INSTABILITIES OF DYNAMIC CRACK USING A PHASE FIELD MODEL OF BRITTLE FRACTURE UNDER INPLANE LOADING

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ABSTRACT
We present a phase field model of the propagation of fracture under plane strain. This model, based on simple physical considerations, is able to accurately reproduce the different behavior of cracks (the principle of local symmetry, the Griffith and Irwin criteria, and mode-I branching). In addition, we test our model against recent experimental findings showing the presence of oscillating cracks under bi-axial load. Our model again reproduces well observed supercritical Hopf bifurcation, and is therefore the first simulation which does so.

1 INTRODUCTION

The current engineering approach of crack propagation relies on the coupling of linear elasticity with some empirical laws of the crack tip motion based on an energy balance. This kind of description of crack propagation cannot describe the richness of dynamic crack behaviors[1]. One of the most striking examples of this, is the branching instability. Indeed, the classical theory of crack propagation only describes a single crack and in order to determine the behavior of a crack separating into two other cracks one needs to introduce branching through phenomenological laws. Therefore, it appears that modeling such phenomenon, in the framework of classical fracture theory, imposes to add further criterion to describe the tip motion and its splitting into two crack tips. Such an approach is unsatisfactory from the physicist point of view.

Based on the work of Karma*et al* [2] we present a phase field model of brittle fracture under general in plane loading. This approach based on a continuous description of bond breaking in the process zone allows us to take into account the dynamic of the process-zone and can actually reproduce the basic phenomenon observed in cracks without the use of any additional law. In this paper we first describe briefly the model we use, emphasizing the modifications of the model presented in [2] that were required to describe properly the propagation of cracks under general plane loading (see also [3]). Then we present results of 2D simulations under mode I loading showing that the model reproduces well the behavior of a single crack under in plane loading. Finally, we discuss preliminary results showing that the model reproduces well the behavior of interacting cracks and we propose further extension of this work that disserve attention.

2 PRESENTATION OF THE MODEL

As described in [1], the phase field model of crack relies on an additional variable φ that describe the degree of brokenness of interatomic bonds. Here φ is taken to be equal to 1 when all inter-atomic bonds are intact and to 0 when they are all broken. Following the approach of [1] we describe the coupling of the linear elasticity with the phase field variable introducing a modified version of the elastic energy:

$$E = \iint dx dy \; g(\varphi)\left(\frac{1}{2}\lambda\varepsilon_{ii}^2 + \mu\varepsilon_{ij}^2\right) \quad (1) \;,$$

where $g=(4\varphi-3)\varphi^3$ is a function chosen to be equal to 0 for $\varphi=0$ and to 1 for $\varphi=1$ with the additional requirement that g vanishes faster than φ^2 when φ goes to 0 so that in a cracked material the strain is actually released [1]. The ε_{ij} tensor is linked to the small displacement fields by the relation: $\varepsilon_{ij}=(\partial_i u_j+\partial_j u_i)/2$. Then the evolution equation of the small displacement fields is prescribed by the relation:

$$\partial_{tt} u_i = -\frac{\delta E}{\delta u_i} \quad (2).$$

We now turn to the evolution equation of φ. Similarly to what was proposed in [1], it can be written as follows:

$$\tau \partial_t \varphi = \Delta\varphi - V'(\varphi) - g'(\varphi)(E_\varphi - \varepsilon_c) \quad (3),$$

where $V(\varphi)$ is a double well potential with minima at $\varphi=1$ and $\varphi=0$, ε_c is an arbitrary constant and $E\varphi$ depends on the ε_{ij} tensor as follows:

$$E_\varphi = \begin{cases} \left(\frac{1}{2}\lambda\varepsilon_{ii}^2 + \mu\varepsilon_{ij}^2\right) & \text{if } \varepsilon_{ii} > 0 \\ \left(\frac{1}{2}\lambda\varepsilon_{ii}^2 + \mu\varepsilon_{ij}^2\right) - (1+\alpha)K_{lame}\varepsilon_{ii}^2 & \text{if } \varepsilon_{ii} < 0 \end{cases} \quad (4)\ ,$$

which expresses the fact that under extension the breaking of bonds can occur, while under pure compression ($\varepsilon_{ii}<0$) the breaking of bonds is impossible and that for a given shear stress, the addition of a small compression may make it more difficult to break bonds (α is positive). In other words the eq. (4) introduces an asymmetry between compression and tension, which was not present in [1] since it considers pure mode III where only shear is present. The parameter τ indicates the amount of energy that is dissipated in the process zone: the higher τ the more energy is dissipated. The model defined we turn to results obtained with numerical simulations of rectangular strip (using grids moving with the crack tip allowed us to consider strips that were infinite in the x direction) of an elastic material (parameter values are….).

