A COUPLED ELASTO-PLASTIC MICROPORE DAMAGE MODEL FOR LOW-CYCLE FATIGUE ANALYSES OF DUCTILE METALS AT FINITE STRAINS

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ABSTRACT

The present paper is concerned with the modeling of low-cycle fatigue of ductile metals at high stress levels. Two models, characterized by a combination of micropore damage models with an elasto-plastic material model incorporating isotropic and kinematic hardening at finite strains are investigated. Phenomena associated with cyclic plasticity such as the BAUSCHINGER-effect, ratcheting or mean stress relaxation, cyclic hardening or softening as well as the accumulation of microdamage are considered in the coupled model. For the modeling of cyclic plasticity, a superposition of several kinematic hardening laws according to ARMSTRONG-FREDERICK is employed. The modeling of damage accumulation under cyclic loading is based on the classical micropore models by GURSON and ROUSSELIER. Both micropore damage models are extended to cyclic loading and to combined isotropic and kinematic hardening using concepts proposed by LEBLOND ET AL. Both micropore damage models differ fundamentally in the functional relationship associated with the hydrostatic stress term in the yield conditions. In the paper, the performance of both coupled models in cyclic uniaxial loading is investigated by means of a numerical benchmark example.

1 INTRODUCTION

Failure of cyclically loaded metallic structures resulting from Low-Cycle Fatigue (LCF) occurs already after a relatively small number of loading cycles and initiates, in general, in highly plasticized zones such as notches of the structure. Typical loading scenaria are filling-, emptying- and refilling-processes of metallic tanks or pressure vessels or cyclic settlements of foundations, caused, e.g. by earthquakes. In general, low-cycle fatigue of polycrystalline materials is characterized by the development of persistent slip bands due to internal dislocation movements on the micro-scale eventually leading to the initiation and growth of microcracks. However, in ductile metals subjected to high levels of stress, damage is also caused by debonding at matrix-particle interfaces (at initial defects like inclusions or precipitates) which eventually leads to the initiation and growth of microvoids (Figure 1). This was observed e.g. in the notch root of notched tensile specimen (see [6] and references therein). Hence, micropore damage models originally developed for the description of the nucleation and growth of microvoids in metals subjected to monotonic loading [4, 9] have recently attracted attention in the context of cyclic loading (see e.g. [7, 11]).

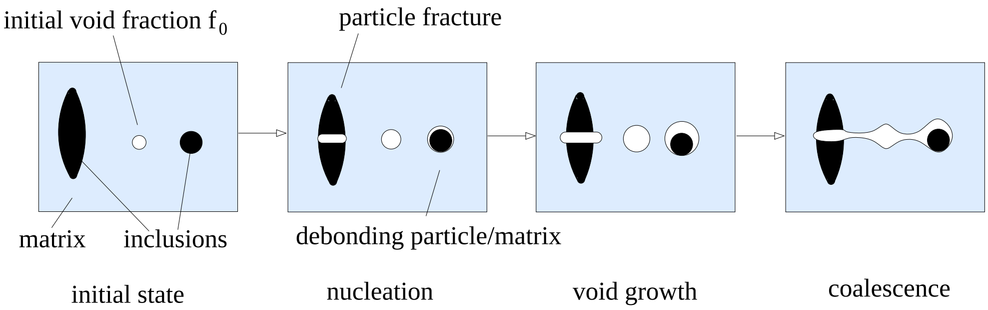

Figure 1: Typical stages of isotropic micropore damage

As far as constitutive modelling of LCF is concerned, two phenomena have to be represented: Firstly, the BAUSCHINGER-effect, ratcheting and cyclic hardening observed during cyclic loading of metals in the plastic regime and secondly, the accumulation of damage induced by microvoid growth. In the paper, two existing models for micropore damage under monotonic loading, the GURSON- [4] and the ROUSSELIER-model [9], are adopted and extended to cyclic loading. Both models are combined with kinematic hardening formulations using, according to CHABOCHE [2], a superposition of several hardening laws. Since, in general, LCF is associated with large plastic deformations, the model is formulated in a finite strain continuum mechanics framework.

2 CYCLIC PLASTICITY MODEL COUPLED WITH FATIGUE DAMAGE

In this section, first the finite strain elasto-plastic material model is presented. Subsequently, the coupling with micropore damage is described.

