DAMAGE AND MACROSCOPIC VANISHING MOTIONS

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ABSTRACT

Damage, for instance damage of concrete, results from microscopic motions. The basic idea we have developed is to account for the power of the microscopic motions in the power of the interior forces. Thus we modify the expression of the power of the interior forces and assume that it depends on the damage rate, which is clearly related to the microscopic motions. Furthermore we assume that it depends also on the gradient of the damage rate to account for microscopic interactions. The consequences of this assumption and a careful treatment of the fact that the damage quantity is a proportion (i.e., quantities with values between 0 and 1) give the basic equations.

The method being based on the separation between the descriptions of the macroscopic and microscopic motions, a natural question arises: what occurs when a macroscopic motion becomes microscopic? We answer this question by showing that the damaging effects of macroscopic vanishing motion remain whereas the macroscopic motion goes to zero. The effects of vanishing macroscopic motions are transferred from the macroscopic equation of motion to the microscopic equation of motion through the intervention of a damage source.

1. INTRODUCTION

Damage, for instance, damage of concrete, results from microscopic motions. The basic idea we have developed is to account for the power of the microscopic motions in the power of the interior forces (e.g. Frémond [4]). Thus we modify the expression of the power of the interior forces and assume that it depends on the damage rate, which is clearly related to the microscopic motions. Furthermore we assume that it depends also on the gradient of the damage rate to account for microscopic interactions. The consequences of this assumption and a careful treatment of the fact that the damage quantity is a proportion (i.e., quantities with values between 0 and 1) give the basic equations. For the sake of simplicity, we assume that the temperature is constant and all thermal effects are deleted.

2. THE EQUATIONS OF MOTION

Let us consider a solid, for instance, a piece of concrete and investigate its damage. Within the framework of continuum mechanics, we want to describe on the macroscopic level the effects of micro-fractures and micro-cavities, which result in the decrease of the material stiffness. Let the scalar $\beta(x,t)$ be the macroscopic damage quantity with value 1 when the material is undamaged and value 0 when completely damaged.

The basic idea of the theory is to refine the power of the interior forces, as described in Frémond [4]. Within the solid, there exist microscopic motions, which produce damage, i.e., the micro-fractures or the micro-cavities. We think that the power of these microscopic motions must be taken into account in the power of the interior forces. Thus the power of the interior forces is chosen to depend on the strain rates $D(\vec{U})$ where $\vec{U}$ is the macroscopic velocity, and also on $d\beta / dt$ and $grad(d\beta / dt)$. Those latter quantities are clearly related to the microscopic motions. The gradient of damage is introduced to take into account the influence of damage at a material point on damage of its neighbourhood. The actual power of the interior forces which takes into account the microscopic motions of domain Ω occupied by the solid is chosen as

$$-\int_{\Omega} \sigma : D(\vec{U}) d\Omega - \int_{\Omega} B \frac{d\beta}{dt} + \vec{H} \cdot grad(\frac{d\beta}{dt}) d\Omega,$$

where σ is the stress tensor. Two new non-classical quantities appear, B, the interior energy of damage, and $\vec{H}$, the flux vector of energy of damage. The power of the exterior forces involves the power

$$\int_{\Omega} A \frac{d\beta}{dt} d\Omega,$$

where quantity A is a volume source of damage, which can be produced by chemical, electrical, or radiative actions, which break the links inside a material, concrete, for instance, without macroscopic deformations. Examples of sources of damage are given in Frémond [4]. The principle of virtual power for quasi-static evolutions gives two sets of equations of motion

$$div\sigma + \vec{f} = 0 \ in \Omega, \ \sigma\vec{N} = \vec{g} \quad in \partial\Omega, \qquad (1),$$

$$div\vec{H} - B + A = 0 \ in \Omega, \ \vec{H} \cdot \vec{N} = \vec{g} \quad in \partial\Omega, \qquad (2),$$

where $\vec{N}$ is the outward unit normal vector to Ω. Equations (2) are new and non-classical. They account for the effects of the microscopic motions.

3. THE CONSTITUTIVE LAWS

The value of the damage quantity β is between 0 and 1

$$0 \le \beta \le 1. \qquad (3),$$

is often thought of as the volume fraction of micro-voids or the quotient of the modulus of the damaged material divided by the modulus of the undamaged material.