3 RESULTS FOR SIMPLE LOADING CONDITIONS

We first tested our model against basic features of crack dynamic in the damped case: does it obey the Griffith and Irwin criteria? Does it obey the principle of local symmetry? Numerical simulations showed that it actually does so. Under pure mode I loading, crack propagation was observed for a strain greater than the critical strain for which the elastic energy stored in the bulk of the medium is greater than the energetic cost of creating a crack (see [2] for the definition of the fracture energy). In addition, as expected from the Irwin criterion, we found that under various mode I loading, the equivalent, for our model, of the stress intensity factor at the tip of a steady crack was constant. The study of pure mode II loading showed that the conditional introduction of $(1+\alpha)K_{lame}\varepsilon_{ii}^2$ term in eq. 4 was crucial to allow the model to respect the principle of local symmetry. Indeed, we could see that without this term, a straight crack would divide into two symmetric branches while when this term was introduced, only a single branch could propagate, making a kink with the initial branch so that the mode II stress intensity factor at the crack tip was set to zero. (See fig 1).

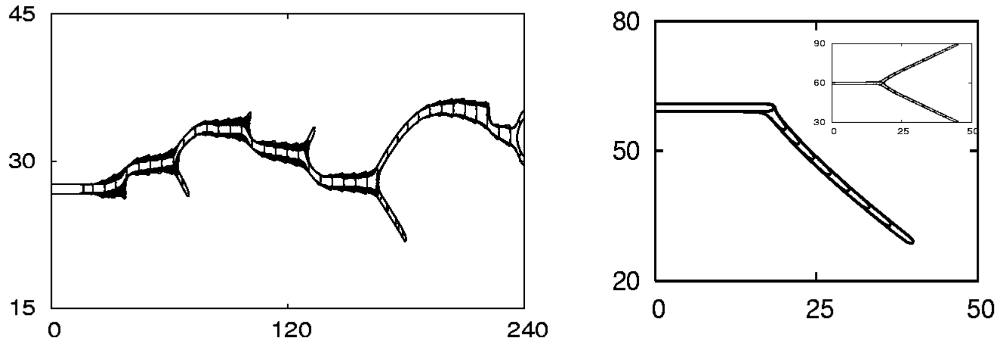


Figure 1

Left: typical contour plot of a ϕ=0.5 line taken at regular time intervals during the branching instability of a single dynamic crack propagating under pure mode 1 stress. The width of the strip is 60 and the value of Δ_y is 16. One can see multiple branches initiating from the main crack with irregular spacing. Right: Typical ϕ=0.5 lines obtained when simulating a crack with overdamped dynamics and pure mode II loading. The crack is initially straight and following time evolutions it changes direction so that the mode II stress intensity facto is set to zero. The inset shows the behavior under the same loading using the model without making difference between tension and compression and extension in the evolution equation of the phase field. One can see that instead of a single crack, two cracks propagate: one because of compression and the other because of extension.

When studying the crack in the dynamic regime, we found that relation between the speed of the stationary crack and the strain was in good qualitative agreement with experimental results and that the crack did exhibit a branching instability for high value of strain as one would expect. The branching angle and the critical speed were in fairly good agreement with theoretical predictions based on the principle of local symmetry for small values of τ.

Hence this model of crack propagation using a phase field approach is able to reproduce, at least qualitatively the basic features of cracks propagating in an elastic medium.

4 OSCILLATING CRACKS

We now turn to the main purpose of our study: the behavior of a crack under biaxial strain. In this case, we consider a strip of elastic material that is extended in both the x and the y direction, with the x size being much larger than the y size. The boundary conditions are chosen so that a crack can not reach the y_{max} and y_{min} boundaries of the medium: $u_x(y_{max})= u_x(y_{min})$ is kept constant in time. In this situation it has been observed that for a given extension in the y direction and increasing extensions in the x direction, the crack is first propagating along a straight line and that for values of the x extension bigger than a critical one, the crack undergoes a hopf bifurcation that leads to oscillating cracks (For a description of the experimental setup see fig. 2).

