MIXED MODE ENERGY RELEASE ANALYSIS BY A BEAM THEORY APPLIED TO TIMBER BEAMS WITH A HOLE

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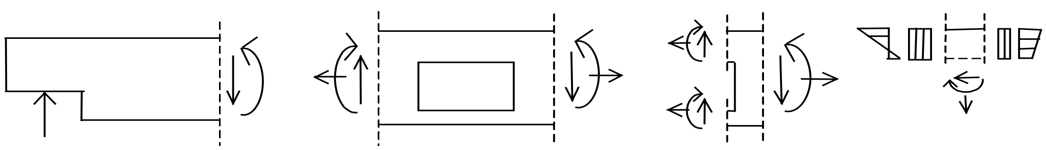
ABSTRACT

The strength of timber structural elements as governed by perpendicular to grain tensile and shear fracture can be rationally analysed using different basic types of material models, Table 1. The applicability of the various models is differently restricted. To enable analysis of elements with a stress singularity or local high stress region consideration to non-zero fracture energy is needed. Empirical strength equations are in codes of timber engineering practice still dominating in relation to perpendicular to grain fracture. The method discussed in this contribution may for timber beams with a stress singularity facilitate the use of rational strength analysis in engineering design. The example of a high stress region considered is that of the corner of a rectangular hole, Figre 1.

Table 1. Methods of strength analysis. G_f denotes fracture energy and f_t material strength.

	Deterministic		Stochastic	
	f_t finite	$f_t \rightarrow \infty$	f_t finite	$f_t \rightarrow \infty$
$G_f = 0$	Conventional stress criteria	--	Weibull weakest link model	--
$G_f \neq 0$	Non-linear fracture mechanics	Linear elastic fracture mechanics	Probabilistic non-linear fract. mech	Probabilistic linear fract.mech.

Figure 1. An end-notched beam, a beam with a hole, a part of infinitesimal length and its upper part.



Simple equations for the energy release rate during crack extension and the corresponding strength of end-notched beams derived by compliance analysis using conventional beam theory were presented and applied to timber beams about 15 years ago. This linear elastic fracture mechanics theory based strength equation is now used in timber engineering codes of practice. Use of the same kind of approach to beams with a hole may seem close at hand but has proved to be more difficult both in relation to engineering analogies with end-notched beams and more basic beam theory studies. This is due to: (1) significant influence of shear makes division of the total energy release rate into modes 1 and 2 necessary, (2) normal force acting on the cross section must be considered and (3) the cross sectional forces and moments acting on the parts above and below the hole are statically indeterminate. Issue (1) is of a basic nature and issues (2) and (3) makes the calculations more comprehensive. An infinitesimally short part of the beam at the end of the hole is considered. The horizontal and vertical forces and the bending moment acting across an infinitesimal horizontal section along the beam part can be calculated by the equations of equilibrium. The energy release rates for modes 1 and 2 can then be obtained by using the method of work of crack closure calculation with consideration to the deformations of the infinitesimal parts below and above the horizontal section. The vertical force contributes to mode 1, the horizontal force to mode 2 and the moment influences both modes. The strength of the beam is then found by using a mixed mode fracture criterion, e.g. the Wu criterion. The accuracy of the beam theory energy release analysis is evaluated by finite element plan stress calculations. Predicted beam strengths are compared to experimental results obtained for glulam timber beams with holes of various size and shape. The general case gives an extensive strength equation, but for various special cases of engineering interest are more user-friendly equations found.