2.1 Elasto-plastic material model for cyclic loading at finite strains

The proposed finite strain elastoplastic model considering isotropic and kinematic hardening is based on the multiplicative decomposition of the deformation gradient $\mathbf{F} = \mathbf{F}_e\mathbf{F}_p$ into an elastic $\mathbf{F}_e$ and a plastic part $\mathbf{F}_p$ (see e.g. [10]). In what follows, tensors related to the intermediate state will be denoted by $(\hat{\ldots})$. The HELMHOLTZ-free energy function is given by

$$\psi = \psi_e + \psi_p, \qquad \psi_e = \psi_e(\hat{\mathbf{C}}, \hat{\mathbf{G}}^{-1}) \ , \qquad \psi_p = \psi_p(\hat{\mathbf{A}}_i, \alpha) \ , \qquad (i = 1, 2, ...n) \ , \tag{1}$$

with the elastic right CAUCHY-GREEN tensor $\hat{\mathbf{C}} = \mathbf{F}_e^*\mathbf{g}\mathbf{F}_e$, where $(\ldots)^*$ denotes the adjoint of a second-order tensor, the metric of the intermediate configuration $\hat{\mathbf{G}}^{-1} = \hat{G}^{ij}\,\hat{\mathbf{G}}_i \otimes \hat{\mathbf{G}}_j$ as well as the internal state variables $\hat{\mathbf{A}}_i$ and α associated with kinematic and isotropic hardening, respectively. The elastic part is defined by a hyperelastic law of NEO-HOOK-type using a deviatoric-volumetric split:

$$\psi_e = \frac{1}{2}\,\kappa\,(ln J_e)^2 + \frac{1}{2}\,\mu\,(J_e^{-2/3}\,tr(\hat{\mathbf{C}}\hat{\mathbf{G}}^{-1}) - 3) \ , \tag{2}$$

where J_e is given by $\sqrt{\det(\hat{\mathbf{C}}\hat{\mathbf{G}}^{-1})}$. For the plastic part of the free energy function the following definition is used:

$$\psi_p = \frac{1}{2}\,\beta\,H\alpha^2 + \frac{\beta}{\delta}\left((e^{-\delta\alpha}+\delta\alpha-1)(\sigma_{Y_\infty}-\sigma_{Y_0})\right) + \sigma_{Y_0}\alpha + (1-\beta)\,\frac{1}{2}\left(\sum_{i=1}^{n} c_i\,tr(\hat{\mathbf{A}}_i^2)\right) \tag{3}$$

with a hardening modulus H, an initial and an asymptotic yield stress σ_{Y_0} and σ_{Y_∞}, respectively, the model parameters δ and c_i, and a parameter β to control the amount of kinematic and isotropic hardening. From the CLAUSIUS-DUHEM-inequality follows the hyperelastic material law and a reduced form of the dissipation inequality :

$$\hat{\mathbf{S}} = 2\frac{\partial\psi_e}{\partial\hat{\mathbf{C}}}\;, \qquad \hat{\mathbf{C}}\hat{\mathbf{S}} : \hat{\mathbf{L}}_p + \sum_{i=1}^{n}\hat{\boldsymbol{\Upsilon}}_i : \dot{\hat{\mathbf{A}}}_i - q\,\dot{\alpha} \geq 0 \tag{4}$$

with the plastic velocity gradient $\hat{\mathbf{L}}_p = \dot{\mathbf{F}}_p\mathbf{F}_p^{-1}$, the back stress tensors $\hat{\boldsymbol{\Upsilon}}_i = -\psi_{p},_{\hat{\mathbf{A}}_i}$ and $q = \psi_{p},_{\alpha}$. The dissipation inequality takes the form:

$$(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}}) : \hat{\mathbf{L}}_p + \sum_{i=1}^{n}\hat{\boldsymbol{\Upsilon}}_i : (\dot{\hat{\mathbf{A}}}_i + \hat{\mathbf{L}}_p) - q\,\dot{\alpha} \geq 0 \tag{5}$$

with $\hat{\boldsymbol{\Upsilon}} = \sum_{i=1}^{n}\hat{\boldsymbol{\Upsilon}}_i$ according to CHABOCHE [2]. By using J_2-plasticity, the principle of maximum dissipation and by setting $(\dot{\hat{\mathbf{A}}}_i + \hat{\mathbf{L}}_p) = \dot{\gamma}\,\frac{b}{c_i}\,\hat{\boldsymbol{\Upsilon}}_i{}^*$, which is motivated by the fact that dissipated energy should always be positive (using positive constants b, c_i and the (positive) consistency parameter $\dot{\gamma}$), the following evolution laws are obtained:

