PREDICTION OF FRACTURE INITIATION AND GROWTH DIRECTION OF ORTHOTROPIC MATERIALS

L. Nobile and C. Carloni

DISTART Department, University of Bologna, Viale Risorgimento 2, 40136 Bologna, ITALY

ABSTRACT

The solution of the elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load has been employed. It is assumed that the crack line does not coincide with an axis of elastic symmetry of the body.

Crack initiation and growth behaviour in anisotropic materials can differ from the isotropic case both qualitatively and quantitatively. Crack initiation behaviour can be readily found by application of the Strain Energy Density Criterion where the failure load and direction of crack initiation can be determined for different properties of orthotropy and directions of biaxial loading. The influence of the non singular terms on the crack initiation angle is also investigated. The Fracture Loci are also represented and compared with the ones obtained from the Maximum Circumferential Stress Criterion.

1 INTRODUCTION

The elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load has been studied in [1]. It is assumed that the crack line does not coincide with an axis of elastic symmetry of the body. The problem must be considered as one of general orthotropy, due in particular to the fact that the elastic coefficients of the material change with rotation of the reference system. The original problem has been transformed with respect to the rotated system of coordinates which is proper of the crack.

One of the main purposes of this paper is to predict the crack growth. The Strain Energy Density Theory [2-4] has been used for isotropic solids and applied by the Authors [5] to orthotropic solids with a crack aligned with an axis of elastic symmetry. It is assumed that the critical strain energy density function has a polar variation [5-8]. In this work, the Strain Energy Density Theory is extended to the general case with the crack not aligned to one of the directions of elastic symmetry. The crack initiation is determined via minimization of the ratio of the strain energy density over the material critical strain energy density.

The effect of the non-singular terms on crack growth is also studied. For isotropic materials, some Authors [9-12] have noted that retaining the second term of the series expansion, can be extremely important to study the effect of biaxial load.

2 THE INCLINED CRACK PROBLEM

An homogeneous, orthotropic and infinite plate, having a central crack, of length 2λ, inclined of an angle φ with respect to the x_1-axis of the Cartesian co-ordinates system O (x_1,x_2) is considered (Fig. 1). The crack is not aligned with one of the orthogonal axes of elastic symmetry of the body, coincident with the co-ordinates system O (x_1,x_2). Admit that the orthotropic body is subjected at infinity to a uniform biaxial load, applied along x_1 and x_2-directions. k is the biaxial load parameter.

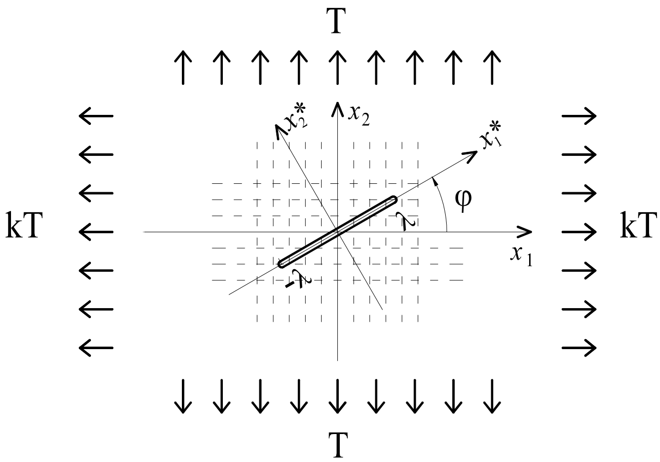

Fig. 1: The inclined crack geometry

The fracture response of an orthotropic plate having a crack not aligned with one of the axes of elastic symmetry has been carried out by Nobile et al. [1]. Other solutions have been proposed by Tsukrov and Kachanov [13], Prabhu and Lambros [14] , among others.

3 STRAIN ENERGY DENSITY THEORY

The Strain Energy Density Criterion [2-4] can be applied to predict the crack propagation in orthotropic materials [5]. The Strain Energy Density can be written as:

$$\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)^{*} = \frac{S^{*}}{r} \qquad (1)$$

where the Strain Energy Density Factor *S* has the following form:

$$S^{*} = a_{11}\mathrm{K}_{\mathrm{I}}^{2} + 2a_{12}\mathrm{K}_{\mathrm{I}}\mathrm{K}_{\mathrm{II}} + a_{22}\mathrm{K}_{\mathrm{II}}^{2} \qquad (2)$$

and the coefficients a_{11}, a_{22} and a_{12} depend on the crack to be aligned or not with one of the directions of elastic symmetry [15,16]. The star is to remind that the strain energy density is calculated referring to the axes x_1^*, x_2^*,

