FATIGUE DAMAGE ACCUMULATION IN UNIDIRECTIONAL COMPOSITE (UD) UNDER APPLIED CYCLING TENSION LOAD

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ABSTRACT

In present report a stochastic fiber break clusters accumulation model for polymer matrix UD composites, subjected to tension-tension fatigue, is under consideration. A stochastic kinetic equation, leads to the development of closed-form analytical solutions for probabilities to obtain adjacent fiber breaks of a particular configuration in a composite material. The chain-of-bundles material model is described. Weibull type function is used to characterize a fatigue lifetime of fiber element. Single fiber fatigue fracture tests were performed with a goal to obtain fiber element lifetime probability function parameters. The continuum damage mechanics with internal state variables was used to describe the constitutive behavior of the composite with damage. The internal state variable that accounts for considered damage is formulated. FEM analysis and fiber breaks accumulation model was incorporated into the formulated constitutive law with damage in order to predict the stiffness loss due to the damage. Theoretical predictions for stiffness degradation due to damage accumulation in fatigue were compared with experimental data for glass fiber and carbon fiber composites.

1. INTRODUCTION

The use of UD composites in a load bearing structures leads to a necessity for investigation of their strength and fatigue properties. For material, loaded by tension-tension, fatigue life diagram consists of three distinct regions. One of them is characterized by a disperse fiber breaks accumulation transforming to macro crack and a final failure. This process has a stochastic nature. The result's scale is dependent on what kind of mechanical behavior are shown by matrix and fibers, was combined in material. For polymer matrix composite, traditionally, fiber stiffness is many times higher than matrix stiffness. At first stage, we appropriate that during such material loading, matrix exhibits close to linear elastic behavior, with possibility to deviate near the point of bulk material failure. Local plasticity and fiber matrix delamination, as well as polymer material viscoelastic properties are changing diffuse damage accumulation kinetics and can play an important role at the final stage of loading and must be described in future.

2. DAMAGE ACCUMULATION

Let consider the unidirectional composite under tension-tension load, applied in fiber direction. Maximal tensile stress (during one cycle) is lower then material strength. Such way loaded sample can carry external force and fail only after hours. Nor matrix material, nor fiber or fiber –matrix interface have internal cracks or fiber- matrix delamination if it will not be noted. UD fibers in cross-section are forming the hexagonal array. We designate the length as L and the number of fibers in specimen as n. Traditionally, the role of fibers in material is to carry an external tension load, the role of matrix is to re-distribute overstress from a place of fiber brake to adjacent fibers and along broken fiber from the place of failure. The length of the area of stress re-distribution along broken fiber δ_f is a basic geometrical parameter to present material as the *chain of bundles*. That's mean the specimen's material is partitioned into a series of m bundles with the bundle length δ_f and *n* fibers in bundle. Composite loading by tension in fiber direction leads to diffuse fiber breakage accumulation. Isolated fiber break is associated with one element failure in community of $n \times m$ elements. All elements have the same dimensions, geometrically each element involve fiber, surrounded by matrix (see Figure 1).

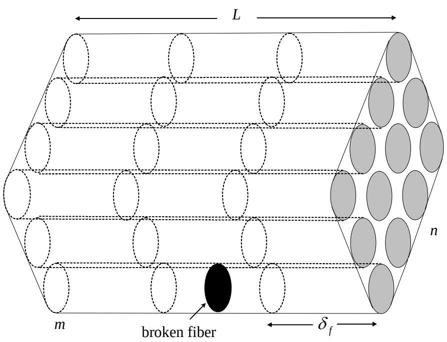

Figure 1. Chain of bundles model. δ_f -element length; *n* – elements number in each bundle; *m* –number of bundles in material.

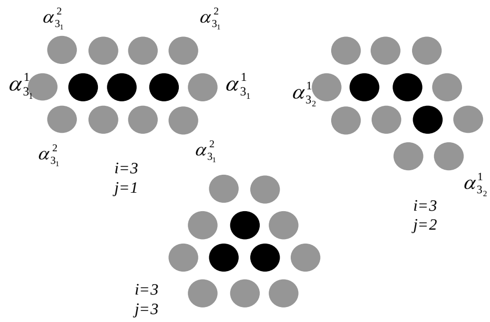

Figure 2. Configurations of $i = 1,2$ adjacent broken fibers. $\alpha_{i_k}^j$ - designation for overloaded nearest fibers.