$$\hat{\mathbf{L}}_p = \dot{\gamma}\,\hat{\mathbf{N}} = \dot{\gamma}\frac{\partial F}{\partial(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})}, \quad \dot{\hat{\mathbf{A}}}_i = -\dot{\gamma}\,\hat{\mathbf{N}} + \dot{\gamma}\frac{b_i}{c_i}\,\hat{\boldsymbol{\Upsilon}}_i{}^*\;, \quad \dot{\alpha} = \dot{\gamma}\sqrt{\frac{2}{3}}\;. \tag{6}$$

By expressing the evolution law for $\hat{\mathbf{A}}_i$ in terms of the back stress $\hat{\boldsymbol{\Upsilon}}_i$ using (3), the final kinematic hardening law of ARMSTRONG-FREDERICK-type [1] formulated with respect to the reference configuration is obtained as:

$$\dot{\hat{\boldsymbol{\Upsilon}}}_i = \dot{\gamma}\,(1-\beta)\,\left(c_i\,\mathbf{N}^* - b_i\,\boldsymbol{\Upsilon}_i\right) + \mathbf{L}_p^*\boldsymbol{\Upsilon}_i - \boldsymbol{\Upsilon}_i\mathbf{L}_p^*\;, \qquad (i = 1, 2, \cdots, n)\;. \tag{7}$$

2.2 Extension to GURSON- and ROUSSELIER-type ductile damage models

Following the work of LEBLOND ET AL. [5] who proposed a consistent extension of the original GURSON-model to combined isotropic and kinematic-hardening, the corresponding yield functions are formulated as:

$$\begin{aligned} F_{Gurson} &= \frac{(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})^2_{eq}}{q_1^2} + 2\,f^*\,q\,cosh\left(\frac{3}{2}\frac{(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})_m}{q_2}\right) - 1 - (q\,f^*)^2 = 0\;, \\ F_{Rousselier} &= \frac{(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})_{eq}}{q_1} + f^*\,q\,exp\left(\frac{3}{2}\,\frac{(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})_m}{q_2}\right) - 1 = 0\;, \end{aligned} \tag{8}$$

with the equivalent relative stress $(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})_{eq} = \sqrt{\frac{3}{2}\left(dev(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})\right)^2 : \hat{\mathbf{I}}}$, the hydrostatic stress $(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}})_m = \frac{1}{3}\left(\hat{\mathbf{C}}\hat{\mathbf{S}} - \hat{\boldsymbol{\Upsilon}}\right) : \hat{\mathbf{I}}^*$, the isotropic hardening parameter q_1 belonging to the deviatoric part, the isotropic hardening parameter q_2 belonging to the hydrostatic part, and the void volume fraction f^* defined according to NEEDLEMAN & TVERGAARD [8]. The yield function F_{Gurson} describes an ellipsoid in the principle stress space, whereas the ROUSSELIER-type yield function is non-symmetric with respect to the plane $tr(\hat{\mathbf{C}}\hat{\mathbf{S}}) = 0$. Hence both models fundamentally differ in their descriptions of damage accumulation in cyclic loading of ductile metals. The hardening parameters q_1 and q_2 are given by:

$$q_1 = \frac{(1-f_0)}{(1-f)}\,\sigma_{Y_0} + \frac{1}{(1-f)}\,\beta\,\sigma_1 \;, \qquad q_2 = \frac{(1-f_0)}{(1-f)}\left(\sigma_{Y_0} - \frac{1}{ln(f)}\,\beta\,\sigma_2\right) \;, \tag{9}$$

where the transformation from CAUCHY stresses to MANDEL stresses in the yield conditions requires the use of the plastic JACOBIan-determinant $J_p = \frac{(1-f_0)}{(1-f)}$. To extend the micropore damage models to non-proportional loading paths a TAYLOR series expansion of the isotropic hardening terms σ_1 and σ_2 is proposed, which for the case of linear isotropic hardening results in:

$$\dot{\sigma}_1 = \int_{a^3}^{b^3} H\,\left(< d_{eq}^2 >_r^{1/2}\right) dr^3 \;, \qquad \dot{\sigma}_2 = \int_{a^3}^{b^3} H\,\left(< d_{eq}^2 >_r^{1/2}\right) \frac{dr^3}{r^3} \;. \tag{10}$$

b and a are the radii of the plasticized spherical volume cell and the center pore (with the coupling $f = \frac{a^3}{b^3}$). $< d_{eq}^2 >_r$ represents the averaged (over a slice of thickness dr) equivalent plastic strain increment $< d_{eq}^2 >_r = \hat{D}_{peq}^2 + 4\,\frac{b^6}{r^6}\,\hat{D}_{pm}^2$ with $\hat{D}_{peq} = \sqrt{\frac{2}{3}dev(\hat{\mathbf{D}}_p) : dev(\hat{\mathbf{D}}_p^\sharp)}$ and $\hat{D}_{pm} = \frac{1}{3}\,tr(\hat{\mathbf{L}}_p)$. $\hat{\mathbf{D}}_p$ is defined by $\hat{\mathbf{D}}_p = \frac{1}{2}(\hat{\mathbf{G}}\hat{\mathbf{L}}_p + \hat{\mathbf{L}}_p^*\hat{\mathbf{G}})$ with $\hat{\mathbf{D}}_p^\sharp = \hat{\mathbf{G}}^{-1}\hat{\mathbf{D}}_p\hat{\mathbf{G}}^{-1}$ being its associated form.

The back stress tensor used in the kinematic hardening model can be split into a volumetric and a deviatoric part as follows:

$$\hat{\mathbf{\Upsilon}} = \sum_{i=1}^{n} dev(\hat{\mathbf{\Upsilon}}_i) + \hat{\Upsilon}_m\,\hat{\mathbf{I}}^* \;. \tag{11}$$

Considering J_p, the volumetric part and the deviatoric part are defined by:

$$\begin{aligned} &\hat{\Upsilon}_m = J_p\,\hat{\Gamma}_m \quad \text{with} \quad \dot{\hat{\Gamma}}_m = (1-\beta)\,\tfrac{4}{3}\,\hat{D}_{pm}\,\textstyle\int_{a^3}^{b^3} H\,b^3\,\frac{dr^3}{r^6} \;, \\ &dev(\hat{\mathbf{\Upsilon}}_i) = J_p\,(1-f)\,dev(\hat{\mathbf{\Gamma}}_i) \;, \;\; dev(\dot{\hat{\mathbf{\Gamma}}}_i) = (1-\beta)\,\left(c_i\,dev(\hat{\mathbf{L}}_p^*) - \dot{\gamma}\,\bar{b}_i\,dev(\hat{\mathbf{\Gamma}}_i)\right) \;. \end{aligned} \tag{12}$$

Note that $\bar{b}_i$ is different for both models and is obtained by considering the asymptotic stress in the uniaxial tension-compression case. Nucleation of voids is modeled according to CHU & NEEDLEMAN [3]. Considering preservation of mass [9], the growth law is obtained as:

$$\dot{f} = \dot{f}_{nucl} + \dot{f}_{growth} \;, \qquad \dot{f}_{growth} = (1-f)tr(\hat{\mathbf{L}}_p) \;. \tag{13}$$

3 NUMERICAL EXAMPLES AND DISCUSSION

To investigate the prediction capability of the micropore damage models one single finite element has been analyzed numerically over 45 cycles under deformation-controlled uniaxial tension (see Figure 2). The results obtained from the GURSON model (see Figure 3) show that the void volume fraction f increases in the first cycle, then drops below the initial

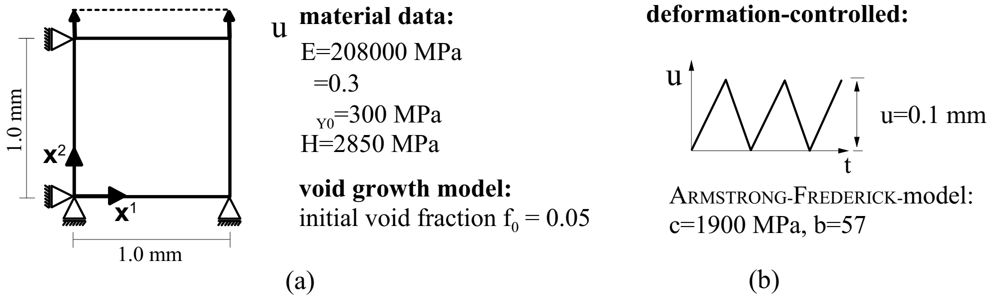

Figure 2: Tensile test of a single element under cyclic deformation-controlled loading: (a) system and material parameters, (b) cyclic loading history

