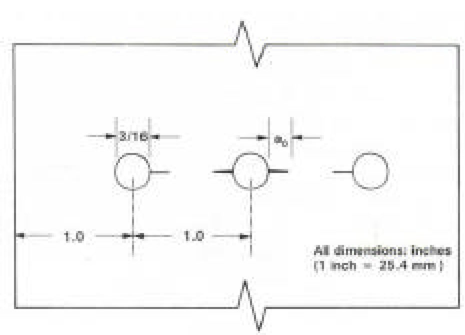

Figure 4. Initial configuration of test coupon.

The objective of the experiment was to measure the crack extension resistance curves analogous to the *R*-curves that characterize long crack behavior. Each coupon was loaded under quasi-static conditions while the extensions of the central cracks were visually monitored through a 20x traveling microscope. Individual extensions Da_L and Da_R were recorded for the left and right cracks, but the average extension; i.e. ½ ($Da_L + Da_R$); is used for comparison with the strain energy density analysis.

Elastic-plastic finite element analysis was carried out on a plane-stress quarter-symmetric model of the coupon, using bi-cubic isoparametric displacement elements in the grid shown in Figure 5, in accordance with the following approximate procedure. The finite element grid was configured to match the initial crack length a_0, and was loaded by a total nominal stress corresponding to the first experimental data point. The two mid-side nodes of the isoparametric elements around the tip of the crack were placed at the one-ninth and four-ninths distances from the crack tip to preserve the $1/r$ character of the strain energy density function [8], see Figure 5(c). The strain energy density factor S was estimated from the numerical results for U, and the corresponding cumulative crack extension was calculated based on the schematic shown in Figure 3. The applied stress was then increased, the mesh updated to the new crack length, and the procedure was repeated until ill-conditioning error intervened.

The results of the strain energy density analysis are compared with the experimental results in Figure 6. Solid and open symbols represent the analytical and experimental data points, respectively. Lines between the data points have been included for visual guidance but have no physical or numerical meaning. The calculated crack extensions appear to be agreement with the experimental observations. The calculations indicate the level of the resistance curve asymptotes but are not able to precisely track the asymptotes as the plastic collapse state is approached.

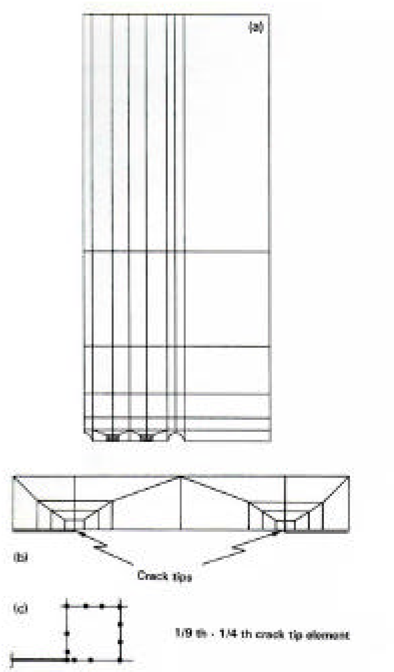

Figure 5. Quarter-symmetric model of test coupon. (a) Full-model. (b) Detail of region between holes. (c) Collapsed isoparametric element at crack tip.

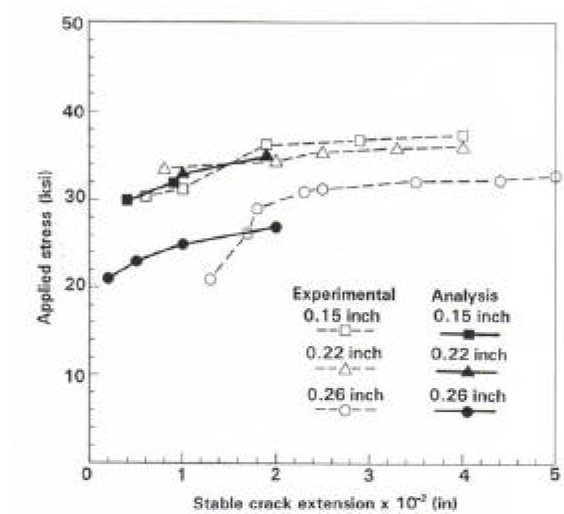

Figure 6. Comparison of results.

4 DISCUSSION

The agreement between the analytical and experimental crack extension was found to be reasonable but not precise. One obvious reason for the lack of better agreement is the approximate procedure used in the elastic-plastic finite element analysis. Although the complications of an incremental analysis were avoided, the approximate procedure must be expected to produce increasing error as the magnitude of the finite crack extensions increases.

Another reason is the presence of some experimental error in the crack extension measurements. The microscope resolution limit mentioned earlier represents a band of uncertainty amounting to 25 to 50% of the measured extensions for which calculations could be made (see Figure 6). Therefore, no final judgement about the accuracy of the strain energy density analysis can be made from the present comparison of results.

Another perspective on the strain energy density analysis is gained by comparison with the net section theory. The net section criterion is a widely accepted conventional method for estimating the strength of mechanical joints under plastic collapse conditions. The critical nominal stress is calculated as:

$$s_{CR} = \frac{A_{net}}{A} f_{tf} \tag{2}$$

where A is the nominal cross-sectional area, A_{net} is the net section area (nominal area less the area occupied by holes, cracks, and notches), and f_{tf} is an assumed material property called the flow stress.

