Transferability and critical distance approaches

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ABSTRACT

An overview of the different local approach is proposed and classified as point, line, area, volume and double volume area. Possibility of transferability of each approach is discussed.

1. INTRODUCTION

It is assumed that the fracture process needs a physical volume. This assumption is supported by the fact that fracture resistance is affected by loading mode, structure geometry and scale effect. Approaching transferability problem with a non local approach means that we have to define an average stress value in a meso volume called the fracture process volume. Several definitions of this fracture process can be found in literature. It seems that the size is not connected to the material microstructure but depends on geometry and loading mode. It is generally two order of magnitude to microstructure and typically a volume at mesoscale as dimension of order of mm. It is considered as the high stress region with different limit (for example 10% of maximum stress decreasing). The value of the"hot spot stress" i.e. the maximum stress value of the stress distribution is unsuitable to explain the influence of these parameters on the fracture resistance. This fracture volume can be considered also as the place where the damage level reach critical value and it cannot be physically localised on a simple point. The size of this volume is always an open discussion. It was first considered as connected with microstructure (see Krassowsky et al 1993) but more and more as the size of the "high stressed region". Knowing the size of the fracture process volume is essential when applying a local fracture criterion to predict critical event because its value combined with a precise description of the stress distribution allow to know the fracture stress (a local value) or the effective stress (the mean of local values over the physical volume). This volume is assumed to be quasi-cylindrical similarly to plastic zone. Experimental investigation by micro hardness, metallographic etching leads to conclusion that this assumption is acceptable. In this case the volume is reduced to a simple cylindrical shape with diameter called under a general term of critical distance X_d. Combining this distance with the stress distribution can be made using point, line, surface and volume approaches with an increasing degree of complexity.

2. EXAMPLE OF POINT APPROACH: RKR CRITERION.

This type of approach was initially proposed by (Neuber 1961) to explain that the real stress concentration factor is less than those deduced from maximum stress.

The most well-known example of "point approach" is the RKR local fracture criterion (Ritchie-Knott and Rice 1973) where critical distance is assumed to have a size of the order of the grain size or bainitic and martensitic laths size. Critical distance is in this case generally called characteristic distance X_c and corresponds to the critical stress σ^*_c. Generally this critical stress is determined on notched tensile or bending specimens as the maximum stress. This local fracture criterion is generally used for brittle materials and the critical stress is generally found closed to yield stress at 0K.

This criterion combined with a description of the HRRR stress distribution (Hutchinson-Rice and Rosengreen 1968) was successfully applied to explain the decreasing of fracture toughness K_{Ic} with yield stress σ_y according to the following relationship:

$$K_{Ic} \cdot \sigma_y^{(N-1)/2} = Cst \tag{1}$$

where N is the strain hardening exponent of the Ramberg-Osgood law.
RKR criterion is also used to predict the shift of transition temperature when increasing loading rate assuming that relationship (1) is valid at any temperature:

$$K_{Ic} \cdot \sigma_y^{(N-1/2)} = K_\mu \cdot \sigma_{y,0}^{(N-1/2)} \tag{2}$$

where K_μ is fracture toughness at 0 K and $\sigma_{y,0}$ yield stress at same temperature. The evolution of yield stress is assumed to be thermo activated. In this model, the critical cleavage stress is considered as a material characteristic and the characteristic distance which is related to the microstructure is independent to the loading mode and geometry. In this case, transferability of fracture toughness results is not possible.

3. EXAMPLE OF LINE APPROACH

This type of approach was first used by Peterson (Peterson 1974) for the same reason than Neuber
In this case the critical stress is not in coincidence with critical distance on stress distribution.
Peterson considered that the critical distance is a material characteristic and function of ultimate stress. An example of application of this approach is done when explain influence of notch radius on fracture toughness. Greager and Paris distribution for notches (Creager and Paris 1967) is given by:

$$\sigma_{yy} = \frac{K\rho}{\sqrt{2\pi r}} \cdot \left(1 + \frac{\rho}{2r}\right) \tag{3}$$

In this approach, critical distance X_d is taken as

$$X_d = \rho/2 \tag{4}$$

By averaging this stress distribution over the critical distance, we get fracture toughness $K_{\rho,c}$

$$K_{\rho,c} = \sqrt{\frac{\pi\rho}{2}} \tag{5}$$

Relationship (5) indicates that fracture toughness measured on notched specimen increases with the square root of notch radius. An example of validity of such relation ship is given in figure where the fracture toughness of glass is plotted versus the square root of notch radius. A threshold value can be distinguish in such a curve indicates that for sharp notches critical distance is in this case, never less than characteristic distance $X_{c.}$
Transferability can be insured by taking into account the fact that the stress distribution is shifted to lower stress value by changing constraint (or geometry). Such approach as been used by Dodds (Dodds 1997) assuming that fracture toughness depends on a parameter Q defined as

$$Q = \frac{\sigma_{yy} - \left(\sigma_{yy}\right)_{reference}}{\sigma_y} \tag{6}$$