The internal constraint (3) on the damage quantity is a physical property. Thus, it must be taken into account by the functions, which describe the whole physical properties, i.e., either the free energy Ψ or the dissipative forces, which can be defined by a pseudo-potential of dissipation Φ. The free energy is chosen because it describes properties related to the state, and because the dissipative forces describe properties related to the velocities. For the sake of simplicity, small perturbations are assumed, and $\varepsilon(\vec{u})$ denotes small deformation where $\vec{u}$ is the small displacement and choose the free energy

$$\Psi(\varepsilon,\beta,grad\,\beta)=\frac{\beta}{2}\left\{2\mu\varepsilon:\varepsilon+\lambda\left(tr\varepsilon\right)^2\right\}+w(1-\beta)+\frac{k}{2}\left(grad\,\beta\right)^2+I(\beta),$$

where I is the indicator function of the set $[0,1]$, ($I(x)=0,$ if $x\in[0,1]$ and $I(x)=+\infty$, if $x\notin[0,1]$) which takes into account internal constraint (3). Thus the free energy has its physical value for any actual or physical value of β. The free energy is equal to $+\infty$ for any value of β which is physically impossible. Due to the expression of the actual power of the interior forces depending on the velocities $d\beta/dt$ and $grad(d\beta/dt)$, it is natural to assume that the free energy depends on β and $grad\,\beta$. The generalised derivatives of the indicator function, $\partial I(\beta)$ the subdifferential set, contains the reaction to the internal constraint (3) which is zero for $0<\beta<1$, positive for $\beta=1$, and negative for $\beta=0$. The parameters λ and μ are the Lamé parameters. The first term of Ψ is a quadratic function with respect to the strain tensor and a linear function with respect to the damage quantity. It gives the simplest model where damage is coupled with elasticity. The quantity w is the initial damage threshold expressed in terms of volumetric energy. It is equivalent to the initial threshold expressed in terms of damage force in the models issued from the classical theory (e.g. Lemaitre Chaboche [7], Lemaitre [8]). The parameter k measures the influence of a material point on its neighbourhood.

For the sake of simplicity, we assume that there is only dissipation with respect to $d\beta/dt$ Thus the pseudo-potential of dissipation depends only on $d\beta/dt$

$$\Phi(\frac{d\beta}{dt})=\frac{c}{2}\left(\frac{d\beta}{dt}\right)^2+I_-\left(\frac{d\beta}{dt}\right).$$

The quantity c is the viscosity parameter of damage. The function I_- is the indicator function of the interval $(-\infty,0]$, ($I_-(x)=0$ if $x\le 0$, $I_-(x)=+\infty$ if $x>0$). The effect of this indicator function is to require that $d\beta/dt$ is negative: the broken microscopic links cannot mend by themselves for the material, which has been chosen. This is the case for concrete. This is not the case of some polymers, which have the property of recovering their strength once they have been damaged. The pseudo-potential of dissipation adapted to describe them must not involve the indicator function I_-. The constitutive relationships are

$$\sigma=\frac{\partial\Psi}{\partial\varepsilon},\ B=\frac{\partial\Psi}{\partial\beta}+\frac{\partial\Phi}{\partial(d\beta/dt)},\ \vec{H}=\frac{\partial\Psi}{\partial(grad\,\beta)}.$$

The equations of evolution are obtained by using the previous constitutive laws and the Equations of motion (1) and (2). They are

$$div\left(\beta\left\{2\mu\varepsilon+\lambda tr\varepsilon 1\right\}\right)+\vec{f}=0 \quad in\,\Omega,$$

$$\sigma\vec{N}=\vec{g} \quad in\,\partial\Omega, \qquad (4),$$

$$w+A-\frac{1}{2}\left\{2\mu\varepsilon:\varepsilon+\lambda\left(tr\varepsilon\right)^2\right\}\in c\frac{d\beta}{dt}-k\Delta\beta+\partial I(\beta)+\partial I_-(\frac{d\beta}{dt}) \quad in\,\Omega,$$

$$k\frac{\partial\beta}{\partial N}=0\, in\,\partial\Omega, \beta(x,0)=\beta_0(x) \quad in\,\partial\Omega, \qquad (5),$$

where $\Delta\beta$ is the Laplacian of β. The function β_0 is the initial value of the damage, with $\beta_0=1$ when the structure is initially undamaged.

