INFLUENCE OF VOLUME FRACTION ON CRACKING CHARACTERISTICS OF MAGNETOELECTROELASTIC COMPOSITES

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ABSTRACT

Analyzed in this work is the crack initiation and growth behavior of a linecrack in a magnetoelectroelastic composite that is made of $BaTiO_3$ and $CoFe_2O_4$. The former represents the inclusions and the latter the matrix. Interaction of the elastic, electric and magnetic effects with the line crack can be exhibited explicitly by the form of the local strain energy density function. This includes the ways with which crack growth could be affected by altering the directions of poling for the electric and magnetic field with respect to those for the applied electric and magnetic field. Presumably, the various material, geometric and loading parameters could be selected to suppress crack extension provided that a suitable fracture criterion could be found. The strain energy density function criterion being positive definite is tested and applied as a possible candidate. The results reveal several previously undiscovered phenomena of crack initiation and growth behavior. A series of new experiments are recommended for future work

1. INTRODUCTION

This work is concerned only with the mechanical behavior of the dual phase $BaTiO_3$ and $CoFe_2O_4$ composite. The electric and magnetic effects can have a significant influence on the ways with which the composite could fail by macro-cracking. Electric and magnetic poling give rise to preferred directions in the composite when they are heated above the ferroelectric transition temperature and/or kept in a DC magnetic field to reach saturation at room temperature. Depending on the directions of the applied electric and magnetic fields with respect to poling, a pre-existing line crack could extend longer or shorter [1-3] in comparison to the reference state when magnetoelectric effect is not present. For the $BaTiO_3$(inclusion) -$CoFe_2O_4$(matrix) composite, it is not obvious how the volume fraction of the inclusions would affect the fracture characteristics of the composite. Even when the composite properties are homogenized for determining the parameters in the constitutive relations, the multiscale nature of the problem prohibits the use of certain fracture criteria that are not foregiving to the different rates of energy release due to mechanical, electrical and magnetic means. Furthermore, it is not adequate to select just one of the stress or strain components and use it as a criterion for determining the failure of the composite for a specific loading. For the same material, the behavior of stress with time and strain with time may be different. The crack tip stress intensity factor may not have the same crack tip characteristics as the strain intensity factor. There is no obvious preference to choose one over the other. The likelihood is that a criterion may be problem specific. That is to say the same criterion may no longer apply when loading direction with reference to the composite microstructure is changed. This is particularly true for multi-functional composites. Briefly stated, the classical fracture mechanics approach limited to isotropic and homogeneous materials should not be taken for granted. It may not be valid for anisotropic and/or nonhomogeneous composites. For piezoelectric materials, the energy release rate approach has yielded negative results [4-6]. Disqualification of criteria could be made by a process of elimination once a criterion encounters contradiction. Agreement with test

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data is only one of the ways of choosing a criterion; it may be necessary but not sufficient. These and other new findings concerned with the effect of the volume fraction of the inclusion are discussed.

2. BASIC FORMULATION

Consider a rectangular Cartesian coordinate system x_j (j=1, 3) that is attached to a linear magnetoelectroelastic medium as shown in Fig. 1. Equal and opposite normal stresses σ_∞ are applied far away from the crack of length 2a in addition to the application of electric field E_∞ and magnetic field H_∞. Poling of E and H are assumed to be normal to the crack in the x_3- or y-direction. They can be reversed by attaching a negative sign to E or H. In what follows, x_1 and x_3 will be denoted by x and y, respectively.

