A MICROMECHANICS APPROACH TO QUANTIFY STRENGTH MISMATCH EFFECTS ON FRACTURE BEHAVIOR OF INTERFACE CRACKS

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ABSTRACT
This study extends the Weibull stress approach to address effects of strength mismatch on macroscopic fracture toughness of interface cracks. The approach builds on the Beremin model to establish a relationship between the microregime of fracture and macroscopic crack driving forces for bimaterial media by adopting the Weibull stress (σ_w) as a probabilistic fracture parameter. Plane-strain, small scale yielding (SSY) reference fields for stationary interface cracks are presented to provide a measure of mismatch effects. The analyses show a strong effect of mismatch level on the magnitude of the Weibull stress which enables assessments of fracture behavior in interface cracks.

1 INTRODUCTION

This study extends the Weibull stress approach [1] to address effects of strength mismatch on macroscopic fracture toughness of interface cracks. The approach builds upon the Beremin model [2] to establish a relationship between the microregime of fracture and macroscopic crack driving forces (such as the stress intensity factor, K, and the J-integral) for bimaterial media by adopting the Weibull stress (σ_w) as a probabilistic fracture parameter. In the context of probabilistic fracture mechanics, the Weibull stress emerges as a near-tip fracture parameter to describe the coupling of remote loading (as measured by K or J) with a micromechanics model describing transgranular cleavage. A key feature of this methodology is that σ_w incorporates *both* the effects of stressed volume (the fracture process zone) and the potentially strong changes in the character of the near-tip stress fields due to strength mismatch in the bimaterial system. Plane-strain, small scale yielding (SSY) reference fields for stationary interface cracks are presented to provide a measure of mismatch effects. The analyses show a strong effect of mismatch level on the magnitude of the Weibull stress which enables assessments of fracture behavior in interface cracks. These SSY results exhibit the essential features of the micromechanics approach in correlating macroscopic fracture toughness with strength mismatch variations.

2 THE WEIBULL STRESS FOR INTERFACE CRACKS

Recent developments in micromechanics methodologies to describe transgranular cleavage fracture have adopted the Weibull stress (σ_w) [1,2] as a fracture parameter which provides a robust coupling between the microregime of fracture (which includes a local failure criterion *and* the stresses that develop ahead of a macroscopic crack) with macroscopic (remote) loading. For a stationary macroscopic crack lying in homogeneous materials, the Weibull stress is given by integration of the (local) principal stress over the fracture process zone in the form [1]

$$\sigma_w = \left[\frac{1}{V_0} \int_{\Omega} {\sigma_1}^m d\Omega \right]^{1/m} , \tag{1}$$

where σ_1 is the maximum principal stress, V_0 is a reference volume, Ω denotes the volume of the (near-tip) fracture process zone defined by the loci $\sigma_1 \geq \lambda\sigma_0$ with $\lambda \approx 2$, and parameter m (the Weibull modulus) define the microcrack distribution.

For bimaterial systems, such as the interface crack along two different materials schematically represented in Fig. 1, the fracture process may result from a complex interplay of the operative failure mechanism for each individual material. For a given remote loading, the *mismatch* in mechanical properties, such as E/σ_0 and ν, between both materials produce crack-tip stress and strain fields quite different than the fields that arise in the corresponding homogeneous material. Moreover, with increased levels of strength mismatch between both media, the fracture process zone for each material differs significantly from those for a homogeneous material.

A simplified form for σ_w applicable to bimaterial systems is adopted in the present work. Referring to Fig. 1 which shows a crack lying along an interface separating material 1 and material 2, we define the Weibull stress corresponding to each media as

$$\sigma_w^k = \left[\frac{1}{V_0} \int_{\Omega_k} {\sigma_1}^{m_k} d\Omega_k \right]^{1/m_k} , \tag{2}$$

where the subscript k denotes the *k-th* material, Ω_k is the fracture process zone for material k and m_k denotes the Weibull modulus for material k. Since the Weibull stress can be associated with a crack-tip driving force incorporating a local failure criterion (see Ruggieri and Dodds [1]), the present definition for σ_w implies that fracture of the entire bimaterial system takes place in *only one* material.

3 COMPUTATIONAL PROCEDURES AND FINITE ELEMENT MODELS

3.1 Small Scale Yielding Model

The modified boundary layer model (MBL) [6] simplifies the generation of numerical solutions for stationary interface cracks under well-defined SSY conditions with varying levels of strength mismatch. Figure 1 shows the plane-strain finite element model for an infinite domain, single-ended interface crack with an initially blunted notch (finite root radius, $\rho = 10\mu$m). The SSY model has one thickness layer of 4130 8-node, 3-D elements with plane-strain constraints imposed on all nodes.