For isotropic materials the relative minimum of d*W*/d*V* is assumed to be associated with the direction of crack growth, that occurs when d*W*/d*V* reaches a critical value $(\mathrm{d}W/\mathrm{d}V)_c$ that depends on the material. In the orthotropic case the critical strain energy density $(\mathrm{d}W/\mathrm{d}V)_c$ depends on the direction. A simpler relation for $(\mathrm{d}W/\mathrm{d}V)_c$ is defined [5]:

$$\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^{\vartheta} = \left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^{x_1} \cos^2(\vartheta+\varphi) + \left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^{x_2} \sin^2(\vartheta+\varphi) \qquad (3)$$

where $(\mathrm{d}W/\mathrm{d}V)_c^{x_1}$ and $(\mathrm{d}W/\mathrm{d}V)_c^{x_2}$ are the critical strain energy densities in x_1- and x_2 - directions, respectively.

The crack initiation angle for orthotropic materials, ϑ_0, can be obtained minimizing the ratio:

$$R_W^* = \frac{\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)^*}{\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^{\vartheta}} \qquad (4)$$

that can be normalized for numerical analysis:

$$\overline{R}_W^* = \frac{\overline{\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)^*}}{\overline{\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^{\vartheta}}} \qquad (5)$$

where:

$$\overline{\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)^*} = \frac{C_{66}}{\mathrm{T}^2}\frac{2r}{l}\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)^* \qquad (6)$$

$$\overline{\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^{\vartheta}} = \frac{\left(\mathrm{d}W/\mathrm{d}V\right)_c^{x_1}}{\left(\mathrm{d}W/\mathrm{d}V\right)_c^{x_2}}\cos^2\left(\vartheta+\varphi\right)+\sin^2\left(\vartheta+\varphi\right) \qquad (7)$$

Referring to an orthotropic material [17] E-Glass Epoxy, with a given $\left(\mathrm{d}W/\mathrm{d}V\right)_c^{x_1}\Big/\left(\mathrm{d}W/\mathrm{d}V\right)_c^{x_2}$ ratio assumed to be equal to the square of the elastic moduli ratio along the axes of elastic symmetry, Figure 2 shows the representation of the dimensionless Strain Energy Density $\overline{\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)^*}$ and the normalized ratio $\overline{R}_W^*$ as a function of the polar angle, for a fixed value of the biaxial load parameter and for a fixed value of φ.

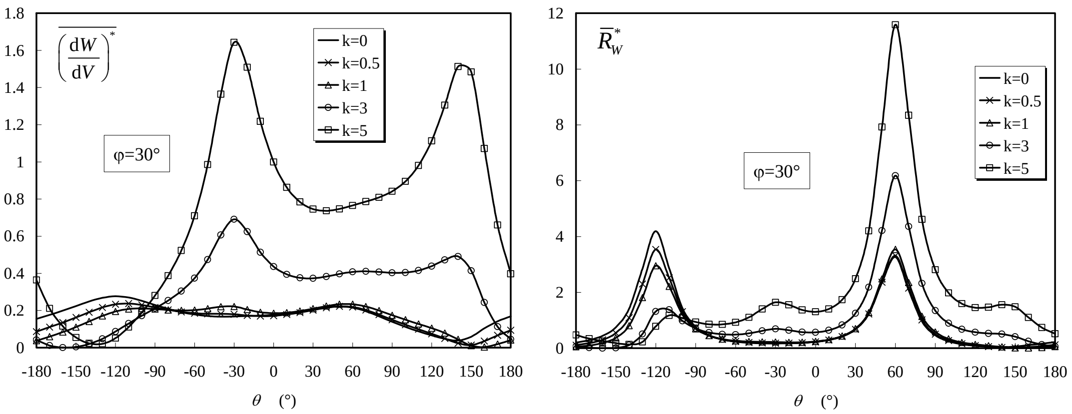

Fig. 2: *E-Glass Epoxy: dimensionless Strain Energy Density* $\overline{\left(\mathrm{d}W/\mathrm{d}V\right)^*}$ *and normalized ratio* $\overline{R}_W^*$ *vs. polar angle* ϑ *, for different values of the biaxial load parameter k, when* φ=30°.

Note that the differenced between $\overline{\left(\mathrm{d}W/\mathrm{d}V\right)}^{*}$ and $\bar{R}_{W}^{*}$ are more evident when the elastic moduli ratio increases as reported in [15,16,18].

Referring again to E-Glass Epoxy (Arcisz and Sih [17]), (Fig. 3), the crack extension angle ϑ_0 versus the crack inclination angle φ, is obtained for different values of the biaxial load parameter, as shown in figures 3.