For plane strain, the displacements u_j, magnetic field potential φ and electric field potential ϕ can be expressed in terms of a single function f(z) of the complex variable z = x + μy as

$$u_x = f(z), \quad u_y = a_1 f(z), \quad \varphi = a_2 f(z), \quad \phi = a_3 f(z) \qquad (1)$$

in which a_j are coefficients to be found for specific problems. Once eqs. (1) are known, the strain, electric and magnetic field can be found from

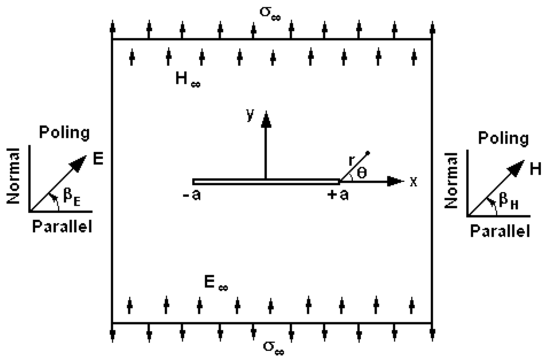

Fig.1 Crack in magnetoelectroelastic material.

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\varphi_{,i}, \quad H_i = -\phi_{,i} \quad \text{for } i = x, y \qquad (2)$$

The quantities ε_{ij}, E_j and H_j in eqs. (2) are related to the stress σ_{ij}, electric displacement D_j and magnetic flux B_j by the constitutive relations:

$$\sigma_{ij} = c_{ijks}\varepsilon_{ks} - e_{sij}E_s - h_{sij}H_s, \quad D_i = e_{iks}\varepsilon_{ks} + \alpha_{is}E_s + \beta_{is}H_s, \quad B_i = h_{iks}\varepsilon_{ks} + \beta_{is}E_s + \gamma_{is}H_s \qquad (3)$$

Note that c_{ijks}, e_{iks}, h_{iks}, and β_{is} are the elastic, piezoelectric, piezomagnetic and electromagnetic constants, respectively. And α_{is} and γ_{is} are dielectric permitivities and magnetic permeabilities.. The quantities in eqs. (3) are required to satisfy the equations of equilibrium in the forms

$$\sigma_{ij,j} = 0, \qquad D_{i,i} = 0, \qquad B_{i,i} = 0 \qquad (4)$$

where body forces have been neglected. The physical constants γ_{ij}, $\in_{ij}$, h_{ij} and e_{ij} determine the elastic, piezoelectric, piezomagnetic of the $BaTiO_3$-$CoFe_2O_4$ composite. For this problem, it suffices to consider the four roots of μ in upper half complex plane such that eqs. (1) may be rewritten as

$$\mathbf{u} = 2\,\mathrm{Re}\left\{\sum_{k=1}^{4} a_k f_k(z_k)\right\}, \quad z_k = x + \mu_k y \qquad (5)$$

The method of solution follows the foot steps in anisotropic elasticity [7]. Knowing that df_k/dz_k must have the $1/(r)^{-1/2}$ stress singularities at the crack tips with r being the distance from the crack tip, the functions f_k in eq. (5) become

$$f_k = M_k z_k + N_k(z_k - \sqrt{z_k^2 - a^2}) \qquad (6)$$

in which the coefficients M_k and N_k (k=1,---,4) can be determined from the boundary conditions

$$D_y = 0 \text{ and } B_y = 0 \text{ for } |x| \le a \text{ and } y = 0^{\pm} \qquad (7)$$

for an impermeable crack subjected to the followings loadings far away:

$$\sigma_{xx}^{\infty}=0,\ \sigma_{xy}^{\infty}=0,\ \sigma_{yy}^{\infty}=\sigma_{\infty},\ E_y^{\infty}=E_{\infty},\ E_x^{\infty}=0,\ H_y^{\infty}=H_{\infty},\ H_x^{\infty}=0 \tag{8}$$

As mentioned earlier, the foregoing governing equations correspond to the constitutive coefficients γ_{ij}, $\in_{ij}$, h_{ij} and e_{ij} in eq. (3). The threshold that accounts for this change will be determined by using the strain energy density function as a criterion [8,9].