With the plastic region limited to a small fraction of the domain radius, $R_p < R/20$ (R is the radius of the outer circular boundary), the general form of the asymptotic crack-tip stress fields well outside the plastic region in polar coordinates (r, θ) is given by [5]

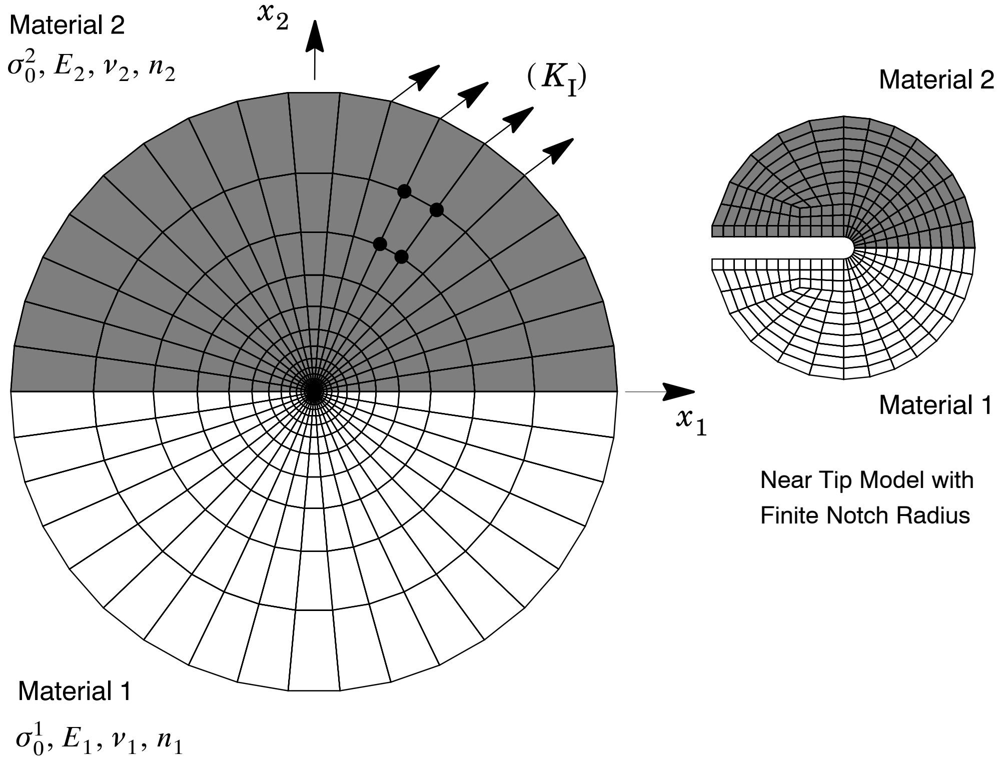

Figure 1 *SSY model for an interface crack with a K-field imposed on boundary.*

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \tag{3}$$

where K is the stress intensity factor, $f_{ij}(\theta)$ define the angular variations of in-plane stress components. In the present investigation, numerical solutions are generated by imposing displacements of the elastic, Mode I singular field on the outer circular boundary ($r = R$) which encloses the crack

$$u(R, \theta) = K_I \frac{1+\nu}{E} \sqrt{\frac{R}{2\pi}} \cos\left(\frac{\theta}{2}\right)(3 - 4\nu - \cos\theta) \tag{4}$$

$$v(R, \theta) = K_I \frac{1+\nu}{E} \sqrt{\frac{R}{2\pi}} \sin\left(\frac{\theta}{2}\right)(3 - 4\nu - \cos\theta)) \tag{5}$$

3.2 Constitutive Models and Finite Element Procedures

The elastic-plastic material employed in the analyses follows a J_2 flow theory with conventional Mises plasticity. The uniaxial true stress-logarithmic strain curve obeys a simple power-hardening model in the form

$$\frac{\epsilon}{\epsilon_0} = \frac{\overline{\sigma}}{\sigma_0} \quad \epsilon \le \epsilon_0 \ ; \qquad \frac{\epsilon}{\epsilon_0} = \left(\frac{\overline{\sigma}}{\sigma_0}\right)^n \quad \epsilon > \epsilon_0 \tag{6}$$

where σ_0 and ϵ_0 are the reference (yield) stress and strain, and n is the strain hardening exponent.