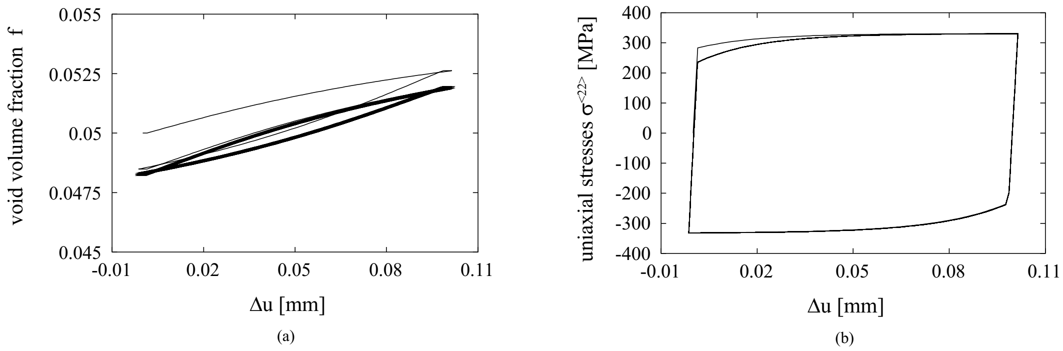

Figure 3: Uniaxial cyclic tensile test of a single element during 45 cycles: Results obtained for the GURSON-model and a ARMSTRONG-FREDERICK kinematic hardening law. (a) Result for the void volume fraction, (b) numerical results for the uniaxial stresses $\sigma^{<22>}$

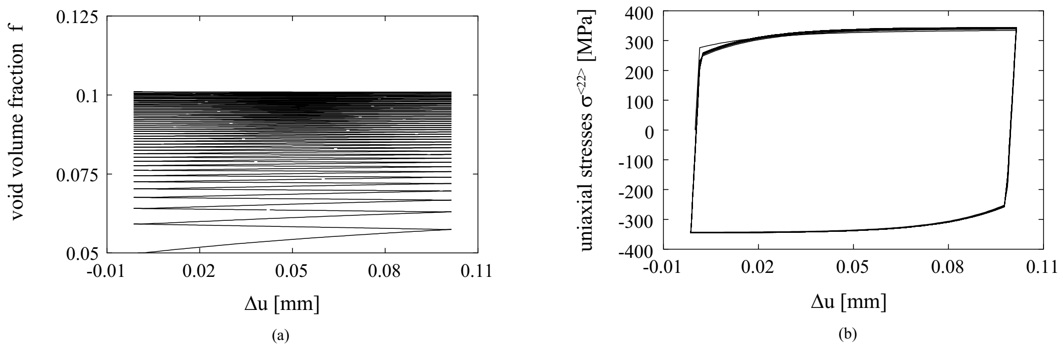

Figure 4: Uniaxial cyclic tensile test of a single element during 45 cycles: Results obtained for the ROUSSELIER-model and a ARMSTRONG-FREDERICK kinematic hardening law. (a) Result for the void volume fraction, (b) numerical results for the uniaxial stresses $\sigma^{<22>}$

void volume fraction of $f_0 = 0.05$ and continues in a stable loop in the $f - \Delta u$-diagram. Accordingly, the softening effect in the stress component $\sigma^{<22>}$ (Figure 3b) is negligible. In contrast, the ROUSSELIER-model leads to a continuously growing void volume fraction f (see Figure 4a). The increments of the void volume fraction, however, are decreasing with increasing number of cycles. The softening effect in the stress component $\sigma^{<22>}$ (Figure 4b) is more pronounced. Hence, from this numerical test, it is concluded that the ROUSSELIER model is better suited to predict accumulation of damage under cyclic loading. However, closure of pores in a plasticized matrix material, as predicted by the GURSON model, has also been observed in unit cell calculations carried out in [11]. Further studies, based on more complex boundary value problems such as notched tensile specimen, have to be performed before further conclusions on the performance of microvoid models in cyclic uni- and multiaxial loading situations can be drawn.

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