UNIFIED FRAMEWORK FOR DAMAGE AND FATIGUE OF METALS, CONCRETE AND ELASTOMERS

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ABSTRACT

A generalized damage model is presented. Built within the thermodynamics framework, it assumes a damage evolution governed by the main dissipative mechanim: plastictity for metals, internal sliding with friction for concrete and filled elastomers. The model applies to different classes of materials but also to different kinds of loadings, monotonic and fatigue loadings. From the Continuum Damage Mechanics point of view, the number of cycles to rupture in fatigue is reached when the damage D equals the critical damage D_c. Examples of calculated fatigue curves are given for different materials.

1. INTRODUCTION

Continuum Damage Mechanics (CDM) is a powerfull tool to deal with failure of materials and structures. Damage is considered as part of the material behavior. CDM gives a framework to write the damage constitutive equations and to extend them to 3D, therefore to structures computations.

Lemaitre damage evolution law of a damage rate governed by plasticity but also enhanced by the elastic energy density is able to deal with many situations: ductile failure, fatigue, creep and creep-fatigue of metals or polymers [1, 2]. For materials such as composites, concrete and filled elastomers, no plasticity occurs as other specific dissipative mechanisms take place and Lemaitre damage law seems useless.

The study of these mechanisms [3, 4, 5] exhibits a general form for the thermodynamics potential (Helmholtz specific free energy). It allows for an extension of plasticity coupled with damage framework (section 3) to more general constitutive models with internal sliding and friction (sections 2, 4 and 5).

2. GENERAL THERMODYNAMICS MODEL

The idea for a unified damage model valid for many materials is to relate the damage rate to the main dissipative mechanism, often internal sliding and friction, and to consider damage as governed by a cumulative measure of the internal sliding. This applies to metals for which internal slips are mainly due to dislocations creation and evolution, but also to non metallic materials such as concrete with internal sliding with friction of the microcracks and such as filled elastomers in which a dissipative phenomenon occurs due to internal sliding of the macro-molecular chains on themselves and on the black carbon filler particules.

2.1. Thermodynamics Variables

Define $\mathcal{V} = [\boldsymbol{\epsilon}^\pi, \boldsymbol{a}, q, D]$ as internal variables associated with $\mathcal{A} = [-\boldsymbol{\sigma}^\pi, \boldsymbol{x}, Q, -Y]$. The physical meaning of these thermodynamics variables depends on the type of material and of the physical dissipative mechanisms. The strain $\boldsymbol{\epsilon}^\pi$ due to internal sliding is an internal inelastic strain (equal to the plastic strain $\boldsymbol{\epsilon}^p$ in plasticity) and Y is the strain energy release rate density.

2.2. State and evolution laws: generalized damage model

The general expression for the state potential allowing to couple damage and internal friction reads [3, 4, 5]:

$$\rho\psi = (1 - D) \cdot w_1(\boldsymbol{\epsilon}) + g(D) \cdot w_2(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^\pi) + w_s(q, \boldsymbol{a}) \tag{1}$$

where w_1 and w_2 define the strain energy density and w_s the stored energy density, function of the scalar variable q and of the tensorial variable $\boldsymbol{a}$. The function $g(D)$ is simply taken as $g(D) = 1 - D$ in the following.

The state laws read $\mathcal{A} = \rho\frac{\partial\psi}{\partial\mathcal{V}}$,

$$\begin{aligned}
\boldsymbol{\sigma} &= \rho\frac{\partial\psi}{\partial\boldsymbol{\epsilon}} = (1 - D)\frac{\partial(w_1 + w_2)}{\partial\boldsymbol{\epsilon}} \quad \longrightarrow \quad \tilde{\boldsymbol{\sigma}} = \frac{\partial(w_1 + w_2)}{\partial\boldsymbol{\epsilon}} \\
\boldsymbol{\sigma}^\pi &= -\rho\frac{\partial\psi}{\partial\boldsymbol{\epsilon}^\pi} = (1 - D)\frac{\partial w_2}{\partial\boldsymbol{\epsilon}} \quad \longrightarrow \quad \tilde{\boldsymbol{\sigma}}^\pi = \frac{\partial w_2}{\partial\boldsymbol{\epsilon}} \\
\boldsymbol{x} &= \rho\frac{\partial\psi}{\partial\boldsymbol{a}} = \frac{\partial w_s}{\partial\boldsymbol{a}} \\
Q &= \rho\frac{\partial\psi}{\partial q} = \frac{\partial w_s}{\partial q} \\
Y &= -\rho\frac{\partial\psi}{\partial D} = w_1 + w_2
\end{aligned} \tag{2}$$

They naturally define the effective stresses $\tilde{\boldsymbol{\sigma}}$, $\tilde{\boldsymbol{\sigma}}^\pi$ such as the elasticity law written in terms of strains and of effective stresses does not depend explicitly upon D (strain equivalence principle).