The three-dimensional computations reported here are generated using the research code WARP3D [4]. The finite element analyses consider material flow properties for the base metal (material 1) representing a typical structural steel with $n = 10$ (moderate hardening) and $E/\sigma_0 = 500$. From these mechanical properties for the base metal, the matrix analysis for the bimaterial system is constructed by adopting the flow properties for material 2 as shown in Table 1. These ranges of properties reflects the upward trend in yield stress with the decrease in strain hardening exponent characteristic of ferritic steels. In all analyses, $E = 206$ GPa and $\nu = 0.3$.

Table 1 Mechanical properties for the bimaterial system employed in the analyses.

	σ_0^1 (MPa)	n^1	σ_0^2 (MPa)	n^2
0.8 Undermatch	412	10	330	5.9
Evenmatch	412	10	412	10
1.5 Overmatch	412	10	618	18

Numerical computations of the Weibull stress used to construct σ_w *vs.* K trajectories for the SSY model are performed using the research code WSTRESS [3] which implements a finite element form of Beremin's formulation [2]. The process zone used here includes all material inside the loci $\sigma_1 \geq \lambda\sigma_0$, with $\lambda = 2$; results for σ_w differ little over a wide range of λ-values.

4 EFFECT OF STRENGTH MISMATCH ON FRACTURE RESISTANCE

The simplified form of the Weibull stress, given by Eq. (2), provides the basis to assess effects of strength mismatch on the fracture resistance for bimaterial systems, such as the one depicted in Fig. 1. The procedure relies on the notion of σ_w as the crack-tip driving force [1] which establishes a robust coupling between the applied load and level of mismatch thereby describing the local, crack-tip response for cleavage fracture. In all analyses, the Weibull modulus is adopted as $m = 20$, which is representative of cleavage fracture in typical ferritic steels [1,2].

Figure 2 displays the evolution of the Weibull stress normalized by the yield stress σ_0^k ($k = 1$, 2) (note that the normalizing stress corresponds to the material upon which σ_w is computed) with increased loading. For all levels of mismatch and material combination, the Weibull stress increases monotonically with the K-levels. The most striking feature of these results is the development of σ_w with increasing loading for the mismatched conditions (see Figs. 2(b-c). The *normalized* levels of σ_w for the *weaker* material are consistently higher than the corresponding σ_w-levels for the stronger material in all material combinations. In the context of the micromechanics approach adopted in the present work, such results provide important features associated with the fracture resistance of bimaterial systems. The physical significance is this: strength mismatch alters the fracture resistance for the bimaterial media. In particular, the effect of strength overmatch has important implications for the fracture behavior of the weaker material. Although strength overmatch ‘‘shields’’ the stronger material, it potentially causes substantial increase in the crack-tip driving force in the weaker material as quantified by σ_w. Such behavior may adversely impact the load-carrying capacity of the entire bimaterial system with an interface crack.

6 CONCLUDING REMARKS

This work has presented a micromechanics-based framework to assess the effects of strength mismatch on fracture resistance of bimaterial systems. To incorporate the pronounced effects of strength mismatch on the crack-tip stress fields and on the near-tip stressed volume (i.e., the fracture process zone ahead of the crack), the methodology adopts the Weibull stress, σ_w, as a near-tip, local fracture parameter. The strength mismatch affects the evolution of σ_w under increasing applied load which reflects on fracture resistance of interface cracks. While the work has not explored an extensive range of material combinations, the relative operational simplicity and robustness of the Weibull stress approach encourages further investigations in procedures for fracture assessments of bimaterial media with interface cracks. Further work is in progress to develop a more refined framework employing the Weibull stress which is more applicable to bimaterial and multimaterial systems.

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REFERENCES

1. Ruggieri, C. and Dodds, R. H., ‘‘A Transferability Model for Brittle Fracture Including Constraint and Ductile Tearing Effects: A Probabilistic Approach,’’ *International Journal of Fracture*, Vol. 79, pp. 309-340, 1996.

2. Beremin, F.M, ‘‘A Local Criterion for Cleavage Fracture of a Nuclear Pressure Vessel Steel,’’ *Metallurgical Transactions,* Vol. 14A, pp. 2277–2287, 1983.

3. Ruggieri, C., ‘‘WSTRESS Release 3.0: Numerical Computation of Probabilistic Fracture Parameters for 3-D Cracked Solids,’’ *BT–PNV–51(Technical Report)*, EPUSP, University of São Paulo, 2002..

