EFFECT OF THE ANISOTROPY ON THE CRACK BIFURCATION ANGLE

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ABSTRACT

The stress intensity factors K_I and K_{II} are developed analytically for the double cantilever beam specimen composed by the anisotropic material. These expressions are applied to orthotropic material in the case, where the material and specimen geometrical axes are not confused. Comparisons with the results obtained by the finite element method showed a good agreement. These analytical expressions of K_I and K_{II} are validated numerically; they serve us to basis for studies on crack propagation in anisotropic material under mixed mode loading.

The majority of the study made before in the mixed mode criterions has concerned only orthotropic materials in the case where its material axes coincide with specimen geometrical axes. The objective of this work is to study the influence of material axes orientation on the crack bifurcation angle θ_C .To carry out this work, strain energy density of Sih has been used; and an approximation of homogeneous linear elastic orthotropic material has been chosen.

The material and geometrical basis are defined by $\left(\vec{l},\vec{r},\vec{t}\right)$ and $\left(\vec{i},\vec{j},\vec{k}\right)$; where $\vec{l}$, $\vec{r}$ and $\vec{t}$ are the orthotropical axes of material and where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are respectively the crack direction, the perpendicular direction to the crack plane and the parallel direction to the crack front. For simplicity reasons, one has envisaged only the cases where the crack direction $\vec{i}$ is always confused with one of the material axes $\vec{l}$, $\vec{r}$ or $\vec{t}$. In each case, we denote by φ_L , φ_R and φ_T the rotation angle around $\vec{l} \equiv \vec{i}$, $\vec{r} \equiv \vec{i}$ and $\vec{t} \equiv \vec{i}$ ($0° \leq \varphi_L, \varphi_R, \varphi_T \leq 90°$). The angle of the load with the crack direction is denoted by β ($\beta = 0°$ in mode II, $0° \prec \beta \prec 90°$ in mixed mode, and $\beta = 90°$ in mode I). the study of the modes (I, I+II and II) allows us to deduce the following conclusions :

<u>For the mode I</u> $\left(\beta = 90°\right)$:

The crack bifurcation angle $\theta_C = 0°$ for each crack length and angle orientation (φ_L , φ_R or φ_T) tested. this is in agreement with the theory (all fissure submit to the mode I propagates in its own plan).

<u>For the mode II</u> $\left(\beta = 0°\right)$:

In the case $\vec{l} \equiv \vec{i}$; the critical angle θ_C is independant on the the orientation angle φ_L and the crack length. Its value is equal to $\mp 77°$. In the cases $\vec{r} \equiv \vec{i}$ and $\vec{t} \equiv \vec{i}$, the critical angle θ_C is independant on the crack legth. But its value varies weackly between $79°$ and $84°$ when the orientation angles φ_R or φ_T cover $\left[0°,90°\right]$.

<u>For the mixed mode</u> $\left(0° \prec \beta \prec 90°\right)$:

In the 3 studied cases; for fixed orientation angle (φ_L , φ_R or φ_T), the critical angle θ_C tends to decrease with increasing crack length. For fixed crack length; variations of the critical angle θ_C are weaks when φ_L varies (case $\vec{l} \equiv \vec{i}$). It's not true on the 2 other cases, where θ_C varies strongly on the orientation angles φ_R and φ_T (cases $\vec{r} \equiv \vec{i}$ and $\vec{t} \equiv \vec{i}$).

1. INTRODUCTION

For the majority of specimen fracture, the mode I and II stress intensity factors are determined only by numerical method. In this paper, we propose theoretical expressions of K_I and K_{II} for anisotropic material. These expressions are then applied to orthotropic material in the case where its material axes and specimen geometrical axes are not confused.

Comparisons with the finite element method showed a good agreement. These analytical expressions of K_I and K_{II} are validated numerically, then they serve us to basis for studies on crack propagation in anisotropic material under mixed mode loading.