Consider the non associated dissipative potential,

$$F = f + F_x + F_D \tag{3}$$

where:

- $f = \|\tilde{\boldsymbol{\sigma}}^\pi - \boldsymbol{x}\| - Q - \sigma_s < 0$ defines the reversibility domain, $\|.\|$ is a norm in the stresses space (not necessary von Mises norm) and σ_s is the reversibility limit.
- the functions $F_x = \dfrac{\gamma}{2C_x}\boldsymbol{x} : \boldsymbol{x}$ and $Q = Q(q)$ model the internal sliding nonlinearity. F_x models nonlinear kinematic hardening for metals (Armstrong-Frederick law), it accounts for strain softening and Mullins effect in elastomers [6].
- $F_D = \dfrac{S}{(s+1)(1-D)}\left(\dfrac{Y}{S}\right)^{s+1}$ is the damage potential with S and s the damage parameters. It leads to Lemaitre damage evolution for metals [1] and to its generalization to other materials.

The evolution laws derive from the dissipative potential through the normality rule $\dot{\mathcal{V}} = -\dot{\mu}\frac{\partial F}{\partial\mathcal{A}}$ with $\dot{\mu}$ a Lagrange multiplier given by the consistency condition $f = 0$ and $\dot{f} = 0$ for non viscous materials or given by a viscosity law for viscous materials. $\dot{\pi} = \dot{\mu}/(1 - D)$ is found equal to the norm of the inelastic strain. This defines the cumulative measure π of the internal sliding,

$$\pi = \int_0^t \|\dot{\boldsymbol{\epsilon}}^\pi\| dt \tag{4}$$

which will recover the accumulated plastic strain as $p = \int_0^t \sqrt{\frac{2}{3}\dot{\boldsymbol{\epsilon}}^p : \dot{\boldsymbol{\epsilon}}^p} dt$ in plasticity.

The generalized damage evolution law is obtained as:

$$\begin{aligned} &\dot{D} = \left(\frac{Y}{S}\right)^s \dot{\pi} \\ &D = D_c \quad \longrightarrow \quad \text{mesocrack initiation} \end{aligned} \tag{5}$$

which corresponds to damage governed by the main dissipative mechanisms through $\dot{\pi}$ and where D_c is the critical damage at mesocrack initiation.

3. CONTINUOUS DAMAGE AND FATIGUE OF METALS

3.1. Elasto-plasticity coupled with damage

For metals set $\boldsymbol{\epsilon}^\pi = \boldsymbol{\epsilon}^p$ (the plastic strain) and consider $r = q$, $R = Q$, $\boldsymbol{X} = \boldsymbol{x}$, $\boldsymbol{\alpha} = \boldsymbol{a}$ as hardening variables. Elasto-plasticity coupled with damage is then the particular case:

- $w_1 = 0$, $w_s(r, \boldsymbol{\alpha}) = G(r) + \frac{1}{3}C\, \boldsymbol{\alpha} : \boldsymbol{\alpha}$ for the thermodynamics potential written

$$\rho\psi = \frac{1}{2}(1-D)(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^p) : \underline{\boldsymbol{E}} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^p) + G(r) + \frac{1}{3}C\, \boldsymbol{\alpha} : \boldsymbol{\alpha} \tag{6}$$

- $f = \left(\dfrac{\boldsymbol{\sigma}}{1-D} - \boldsymbol{X}\right)_{eq} - R - \sigma_y$ for the yield fonction. The von Mises norm $(.)_{eq}$ is used and $\sigma_y = \sigma_s$ is the yield stress of the material.

Lemaitre damage evolution law is recovered:

$$\dot{D} = \left(\frac{Y}{S}\right)^s \dot{p} \tag{7}$$

3.2. Calculation of Manson-Coffin curve for metals

The damage model allows also to calculate the failure conditions for low cycle fatigue loading. Assume here a symmetric periodic loading between $\sigma_{\min} = -\sigma_{\text{Max}}$ and σ_{Max}.

The increment of damage per cycle $\frac{\delta D}{\delta N}$ comes from a first integration of the uniaxial damage law, in which the damaged material is considered as perfectly plastic: $\frac{\sigma}{1-D} \approx \sigma_{\text{Max}} = const$, $Y = \sigma^2/2E(1-D)^2 \approx \sigma^2_{\text{Max}}/2E$,

$$\frac{\delta D}{\delta N} = \int_{1\ cycle} \dot{D}\, dt = \left(\frac{\sigma^2_{\text{Max}}}{2ES}\right)^s 2\Delta\epsilon_p \tag{8}$$

where $\Delta\epsilon_p$ is the plastic strain increment over half a cycle. A second integration gives the number of cycles of rupture N_R corresponding to the critical value of the damage D_c,

$$N_R = \frac{D_c}{2\Delta\epsilon_p}\left(\frac{2ES}{\sigma^2_{\text{Max}}}\right)^s \tag{9}$$

which, considered altogether with a cyclic plasticity law, allows to plot the calculated Manson-Coffin curve of the material $\Delta\epsilon_p$ vs N_R [1, 2].