This Q value is determined at a non dimensional (critical) distance equal to

$$\frac{X_d \cdot J}{\sigma_y} = 2 \tag{7}$$

Dodds assumes that the stress distribution is only shifted parallel to stress axis and gives a condition on the gradient of the distribution by the following equation:

$$gradQ = \frac{Q_{(1)} - Q_{(5)}}{4} \leq 0{,}1 \tag{8}$$

where $Q_{(1)}$ and $Q_{(2)}$ are respectively Q value determined at non dimensional distance 1 and 5. However the assumption of Dodds is too restrictive and the change of the stress distribution is more complex as it can be seen on figure

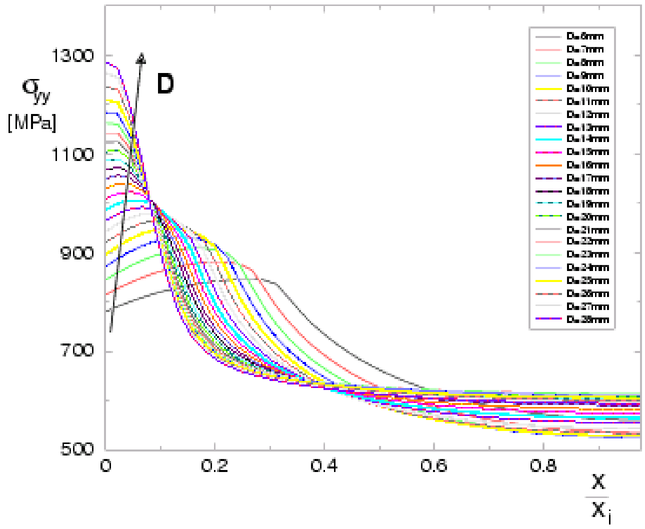

Figure 1: Evolution of Stress distribution with size (SENB specimen).

4. EXAMPLE OF AREA APPROACH : THE BEREMIN APPROACH

In the Beremin approach (Beremin 1983) approach, the probability of fracture is proportional to the area or volume if thickness is constant) of the fracture zone equal to plastic zone:

$$\ln\left(1 - P_f\right) = -\sum_{i=1}^{n} \frac{V_i}{V_0} \cdot \left(\frac{\sigma_I^i}{\sigma_{no}}\right)^{m_w} \tag{9}$$

where V_i is the current volume and V_0 the reference volume

This expression can be write as

$$P_f = 1 - \exp\left[-\left(\frac{\sigma_w}{\sigma_{no}} \right)^{m_w} \right] \tag{10}$$

where σ_w is the Weibull stress, σ_{n0}, the reference stress and m_w the Weibull stress. It can be noted that the Weibull stress is connected to the reference volume and has no connection with the critical stress.

The Weibull‘s distribution is used in the Toughness Scaling Model (TSM) from Kopenhoffer (Kopenhoffer and Dodds 1997). In this model, influence of loading mode and geometry is characterised by the modification of fracture process volume and also by those of the Weibull modulus of the distribution. In this case, there is no a unique curve for transferability but several associated with each discrete value of the Weibull modulus. One example of such a method is given by Dlouhy (Dlouhy and al 2001).

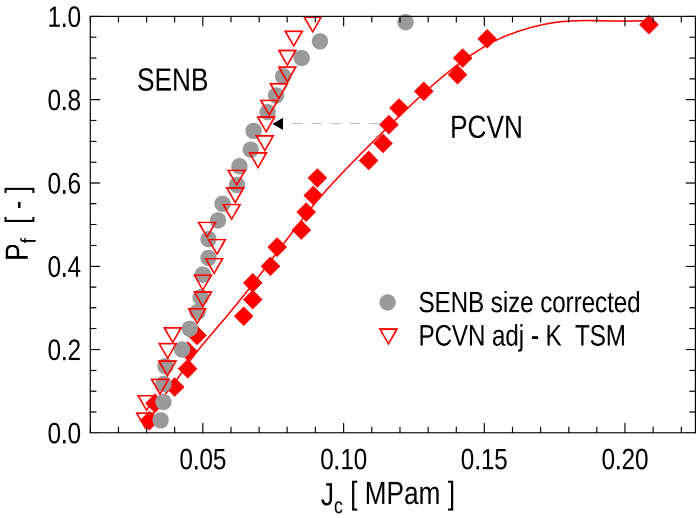

Figure 2: Example of transferability with the Kopenhoffer method (Dlouhy 2001).