Note that, the major part of studies made before in the mixed mode criterions has concerned only orthotropic material in the case where its material axes coincide with specimen geometrical axes. The objective of this work is to study, the influence of the material axes orientation on the crack bifurcation angle θ_C. For this, criterion of Sih has been used.

To carry out this study, the choice has been made on a specimen commonly used in the fracture mechanics; it is about the double cantilever beam specimen (D.C.B.) (f. Lahna [1]) . Its geometrical axes $\vec{i}$, $\vec{j}$ and $\vec{k}$ define respectively the direction of the crack, the perpendicular to the plan of the crack and the normal to the plan $(\vec{i},\vec{j})$ (Figure 1).

The tested material is supposed to be orthotropic; its symetrical material axes are the longitudinal, radial and transversal axes $(\vec{l},\vec{r},\vec{t})$. The behavior's law is supposed homogeneous, elastic and linear.

In this study, we will suppose that a material axis $\vec{l}$, $\vec{r}$ or $\vec{t}$ belongs to the geometrical plan of the crack $(\vec{k},\vec{i})$. In this case, the material basis $(\vec{l},\vec{r},\vec{t})$ is deduced from the geometrical basis $(\vec{i},\vec{j},\vec{k})$ with the help of a rotation function of 2 angles θ and φ (Figure 1) (b. a. Jayne [2]).

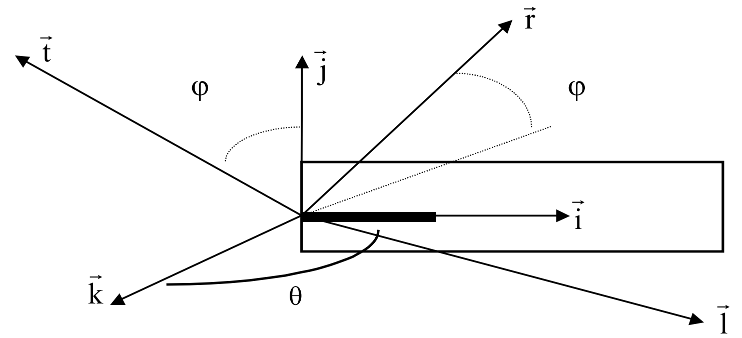

Figure 1 : Defiinition of the geometrical and material basis

$(\theta,\varphi) = (\theta_L,\varphi_L)$, (θ_R,φ_R) or (θ_T,φ_T) according to $\vec{l}$, $\vec{r}$ or $\vec{t}$ belongs plan $(\vec{k},\vec{i})$ (Figure 1)

The compliance tensors $[S]$ et $[\overline{S}]$ according to material and geometrical basis are given by the following matrix [3].

$$[S] = \begin{pmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{pmatrix} \quad ; \quad [\overline{S}] = \begin{pmatrix} \overline{S}_{11} & \overline{S}_{12} & \overline{S}_{13} & \overline{S}_{14} & \overline{S}_{15} & \overline{S}_{16} \\ \overline{S}_{12} & \overline{S}_{22} & \overline{S}_{23} & \overline{S}_{24} & \overline{S}_{25} & \overline{S}_{26} \\ \overline{S}_{13} & \overline{S}_{23} & \overline{S}_{33} & \overline{S}_{34} & \overline{S}_{35} & \overline{S}_{36} \\ \overline{S}_{14} & \overline{S}_{24} & \overline{S}_{34} & \overline{S}_{44} & \overline{S}_{45} & \overline{S}_{46} \\ \overline{S}_{15} & \overline{S}_{25} & \overline{S}_{35} & \overline{S}_{45} & \overline{S}_{55} & \overline{S}_{56} \\ \overline{S}_{16} & \overline{S}_{26} & \overline{S}_{36} & \overline{S}_{46} & \overline{S}_{56} & \overline{S}_{66} \end{pmatrix} \tag{1}$$