Equation (2) represents the physical hypothesis that the stress distribution is uniform tension across the net section at the collapse state. The hypothesis is justified by the known tendency of plastic deformation to smooth out uneven stress distributions along fastener rows in skins. No universally accepted rule exists to define flow stress, which is generally assumed to lie somewhere between the yield strength f_{ty} and the ultimate tensile strength f_{tu}.

The net section strength estimates for coupons with different initial crack lengths a_0 are summarized in Table 1. Comparison with Figure 6 shows that the strain energy density analysis produces essentially the same strength estimates as the net section criterion with $f_{tf} = f_{ty}$.

Table 1. Net section strength estimates.

a_0 in. (mm)	s_{CR} ksi (MPa)		
	Based on f_{ty} = 49.8 ksi (343 MPa)	Based on f_{tu} = 67.2 ksi (463 MPa)	Experiment
0.15 (3.8)	35 (241)	48 (331)	37 (258)
0.22 (5.6)	32 (220)	43 (295)	36 (248)
0.26 (6.6)	30 (207)	40 (277)	33 (226)

On one hand, the preceding comparison is encouraging because the strain energy density analysis produces results in good agreement with a widely accepted conventional analysis method. On the other hand, why make the extra effort to carry out a strain energy density analysis? The answer is that multiple site damage in actual structures often propagates under mixed mode conditions. The net section criterion cannot be applied to such cases because there is no obvious way to define the net section.

The jet transport fuselage failure mentioned in the introduction provides a good example. This failure occurred along a splice at mid-fuselage height, where the skin is subjected to combined biaxial pressurization and transverse shear due to body bending. Multiple site cracks still in the slow propagation stage found at similar locations elsewhere on this fuselage were oriented at about 20 degrees from the spline line (Figure 7). Inspections of other aircraft following this accident revealed some multiple site cracks, at a later point in the slow growth stage, with curved trajectories like the schematic in Figure 8.

5 CONCLUSIONS

Within the limits of experimental and numerical error, the strain energy density analysis appears to provide reasonable predictions of the stable extension observed in Mode I cracks similar to multiple site cracks.

The predictions of crack extension are limited by the capability of the numerical stress analysis. In the present case, the numerical analysis could not provide valid results for crack extensions which closely approached the stability limit. Nevertheless, the crack extension which was predicted was sufficient to allow estimates of the critical applied stress asymptote.

The stress asymptote estimates were found to be in good agreement with estimates made from the net section failure criterion when the material yield strength was used to represent the net section flow stress.

Strain energy density analysis is thus a logical and credible approach to damage tolerance evaluation of multiple site cracking in actual structures where mixed mode conditions prevail, and where the stability limit may be either controlled by plastic collapse or on the borderline between plastic collapse and ductile fracture.

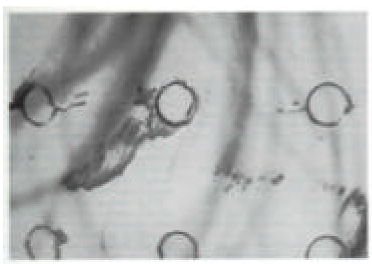

Figure 7. Inclined multiple site fatigue cracks in failed fuselage.

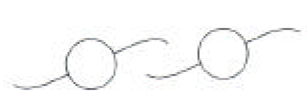

Figure 8. Typical trajectories of multiple fatigue cracks.

REFERENCES

[1] Swift, T., Damage tolerance in pressurized fuselages, *14th ICAF Symp. on New Materials and Fatigue Resistant Aircraft Design*, International Committee on Aeronautical Fatigue, Ottawa, Canada, 1987.

[2] McCabe, D.E. (ed.), *Fracture Toughness Evaluation by R-Curve Methods*, ASTM STP 527 (American Society for Testing and Materials, Philadelphia, PA, 1973).

[3] National Transportation Safety Board, Aircraft accident report – Aloha Airlines, Flight 243, Boeing 737-200, N73711, near Maui, Hawaii, April 28, 1988, NTSB/AAR-89/03, PB89-910404 (1989).

[4] Samavedam, G., and Hoadley, D., Fracture and fatigue strength evaluation of multiple site damaged aircraft fuselages – curved panel testing and analysis, Final report, DOT/FAA/CT-94/10 (1994).

[5] Sih, G.C., Strain energy density factor applied to mixed mode crack problems, *International Journal of Fracture* 10, 305-321 (1972).

[6] Sih, G.C, and Madenci, E., Fracture initiation under gross yielding, strain energy density criterion, *Engineering Fracture Mechanics* 18, 667-677 (1983).

[7] Sih, G.C., and Madenci, E. Crack growth resistance characterized by the strain energy density function, *Engineering Fracture Mechanics* 18, 1159-1171 (1983).

[8] Pu, S.L., Hussain, M.A., and Lorensen, The collapsed cubic isoparametric element as a singular element for crack problems, *International Journal for Numerical Methods in Engineering* 12, 1727-1742 (1978).