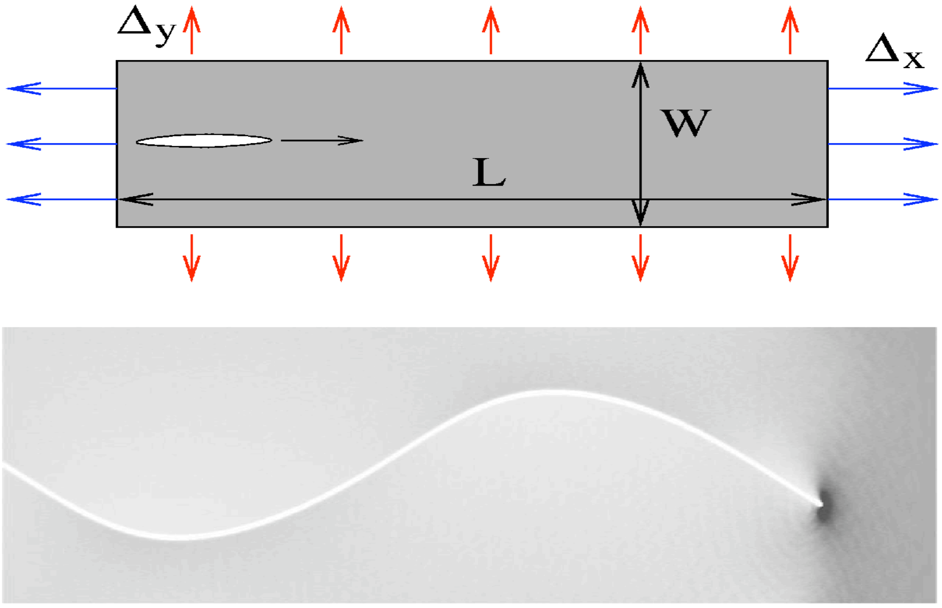


Figure 2

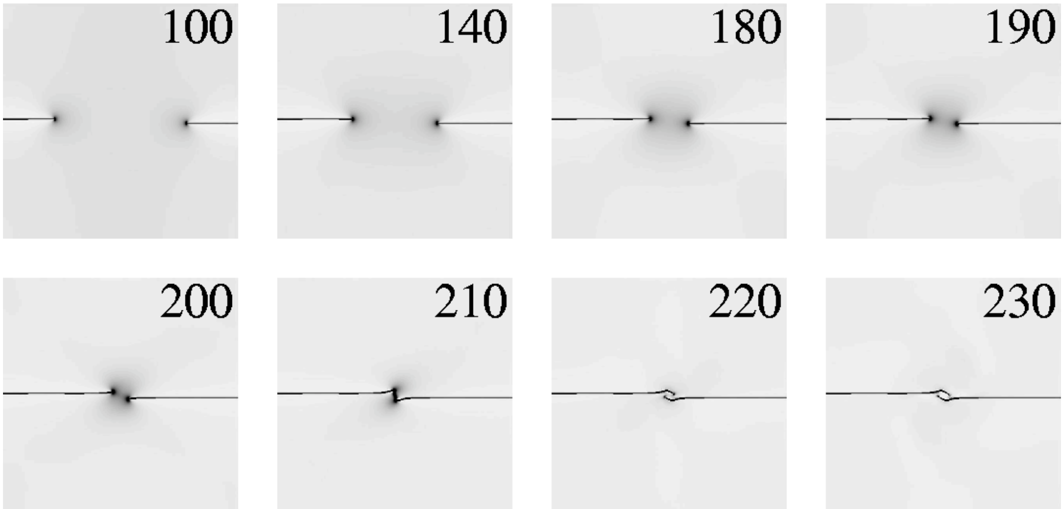
Top: Experimental setup: the sheet of width W and length L is extended in the y direction of Δ_y, while it is extended in the x direction of Δ_x. Hence the ratio of local extensions is $r=(\Delta_x/W)/(\Delta_y/L)$. The boundary conditions at the top and bottom of the strip are chosen so that the material points at these boundaries cannot move. Hence the crack can not propagate toward the top or the bottom of the elastic sheet. Bottom: typical crack observed using this setup with $r>r_c$. The crack (white region) oscillates and as expected, the energy remaining between the arches of its trajectory is small. (The grey scale corresponds to a color plot of the elastic energy density: the darker the more energy).

Using our model and numerical simulations we were able to reproduce qualitatively the phenomenology experimentally observed. Namely, we were first able to check that for a given value of the y extension of the strip, the relevant parameter was actually the local x extension, i.e. the total x extension divided by the length of the strip and that changing this later parameter, keeping the local x extension constant had little effect on the crack tip motion. This being checked, we introduce $r=(\Delta_x/W)/(\Delta_y/L)$ to describe the ratio of local extension in the x and y directions.
Then numerical simulations using increasing values of r show that the system first presents damped oscillations with finite wavelength, and that beyhond a critical value of r (approximately 0.77), the oscillations are no longer damped but amplified. The following time evolution of the system leads to a restabilized state where the crack tip oscillates at a finite wavelength and amplitude following a quasi-sinusoidal line. We were able to check that the amplitude behaves like the square root of the distance from threshold as expected in the case of a Hopf bifurcation and that the wavelength of the oscillating crack was almost independent of the model parameters… and increased linearly with the y size of the strip. These results are in very good qualitative agreement with experimental findings [4].

5 CONCLUSION AND PERSPECTIVE

Hence the model we have presented here and elsewhere reproduces well the behavior of dynamic brittle fracture under various loading conditions. It is easy to implement numerically and allows numerical simulations of large systems. The extensions of this work are numerous and we think it is interesting to mention some of them. First, this model could be used to derive the equations of the crack tip motion and link them with the values of the stress intensity factors. This would allow us to check if this model is consistent with the classical theory of fracture: is there actually a unique relation between the crack tip speed evolution equation and the nature of the singularity at the tip of the crack. We could also extend the model to fully three-dimensional systems and thus derive the motion equations of the crack front. And last, this model should be quite useful to deal with interacting cracks since the phase-field method is well adapted to deal with multiple interfaces that can meet each other. In this later case preliminary simulations have shown that results are in a good qualitative agreement with experimental findings. Hence the model should be quite useful to understand better the dynamic of interacting cracks (see example figure 3).

Figure 3



Typical pattern when considering two cracks propagating toward each other. The applied strain is hydrostatic and the simulation box is square. The cracks (black region) first avoid each other and then meet each other with a right angle.

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