Note that the crack initiation angle depends on the elastic properties of the orthotropic material. For a fixed value of the crack inclination angle φ, the crack initiation angle is an increasing function of k. Note also that for k>1 the crack initiation angle can be negative.

Figure 3 point out the effect of the non-singular terms on the crack initiation angle. The angle of incipient growth is also obtained neglecting the non-singular terms inside the stress components (dot lines). The influence of the non-singular terms is more evident when the mechanical behaviour of the orthotropic material is close to the isotropic one [16]. The effect of the non-singular terms depends on the value of k. For k>1 the effect of the non-singular terms increases when k increases. For k<1 the effect of the non-singular terms increases when k decreases. The influence of the non singular terms seems to be emphasised when the crack inclination angle is close to 90°.

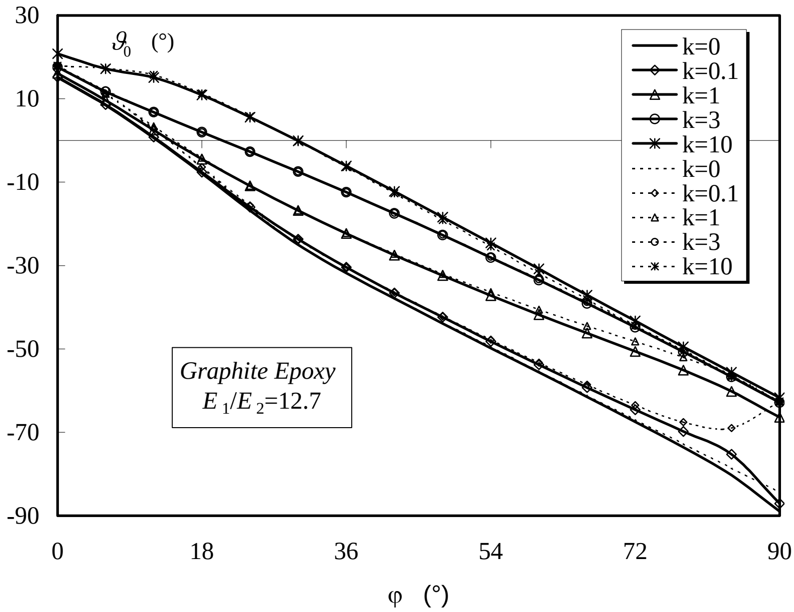

Fig. 3: E-Glass Epoxy: crack initiation angle ϑ_0 vs. crack inclination angle φ, for different values of the biaxial load parameter. Evaluation of the non singular terms effect.

4 FRACTURE LOCI

The Strain Energy Density Theory for orthotropic materials states that crack extension angle ϑ_0 satisfies the following fracture conditions:

$$\frac{\partial}{\partial\theta}\left(\frac{dW}{dV}\Big/\left(\frac{dW}{dV}\right)_c^{\theta}\right)\Bigg|_{\theta=\theta_0}=0 \qquad (8)$$

$$\frac{dW}{dV}\Bigg|_{\theta=\theta_0}=\left(\frac{dW}{dV}\right)_c \qquad (9)$$

Making use of equations (8) and (9), the parametric expression of the fracture loci can be found, as function of λ.

Fig. 4 shows the fracture loci for E-Glass Epoxy for different values of λ. The results can be compared with the ones obtained with the Maximum Circumferential Tensile Stress Criterion [16]. The fracture loci obtained through the Strain Energy Density Theory allow to consider this criterion safer and the effect of λ more emphasized.