3. EFFECT OF VOLUME FRACTION OF BaTiO3

The piezoelectric and piezomagentic properties of the $BaTiO_3$-$CoFe_2O_4$ composite with different volume fraction V_f of the inclusions can be found in [1,2]. Using the energy density function as the fundamental quantity for characterizing the response of $BaTiO_3$-$CoFe_2O_4$, the influence of loading, microstructure parameter and defect growth will be examined. Piezomagnetic and/or piezoelectric properties are determined by the composite microstructure and they are governed by the macroscopic constitutive coefficients, say h_{ij} and e_{ij}. When the volume fraction of the inclusions is changed, the energy density factor

$$S=r\left(\frac{dW}{dV}\right) \tag{9}$$

For the magnetoelectroelastic material, the volume energy density function dW/dV can be computed from

$$\frac{dW}{dV}=\frac{1}{2}\sigma_{ij}\varepsilon_{ij}+\frac{1}{2}E_iD_i+\frac{1}{2}H_iB_i \tag{10}$$

Failure by stable crack growth is assumed to occur when dW/dV becomes critical and by unstable crack extension when S becomes critical. The direction of stable and unstable crack growth is assumed to correspond with the minimum of dW/dV and S, respectively. A detail account of the theory can be found in [9]. It suffices to present some of the results using the normalized strain energy density factor $S_{min}/\sigma_{\infty}^2a$. This will be illustrated for h_{ij} magnified by a factor of 100. Plotted in Fig. 2 are the variations of $S_{min}/\sigma_{\infty}^2a$ with $E_{\infty}/\sigma_{\infty}$ for $H_{\infty}/\sigma_{\infty} = -10^{-4}\ C^2/(Ns^2)$. The influence of the volume fraction is small for negative $E_{\infty}/\sigma_{\infty}$ ratio. As $E_{\infty}/\sigma_{\infty}$ becomes positive $S_{min}/\sigma_{\infty}^2a$ would increase much faster for high V_f. This effect is quite noticeable in Fig. 2. As the magnetic poling is

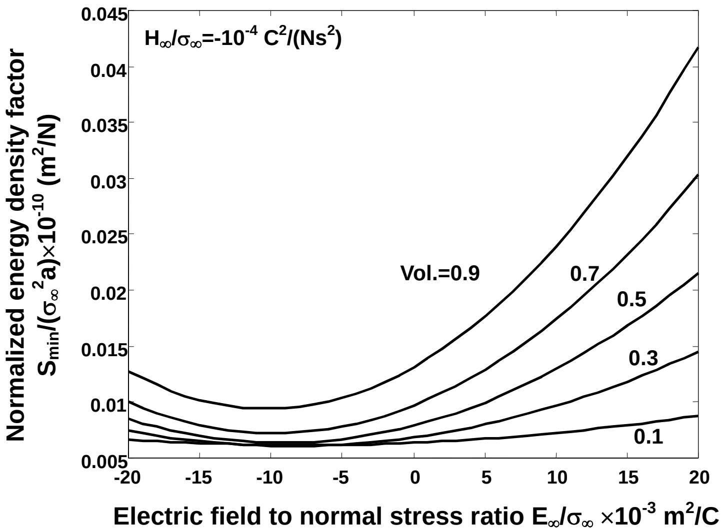

Fig. 2. Normalized $S_{min}/\sigma_{\infty}^2a$ with $E_{\infty}/\sigma_{\infty}$ for $H_{\infty}/\sigma_{\infty} = -10^{-4}\ C^2/(Ns^2)$.

changed from negative to positive, i.e., with $H_\infty/\sigma_\infty = 10^{-4}\ C^2/(Ns^2)$, the curves for different V_f would intersect one another. Increase in V_f would further benefit the critical normal stress because this would decrease $S_{min}/\sigma_\infty{}^2a$ giving rise to even lower critical stress. The increase in critical stress with V_f starts when the applied electric field to normal stress ratio becomes larger than $5 \times 10^{-3}\ m^2/C$. Refer to Fig. 3.