Equations (5) are the equations of evolution of damage in domain Ω. The elements of $\partial I(\beta)$ and $\partial I_-(d\beta/dt)$ are reactions which force β to remain between 0 and 1 and $d\beta/dt$ to be negative. In the first equation (5), the source of damage in the left-hand side is the external source A and a deformation energy. That agrees with the experimental observations. This model is sufficient to describe the damage phenomena during multi-axial loading and unloading without changing the sign of the exterior actions. Examples, applications and upgraded models can be found in Frémond [4], Frémond Nedjar [6], Nedjar [9].

4. A MACROSCOPIC MOTION BECOMES MICROSCOPIC

There are two equations of motion: the first equation (4) accounts for the macroscopic motion whereas the second equation (5) accounts for the microscopic motion. One can wonder what occurs when a macroscopic motion becomes microscopic. In order to answer this question let apply exterior actions to a structure in such a way that the amplitude of the resulting macroscopic motions become smaller and smaller. The macroscopic motions vanish and it is no longer possible to consider that they are macroscopic: one has to consider them as microscopic. One may wonder if their damaging effects also vanish. If they do not, how are they taken into account by the theory? Is there a transfer from the equation which describes the macroscopic motions towards the equation which describes the microscopic motions?

We are interested to consider the behaviour of the solutions β_τ, $\vec{u}_\tau$, to the initial and boundary values problem associated with eqs. (4), (5), in the case when a vanishing sequence of external forces $\vec{f}_\tau$ is applied, i.e. $\vec{f}_\tau\to 0$ as τ tends to 0. The partial differential equations (4), (5) are difficult to solve, (e.g. Bonetti Schimperna [2]) although there are results in dimension one (e.g. Frémond Kuttler Shillor [3]). They may be modified by adding some non-linear viscosity (e.g. Bonetti Frémond [1]). We can exploit an a priori estimates-passage to the limit procedure to perform the required asymptotic analysis. Thus, at a first step, we aim to find existence and uniqueness of solutions to the system for any fixed τ and, on a second step, we prove properties on these solutions which allow us to pass to the limit as τ tends to 0 (e.g. Bonetti Frémond [2]).

Then, as one can prove that the displacement vanishes

$$\lim_{\tau\to 0}\vec{u}_\tau = 0,$$

namely there are not macroscopic displacements or deformations at the limit. The limit of the mechanical source of damage, $\left\{2\mu\varepsilon:\varepsilon+\lambda\left(tr\varepsilon\right)^2\right\}/2$ may be different from 0

$$\lim_{\tau\to 0}\frac{1}{2}\left\{2\mu\varepsilon:\varepsilon+\lambda\left(tr\varepsilon\right)^2\right\} = D \geq 0.$$

The limit damage, $\beta = \lim_{\tau\to 0}\beta_\tau$ satisfies

$$w + A - D \in c\frac{d\beta}{dt} - k\Delta\beta + \partial I(\beta) + \partial I_-(\frac{d\beta}{dt}) \quad in\,\Omega. \qquad (6),$$

Hence, let us discuss the mechanical meaning of D. It is the weak limit of deformation energies associated to the vanishing sequence of macroscopic motions and, in general, $D \geq 0$. Thus, this function, representing a source of damage in (6), can be interpreted as the remaining damaging effect of macroscopic motions, acting at a microscopic level. It follows that a sequence of vanishing macroscopic motions can retain its damaging effect, at the limit, as a source of damage in the equation of microscopic motions. A closed form example in dimension one is given in Frémond [5]. In order to support this limit analysis, in the next section we briefly discuss the balance of the energy of the problem.

5. ENERGY PHENOMENA

The work which is provided to the structure

$$T(t) = \lim_{\tau\to 0} T_\tau(t) = \int_0^t\int_\Omega \left\{\vec{f}_\tau\frac{d\vec{u}_\tau}{dt} + A\frac{d\beta_\tau}{dt}\right\} d\Omega dt,$$

is divided between damaging external work

$$W(t) = \int_0^t\int_\Omega \left\{A - D\right\}\frac{d\beta}{dt} d\Omega dt,$$

external source of heat $Q(t)$, and stored energy

$$S(t) = \int_\Omega D(t)^2 d\Omega + \int_\Omega D(t)\beta(t) d\Omega. \qquad (7),$$

In particular, when no instantaneous damage work is applied at the final time $D(t) = 0$, from (7) it results $S(t) = 0$ and, consequently, the work which has been provided is exactly the sum of the damaging work $W(t)$ and of the heat sources $Q(t)$ resulting from the dissipative phenomena (e.g. Bonetti Frémond [1]).

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