4. CONTINUOUS DAMAGE AND FATIGUE OF CONCRETE

4.1. Elasticity with internal friction coupled with damage

The general thermodynamics framework of section 2 allows to derive constitutive equations for quasi-brittle materials describing damage induced by mechanical loadings and its consequences in terms of hysteresis, internal friction and rupture.

- The potentials w_1 and w_2 are quadratic functions so that

$$\rho\psi = (1-D)\left[\frac{1}{2}\boldsymbol{\epsilon} : \underline{\boldsymbol{E}}_1 : \boldsymbol{\epsilon} + \frac{1}{2}(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{\pi}) : \underline{\boldsymbol{E}}_2 : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{\pi})\right] + G(q) + \frac{1}{2}C_x\, \boldsymbol{a} : \boldsymbol{a} \tag{10}$$

  and Hooke tensor is $\underline{\boldsymbol{E}} = \underline{\boldsymbol{E}}_1 + \underline{\boldsymbol{E}}_2$.

- The internal sliding criterion f, the potential F_x and F_D are those defined in section 2.2.

$\underline{\boldsymbol{E}}_1$, $\underline{\boldsymbol{E}}_2$, σ_s, C_x, γ, S, s are the material parameters (tensorial or scalar) to which one adds the critical damage D_c for mesocrack initiation.

The damage evolution law for concrete submitted to monotonic, cyclic or seismic loading is the generalized damage law,

$$\dot{D} = \left(\frac{Y}{S}\right)^s \dot{\pi} \tag{11}$$

4.2. Monotonic response and fatigue curve of concrete in compression

For a monotonic compression test, the response of this model is presented in figure 1a. The crack initiation criterion $D = D_c = 0.9$ allows to compute the fatigue curve from Continuum Damage Mechanics equations (figure ??b).

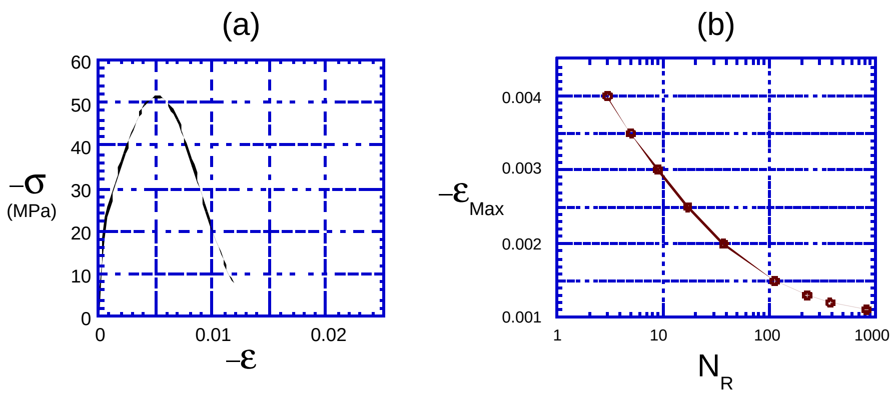

Figure 1: (a)-Monotonic response in compression, (b)-Computed fatigue curve

5. CONTINUOUS DAMAGE AND FATIGUE OF ELASTOMERS

Elastomers may be considered as hyperelastic with internal friction coupled with damage [6]. Internal viscosity is an additional dissipative mechanism and is not taken into account here.

5.1. Hyperelasticity with internal friction coupled with damage

Lemaitre damage law governed by the accumulated plastic strain rate is useless. Fortunately, its generalization to any dissipative phenomenon described in section 2.2 applies. It just needs to be formulated within the finite strains framework. The strain $\boldsymbol{E} = \frac{1}{2}(\boldsymbol{C} - \mathbf{1})$ is the Green-Lagrange strain tensor, with $\boldsymbol{C} = \boldsymbol{F}^T \cdot \boldsymbol{F}$ the dilatation tensor and $\boldsymbol{F}$ the tranformation gradient. The associated stress is the second Piola-Kirchhoff stress tensor $\boldsymbol{S}$.

Modeling internal friction of elastomers needs two internal variables. They are the internal inelastic strain $\boldsymbol{E}^\pi$ (instead of $\boldsymbol{\epsilon}^\pi$ in case of small strains) associated with the opposite of a stress denoted $\boldsymbol{S}^\pi$ and the internal sliding variable $\boldsymbol{a}$ associated with the residual micro-stress tensor $\boldsymbol{x}$.