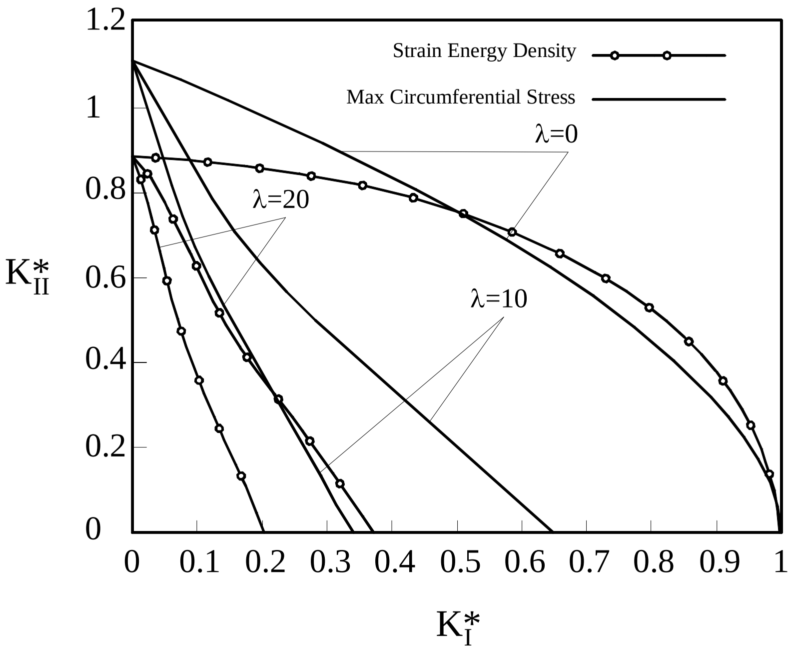

Fig. 3: E-Glass Epoxy: dependence of the fracture loci on parameter λ

5 CONCLUSIONS

The elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load has been employed. The crack line does not coincide with an axis of elastic symmetry of the body. The Strain Energy Density Theory has been extended to orthotropic materials in order to predict the crack initiation angle, defining a polar variation for the critical intensity factor for Mode-I. The crack initiation angle has been represented as a function of the crack inclination angle, for different values of the biaxial load parameter k. The dependence of crack initiation angle on the elastic properties of the orthotropic material and on the biaxial load parameter are underlined. The influence of the non-singular terms is also related to the orthotropic behaviour of the material.

The Fracture Loci are reported and compared with the ones obtained from the Maximum Circumferential Stress Criterion.

5 REFERENCES

[1] Nobile L, Piva A, Viola E (2003). On the crack problem in an orthotropic medium. *Engng. Fract. Mech.*, **71**, 529-546.

[2] Sih G.C., *Strain density factor applied to mixed mode crack problems*, Int. J. Fract. (1974); **10**:305-321.

[3] Sih G.C., *A special theory of crack propagation: methods of analysis and solutions of crack problems*, Mechanics of Fracture I, Noordhoff, Leyden, 1973.

[4] Sih G.C., *Mechanics of Fracture initiation and propagation*, Kluwer Academic Publisher, 1991.

[5] Carloni C., Nobile L., *Crack Initiation in orthotropic solids as predicted by the strain energy density theory*, Theor. Appl. Fract. Mech.(2002); **38**(2):109-119.

[6] Saouma V, Ayari ML, Leavell D (1987). Mixed mode crack propagation in homogeneous anisotropic solids. *Engng. Fract. Mech.*, **27**, 171-184.

[7] Beuth JL, Herakovich CT (1989). Analysis of crack extension in anisotropic materials based on local stress. *Theor. Appl. Fract. Mech.*, **11**, 27-46.

[8] Ayari ML, Zhiming Ye (1995). Maximum strain theory for mixed mode crack propagation in anisotropic solids. *Engng. Fract. Mech.*, **52**, 389-400.

[9] Eftis J., Subramonian N., Liebowitz H., *Crack Border Stress and Displacement Equations Revisited*, Engng. Fract. Mech. (1977); **9**:189-210.

[10] Eftis J., Subramonian N., Liebowitz H., *Biaxial Load Effects on the crack border elastic strain energy and strain energy rate*, Engng. Fract. Mech. (1977); **9**:753-764.

[11] Eftis J., Subramonian N., *The Inclined Crack under Biaxial Load*, Engng. Fract. Mech. (1978);**10**:43-67.

[12] Liebowitz H., Lee J.D., Eftis J., *Biaxial Load Effects in Fracture Mechanics*, Engng. Fract. Mech. (1978);**10**:315-335.

[13] Tsukrov I., Kachanov M. (1998). Anisotropic material with arbitrarily oriented cracks and elliptical holes: effective elastic moduli, *Int. J. Fract.*, **92**, L9-L14.

[14] Prabhu S., Lambros J. (2002). Mixed mode asymptotic crack tip fields in orthotropic materials: Derivation and range of dominance, *Int. J. Fract.*, **118**, 339-361.

[15] Nobile L, Carloni C (2003). Crack initiation in orthotropic solids. Technical Report n° 118, DISTART Department, University of Bologna.

[16] Carloni C., *Mixed Mode Fracture Response of Orthotropic Matrials*, PhD Thesis, University of Bologna, 2004.

[17] Arcisz M, Sih GC (1984). Effect of orthotropy on crack propagation. *Theor. Appl. Fract. Mech.*, **1**, 225-238.

[18] Carloni C., Nobile L., (2004). The Influence of the non-singular terms on the fracture response of cracked orthotropic plates. Proceeding of the International Conference on Restoration Recycling and Rejuvenation Technology for Engineering and Architecture Application, held in Cesena (Italy), June 7-11, 2004, pp. 241-249.