Modeling of damage accumulation in composite material, under applied tension-tensile load is based on the assumption, that failure is a complex stochastic process starting with scattered, isolated fiber brakes (at fiber's internal flow or, structure heterogeneity) overstress redistribution on adjacent to broken fibers, failure of overstressed neighbors, forming breaks clusters and the breaks clusters growth and coalescence in the range of each bundle, orthogonally to the fiber direction. This process starting relatively slowly will transforms to catastrophic ultimate cluster growth, when overstress distributed on closest unbroken neighbors will immediately initiate one of the next fiber break. We can introduce two random variables $I\left(\sigma_a, N_f\right)$ and $J\left(\sigma_a, N_f\right)$ with joint probability function $P\left(I\left(\sigma_a, N_f\right) = i, J\left(\sigma_a, N_f\right) = j\right) = H_i^j\left(\sigma_a, N_f\right)$, $\sigma_a \geq 0, N_f \geq 0$, $\sigma_a = \sigma_{\max}, \sigma_{\min} \to +0$, to find cluster which was born and after that was stayed conservative till number of cycles N_f, as the cluster consisting of i adjacent fiber breaks and having a form number *j*. $W(\sigma_a, N_f)$ is a probability of fiber failure. These function parameters are obtained from a single fiber fatigue fracture test. For example three adjacent breaks, in material with a hexagonal fiber array, may form three different geometrical configurations (see Figure 2.). If the geometrical configuration of three adjacent breaks is forming a line, we are designating it like a cluster with a form number $j = 1$. Cluster with the geometrical configuration forming curved line will have the form number two ($j = 2$) and cluster with compact configuration - number three ($j = 3$). Chance for double break cluster to grow and to form tree break cluster with the form index equal to one, is the chance to fail one of two overstressed fibers belonging to the set of closest neighbors around a two broken fibers. Here we must to note that overstress distributed on adjacent fibers α_2^k, $k = 1,2,3$ coming from two breaks is different for fibers around broken and depends on k. Two break cluster growth to three breaks cluster's set with different geometry, will happen by fiber element failure under different overstresses. Probability to obtain cluster, consists of r adjacent fiber breaks and have form number j, is:

$$H_r^j(\sigma_a, N) =$$

$$= \sum_{l:(r-1)i+\alpha_{(r-1)i}^k \to r_j} \int_0^N \frac{dH_{r-1}^l(\sigma_a, N^*)}{dN^*} m_{(r-1)_l}^k W\left(k_{(r-1)l}^k \sigma_a, N - N^*\right) \times$$

$$\times \left(1 - W\left(k_{(r-1)_l}^k \sigma_a, N - N^*\right)\right)^{m_{(r-1)_i}^k - 1} dN^* \qquad (1)$$

These are kinetic equations, in recurrent form, for stable states integral probabilities. Using non-effective length like the governing parameter for material partition we obtain a large number of fiber elements in any real mechanically loaded composite volume. That's mean we are interesting in a small clusters probabilities or in a lower tail for probabilities distributions. Looking on obtained formulae we can conclude that all clusters probabilities $H_r^j(\sigma_a, N_f)$ can be represented multiplying function dependant only on stress and geometry by function dependent on number of cycles $H_r^j(\sigma_a, N_f) = H_r^j(\sigma_a) \cdot H_r(N_f)$.

3. DEGRADATION OF MECHANICAL PROPERTIES

The thermodynamically consistent constitutive relationships for media with cracks can be written in form as

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + I_{ijkl}^{\xi}\beta_{kl}^{\xi}, \qquad (2)$$

where $I_{ijkl}^{\xi} = I_{ijkl}^{\xi}(H_i^j)$ is damage dependent modulus, and $\xi = 1,...,\Omega$ accounts for different damage modes. The locally averaged internal state variables associated with energy dissipation due to the cracking is defined as [1,2]

$$\beta_{kl} = \frac{1}{V}\int_{S_2} u_k n_l dS, \qquad (3)$$

where β_{kl} are components of internal state variable tensor, V is a local volume in which statistical homogeneity can be assumed, u_k and n_l are crack face displacement and normal respectively, S_2 is a crack surface area.

The constitutive relationships for material with damage can be obtained from (2) and written in the matrix form

$$\{\psi\} = [A]\{\varepsilon\} + [B]\{k\} + \{f^{(1)}\}, \tag{4}$$

where $[A]$ and $[B]$ are extensional stiffness matrix, and the coupling stiffness matrix of the specimen. The effect of the fiber fracture that takes place due to the loading in fiber direction is described by $\{f^{(1)}\} = F([I^{(1)}], \{\beta^{(1)}\})$, where $[I^{(1)}]$, is a damage stiffness tensor, and $\{\beta^{(1)}\}$ is internal state variable accounting for fiber fracture damage. Generally, it can be determined experimentally, or calculated using micromechanics approach. The measurements of stiffness degradation of UD laminate are used in order to determine $\{\beta^{(1)}\}$ experimentally.

4. DETERMINATION OF DAMAGE TENSOR $\{\beta^{(1)}\}$

The progressive fiber fracture can be described as fiber failure in a representative unit element as illustrated in Figure 3. Debonding effects and matrix cracking effects can be accounted in the considered unit element changing the crack opening displacement.