The state potential is written in the reference configuration,

$$\rho_0 \psi = (1 - D)\left[w_1(\boldsymbol{E}) + w_2(\boldsymbol{E} - \boldsymbol{E}^\pi)\right] + \frac{1}{2} C_x \boldsymbol{a} : \boldsymbol{a} \tag{12}$$

with ρ_0 the density of the underformed material and where:

- w_1 is an hyperelastic energy density such as Mooney or Hart-Smith densities,
- for simplicity w_2 is the first term (the second) of Mooney-Rivlin development with inelastic strain $\boldsymbol{E}^\pi$ giving a non constant derivative,

$$w_2 = 4C_{20}\left[\mathrm{trace}(\boldsymbol{E} - \boldsymbol{E}^\pi)\right]^2 = C_{20}\left(I_1 - 2\ \mathrm{trace}\ \boldsymbol{E}^\pi - 3\right)^2 \tag{13}$$

For incompressible materials the stresses are obtained as

$$\begin{aligned} \boldsymbol{S} &= \rho_0 \left.\frac{\partial \psi}{\partial \boldsymbol{E}}\right|_{det\ \boldsymbol{F}=1} = (1 - D)\frac{\partial(w_1 + w_2)}{\partial \boldsymbol{E}} - P\boldsymbol{C}^{-1} \\ \boldsymbol{S}^\pi &= -\rho_0 \frac{\partial \psi}{\partial \boldsymbol{E}^\pi} = (1 - D)\frac{\partial w_2}{\partial \boldsymbol{E}} \end{aligned} \tag{14}$$

with P the internal pressure due to incompressibility. The state laws define the effectives stresses $\tilde{\boldsymbol{S}} = \boldsymbol{S}/(1 - D)$, $\tilde{\boldsymbol{S}}^\pi = \boldsymbol{S}^\pi/(1 - D)$, the residual internal stresses $\boldsymbol{x}$ and the energy density release rate $Y = w_1 + w_2$. The reversibility criterion is $f = \|\tilde{\boldsymbol{S}}^\pi - \boldsymbol{x}\| - \sigma_s < 0$ and σ_s is the reversibility limit. The evolution laws are obtained as in section 2.2 from the normality rule with the same definition for F_x and F_D, the damage law being still given by eqn (5) but with $\pi = \int_0^t \|\dot{\boldsymbol{E}}^\pi\| dt$ the cumulative measure of the internal sliding.

Note that with neither damage nor viscosity the model represents the hysteresis and the stress softening of filled elastomers [6, 7].

5.2. Calculation of the fatigue curve for elastomers

In the same manner the model of hyperelasticity with internal friction coupled with damage allows to calculate the number of cycles to rupture N_R in fatigue of elastomers. A periodic elongation $\lambda(t)$ is applied (it is the larger principal component of $\boldsymbol{F}$) with constant amplitude $\Delta\lambda$ and mean elongation λ_{moy}.

The calculated fatigue curve $\Delta\lambda$ vs N_R is given in figure 2 for a filled Styrene Butadiene Rubber (SBR). The measured elongation at rupture in tension $\lambda_R = 7.2$ is also reported on the diagram.

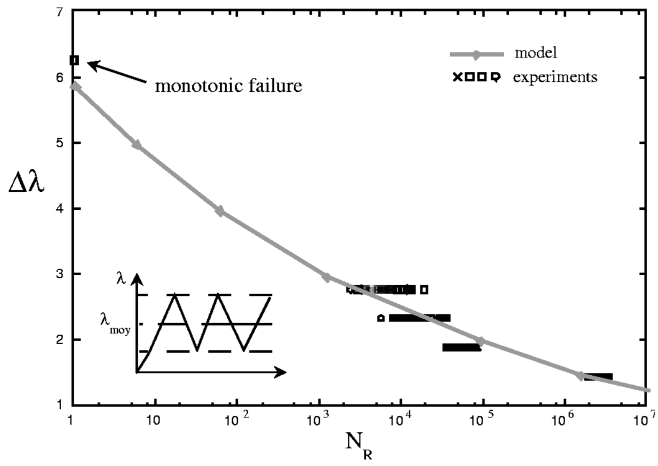

Figure 2: Fatigue curve of a filled SBR for $\lambda_{\text{moy}} = 2.53$

6. CONCLUSION

The present work extends elasto-plasticity coupled with damage framework built mainly for metals and polymers to other materials such as concrete and filled elastomers. The proposed model applies for monotonic as well as fatigue loadings. Written in the thermodynamics framework and in 3D, it will allow for the computation of failures of structures submitted to complex loadings (multiaxial, non proportional, multilevel or random fatigue. . .).

The model introduces a few damage parameters: S, s and the critical damage D_c.

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