4. Koppenhoefer, K., Gullerud, A., Ruggieri, C., Dodds, R. and Healy, B., ‘‘WARP3D: Dynamic Nonlinear Analysis of Solids Using a Preconditioned Conjugate Gradient Software Architecture’’, *Structural Research Series (SRS) 596*, UILU–ENG–94–2017, University of Illinois at Urbana–Champaign, 1994.

5. Williams, M.L., ‘‘On the Stress Distribution at the Base of a Stationary Crack’’, *Journal of Applied Mechanics,* Vol. 24, pp. 109–114, 1957.

6. Larsson, S. G. and Carlsson, A. J., ‘‘Influence of Non–Singular Stress Terms and Specimen Geometry on Small Scale Yielding at Crack–Tips in Elastic–Plastic Materials’’, *Journal of the Mechanics and Physics of Solids*, Vol. 21, pp. 447–473, 1973.

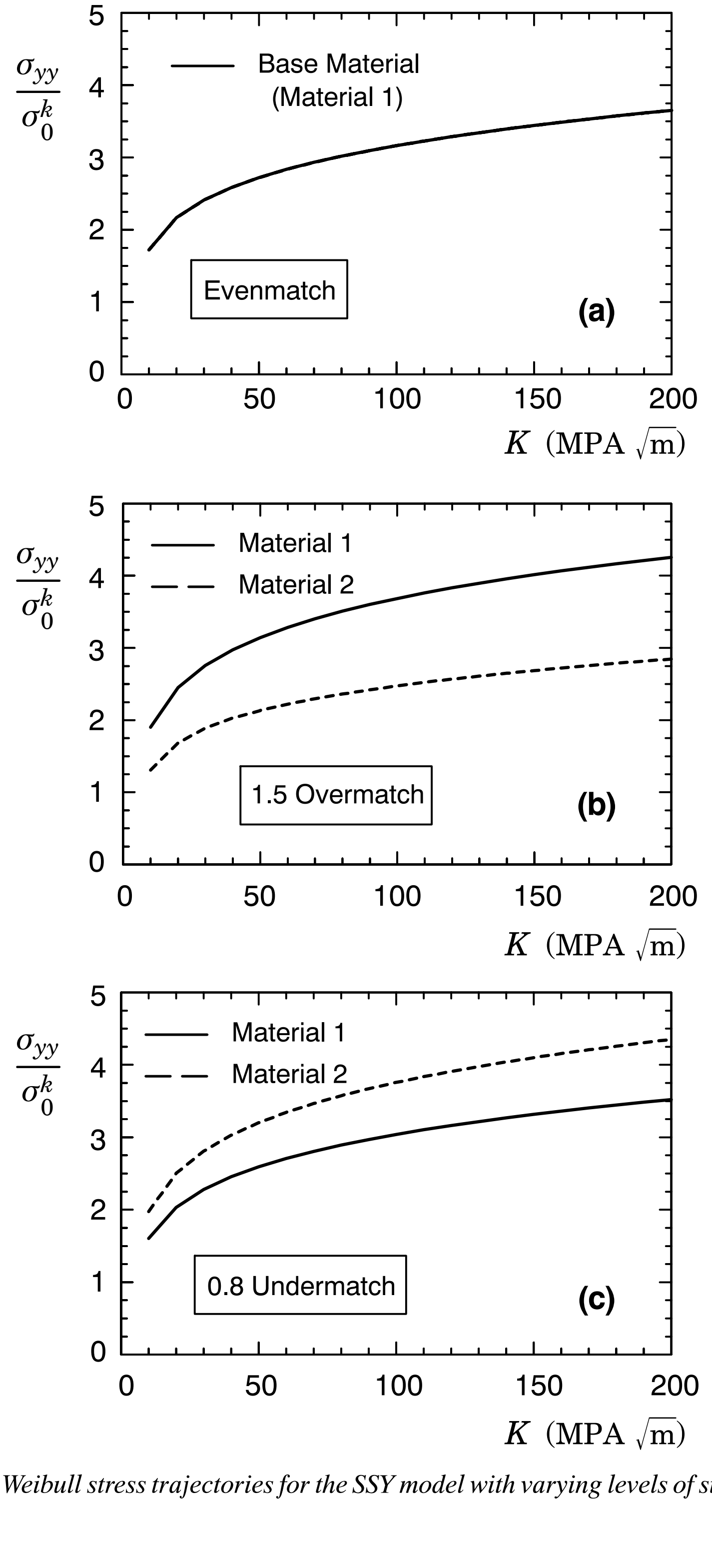

Figure 2 *Weibull stress trajectories for the SSY model with varying levels of strength mismatch.*