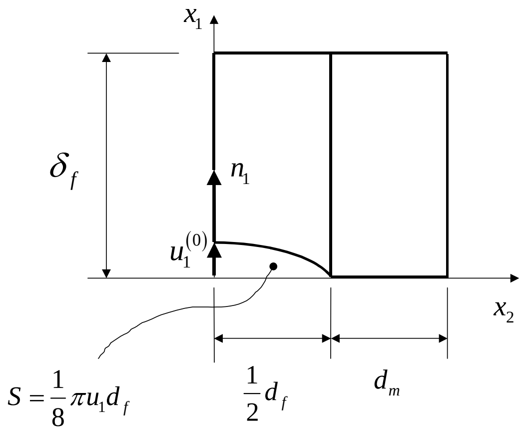

Figure 3. Broken fiber displacement in the repeating unit element (for single fiber break). d_f is a diameter of the fiber, d_m is a dimension of the surrounding matrix, and S is ¼ of the area of the ellipse under the crack profile line.

The general definition of internal state variable β_{kl}, expression (3), can be formulated for considered damage as

$$\beta_{kl}^{(1)} = \frac{1}{V_i}\sum_{m=1}^{M}\int_{S_2} u_k^i n_l dS \tag{5}$$

where S_2 is crack surface area, V_i is repeating unit element for a cluster consisting of *i* adjacent broken fibers. If the crack (see Figure 2), does not change the orientation of the normal we have only one β_{kl} component. It leads to the $\beta_{kl}^{(1)} = \{\beta_{11}^{(1)} 0 \quad 0 \quad 0 \quad 0 \quad 0 \, \beta_{31}^{(1)} 0 \quad \beta_{21}^{(1)}\}^T$, where the $\beta_{11}^{(1)}$ accounts for crack opening in Mode-I, and according to (5), can be expressed for a single fiber break in form

$$\beta_{11}^{(1)} = \frac{1}{8}\frac{m\sqrt{V_f}}{2M\delta_f k}\pi \, u_1^1 . \tag{6}$$

The dimensions may be expressed through the fiber volume fraction V_f and fiber ineffective length, δ_f. The coefficient $k = 0.8244$ yields to form geometry of the hexagonal fiber packing. Both, ineffective length, and number of cracks per unit fiber length as a function of applied strain, can be estimated experimentally from the Single Fiber Fatigue Fragmentation Test (SFFT). On the other hand $H_i^j(\sigma_a, N_f)$ statistics is used for generating the crack density function. FEM can be used to determine ineffective length and crack opening displacements.

5. EXPERIMENTAL CONSIDERATIONS

Let's consider the unidirectional laminate with fiber orientation in loading direction, subjected to fatigue loading $\{\psi(N_f)\} = \{\psi_x(N_f) \quad 0 \quad 0\}^T$,. There is only one type of damage expected, and it can be described with the internal state variable $\beta_{11}^{(1)}$. Further, the constitutive relationships for the considered material can be written in a matrix form as $\{\psi\} = [A]\{\varepsilon\} + [I^{(1)}]\{\beta^{(1)}\}h$. Considering loading conditions, and using $-C_{ijkl} = I_{ijkl}$ [1,2], and $A_{ij} = C_{ij}h$, matrix equation can be reduced to, $\psi_1 = A_{11}\varepsilon_1 + A_{12}\varepsilon_2 + C_{11}\beta_{11}^{(1)}h$. Further, introducing the definition of the stiffness, $E_1(\varepsilon) \equiv \frac{1}{h}\frac{\partial\psi_1}{\partial\varepsilon_1}$, (E_1 is averaged over the loading cycle) and using the expression for ψ_1 in it, the relationship for $\beta_{11}^{(1)}$ as function of measured stiffness for particular applied strain level is obtained,

$$\beta_{11}^{(1)}(\varepsilon_1, N_f) = \frac{h\int_{\varepsilon_1} E_1(\varepsilon_1, N_f)d\varepsilon_1 + \frac{A_{12}A_{21}}{A_{22}}\varepsilon_1 - A_{11}\varepsilon_1}{\frac{A_{12}A_{21}}{A_{22}} - A_{11}} . \tag{7}$$

Calculations were made, and described approach was realized for carbon fiber epoxy and glass fiber polyester composite.

6. CONCLUSIONS

Comprehensive fiber breaks accumulation modeling was performed for UD composite under fatigue. The fatigue SFF tests were carried out to obtain fiber element fracture parameters. The ISV approach was used to obtain stiffness degradation of UD composite during the damage accumulation. The methodology is proposed how to measure the considered ISV experimentally. Theoretically calculated and experimentally measured values of the ISV are compared.

ACKNOWLEDGMENT

Part of the work is done within frames of the EU 5th frame program, OPTIMAT BLADES, Contract Nr. ENK 6-CT-2001-00552. It is also a subject of the Nordic Co-operation Network, supported by Nordic Council of Ministers. Contract j.nr. 2003-8003-23-Modris Megnis, DK.

REFERENCE

1. Allen, D.H., Harris C.E., and Groves, S.E. (1987a): "A Thermodynamical Constitutive Theory for Elastic Composites With Distributed Damage-I. Theoretical Development" *Int. J. Solids Structures* **Vol. 23, No. 9**, pp. 1301-1318.

2. Allen, D.H., Harris C.E., and Groves, S.E. (1987b): "A Thermodynamical Constitutive Theory for Elastic Composites With Distributed Damage-II. Theoretical Development" *Int. J. Solids Structures* **Vol. 23, No. 9**, pp. 1319-1338.