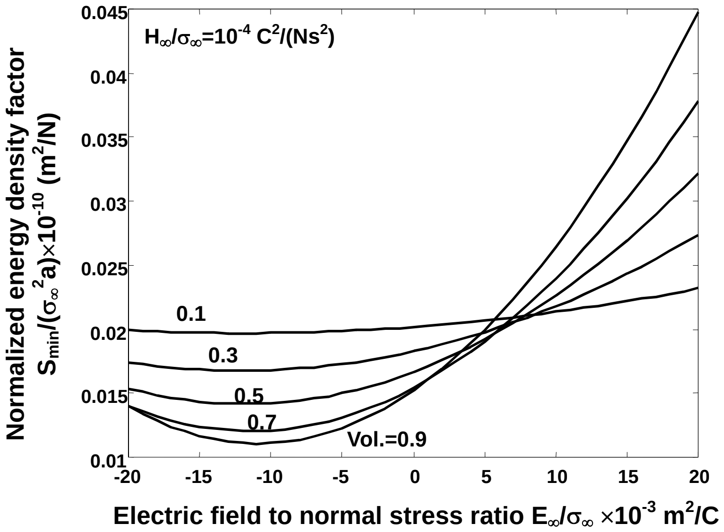

Fig. 3. Normalized $S_{min}/\sigma_\infty{}^2a$ with E_∞/σ_∞ for $H_\infty/\sigma_\infty = 10^{-4}\ C^2/(Ns^2)$.

4. CONCLUDING REMARKS

The multiscaling character of the problem is inherent in the piezoelectric and piezomagnetic material due to the coupling of mechanical, electrical and magnetic effects. When the influence of material inhomoneity becomes time dependent, the behavior becomes one of non-equilibrium. This implies that the local properties can no longer be homogenized in size and time. This fundamental character of small specimen behavior cannot be explained by using equilibrium mechanics and introducing non-linearity.

This work provides some initial thoughts on how to distinguish and related quantities at the different size scales by using the magnetoelectroelastic related quantities at the different size scales by using the magnetoelectroelastic material as an example. Clearly, additional fundamental works need to be done to better understand how the various electromagnetoelastic parameters could be adjusted to retard crack growth.

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REFERENCES

[1] G. C. Sih (ed.), Mechanics of fracture, vol. I – VII, Noordhoff International Publishing, Leyden, 1973 –1981.

[2] G. C. Sih, Implication of scaling hierarchy associated with nonequilibrium: filed and particulate, J. of Theoretical and Applied Fracture Mechanics, 37(3) (2001) 335-369.
[3] G. C. Sih, Micromehanics associated with thermal/mechanical interaction of polycrystals, in: G. C. Sih (Ed.), Mesomechanics 2000: Role of Mechanics for Development of Science and Technology, vol. 1, Tsinghua University Press, 2000, 143-152.
[4] G..C. Sih, Thermomechanics of solids: nonequilibrium and irreversibility, J. of Theoretical and Applied Fracture Mechanics, 9(3) (1988) 175-198.
[5] G. C. Sih, Mechanics and physics of energy density and rate of change of volume with surface, J. of Theoretical and Applied Fracture Mechanics, 4(3) (1985) 157-173.
[6] G. C. Sih, Some basic problems in non-equilibrium thermomechanics, In: S. Sienietyez and P. Salamon (Eds), Taylor and Franciss,New York, 1992, 218-247.
[7] G. C. Sih and H. Liebowitz, Mathematical theories of brittle fracture, Mathematical fundamentals of fracture, in: H. Liebowitz (Ed.), Academic Press, New York, 2 (1968) 67-190.
[8] G. C. Sih, Fracture mechanics of engineering structural components, eds. G. C. Sih and L. Faria, Fracture Mechanics Methodology, vol.I, Martinus Nijhoff Publishers, The Netherlands, 1984, 35-101.
[9] G..C. Sih, Mechanics of Fracture Initiation and Propagation, Kluwer Academic 1991