5. EXAMPLE OF VOLUME APPROACH : THE VOLUMETRIC APPROACH

In this fracture process volume or effective volume V_{ef} the effective strain or stress can be defined as the average of the weighted distribution (Pluvinage 1998). In order to take into account the essential role of stress gradient, stress distribution is weighted by the weight function ϕ. Following this effective strain and strain are defined as follow:

$$\varepsilon_{ef} = \frac{1}{V_{ef}} \int_V \phi(x-s).\varepsilon(s).dV_{ef}(s)$$
$$\sigma_{ef} = \frac{1}{V_{ef}} \int_V \phi(x-s).\sigma(s).dV_{ef}(s) \tag{11}$$

where $\varepsilon(s)$ or $\sigma(s)$ are stress or strain in one point, V_{ef} effective volume and ϕ a weight function Several kind of weight function can be used and have the following forms:

$\phi = (1\text{-}r\chi)$ or $\phi = (e^{r\chi/2})$ (12)

“bell” function

$$\phi = \left[1-\left(\frac{r}{C\ \ X_{ef}}\right)^{[2]}\right]^{2} \tag{13}$$

where r is distance and c is the relative stress gradient defined:

$$\chi = \frac{1}{\sigma_{yy}}.\frac{d\sigma_{yy}}{dr} \tag{14}$$

C is a constant; X_{ef} is the effective distance characteristic of the zone over which stress or strain is averaging.This method has been applied success fully for transferability necessary by change of the loading mode.

6. EXAMPLE OF DOUBLE VOLUME APPROACH

Similarly to Broberg (Broberg 1974), we consider that the fracture process volume is surrounded by a screening zone which controls the flux of elastic energy coming from the structure to the fracture process zone. This screening zone can be considered as circular with diameter equal to the ligament size.

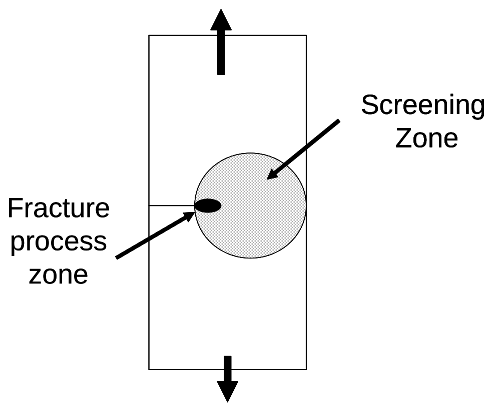

Figure 4: Schematic presentation of double volume approach with fracture process zone and screening zone.

Aifantis (Aifantis 1987) has proposed a modification of plastic flow rule including a plastic strain Laplacian.

$$f = \sigma_{eq} - \left(\Phi\left(\varepsilon_{pl,eq}\right) - c\nabla^{2}\varepsilon_{pl,eq}\right) = 0 \tag{15}$$

σ_{eq} is the Von Mises equivalent stress and $\varepsilon_{pl,eq}$ the plastic equivalent strain.

$$\bar{\varepsilon}_{pl,eq} = \frac{1}{l_c}.\int_{-\infty}^{\infty}\alpha(u)\varepsilon_{pl,eq}.(x+u)du \quad \text{with } u = s\text{-}x \tag{16}$$

If we assume that $\varepsilon_{pl,eq}$ varies slowly, we can approximate $\varepsilon_{pl,eq}.(x+u)$ by a Taylor development:

$$\bar{\varepsilon}_{pl,eq} = \varepsilon_{pl,eq}(x) + \frac{\partial \varepsilon_{pl,eq}}{\partial x}(x) l_c \mu_1 + \frac{\partial^2 \varepsilon_{pl,eq}}{\partial^2 x}(x) l_c^2 \mu_2 + \ldots + \frac{\partial^n \varepsilon_{pl,eq}}{\partial^n x}(x) l_c^n \mu_n \quad (17)$$

with $\mu_i = \int_{-\infty}^{\infty} \alpha(s) \frac{s^n ds}{l_c^{n+1}}$

$\alpha(s)$ is a pair function μ_i values are zero for odd values of i. By limiting Taylor development to two terms

$$\bar{\varepsilon}_{pl,eq} \approx \varepsilon_{pl,eq}(x) + \frac{\partial \varepsilon_{pl,eq}}{\partial x}(x) l_c \mu_1 + \frac{\partial^2 \varepsilon_{pl,eq}}{\partial^2 x}(x) l_c^2 \mu_2 \quad (18)$$

An example of such new method has been proposed by Malmberg (Malmberg 1998) to explain size effect of bars submitted to torsion.

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