In this study, the analytical and numerical problems, that we tend to solve are supposed plans $(\vec{i},\vec{j})$. In this case, the compliance tensor is defined by :

$$[d] = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \tag{2}$$

Where $d_{ij} = \overline{S}_{IJ}$ in the plane stress; and $d_{ij} = \overline{S}_{ij} - \dfrac{\overline{S}_{i3}\overline{S}_{j3}}{\overline{S}_{33}}$ in the plane strain. (3)

2. THEORETICAL FORMULATION

2.1 Analytical expressions of K_I and K_{II}

The stress intensity factors K_I and K_{II} are developed analytically for the double cantilever beam specimen composed by the anisotropic material.These expressions are given by the following formulas(f. Lahna [4,5,6,7])

$$K_I(a)=\left(\frac{P}{b}\right)\left(2d_{11}\right)^{1/2}\left\{\frac{\left\{\frac{12}{\lambda^3h^3}\left[\lambda a\left(\frac{sh^2\lambda c+\sin^2\lambda c}{sh^2\lambda c-\sin^2\lambda c}\right)+\left(\frac{sh\lambda cch\lambda c-\sin\lambda c\cos\lambda c}{sh^2\lambda c-\sin^2\lambda c}\right)\right]^2+\left(\frac{6d_{33}}{5hd_{11}}\right)\right\}}{-\operatorname{Im}\left[d_{22}\left(\frac{\mu_1+\mu_2}{\mu_1\mu_2}\right)\right]}\right\}^{1/2} \tag{4}$$

$$K_{II}(a)=\left(\frac{2P}{bh^{1/2}}\right)\left(\frac{d_{11}}{\frac{1}{2}\operatorname{Im}\left[d_{11}\left(\mu_1+\mu_2\right)\right]}\right)^{\frac{1}{2}}$$

Where the d_{ij} are the components of the compliance tensor. P is the applied load in mode I or in mode II, and b and h are respectively the thickness and half heigth of the specimen.
μ_1 and μ_2 are the roots of the equations : $d_{11}\mu^4-2d_{16}\mu^3+\left(2d_{12}+d_{66}\right)\mu^2-2d_{26}\mu+d_{22}=0$ (5)
Im are the imaginary parts of the complex $d_{22}\left(\frac{\mu_1+\mu_2}{\mu_1\mu_2}\right)$ and $d_{11}\left(\mu_1+\mu_2\right)$

2.2 The criterion of Sih

The criterion for strain energy density factors proposed by Sih is as follows (g. c. Sih [8,9]) :
(i) The direction of crack propagation coincides with the direction θ_C of the minimum strain energy density around the crack tip should satisfy :

$$\left.\frac{\partial S}{\partial\theta}\right|_{\theta_C}=0 \quad \text{and} \quad \left.\frac{\partial^2 S}{\partial\theta^2}\right|_{\theta_C}\succ 0 \tag{6}$$

(ii) A crack extension starts in the θ_C direction when S reaches the critical value S_C, where

$$S=M_{11}K_I^2+2M_{12}K_IK_{II}+M_{22}K_{II}^2 \tag{7}$$

and

$$M_{11}=\frac{1}{4}.\left[d_{11}A^2+d_{22}C^2+d_{66}E^2+2d_{12}AC+2d_{16}AE+2d_{26}CE\right]$$

$$M_{12}=\frac{1}{4}.\left[d_{11}AB+d_{22}CD+d_{66}EF+d_{12}\left(AD+BC\right)\right]+d_{16}\left(AF+BE\right)+d_{26}\left(CF+DE\right)$$

$$M_{22}=\frac{1}{4}.\left[d_{11}B^2+d_{22}D^2+d_{66}F^2+2d_{12}BD+2d_{16}BF+2d_{26}DF\right] \tag{8}$$

and where A , B, C , D , E and F are given by :

$$A=\operatorname{Re}\left[\frac{\mu_1\mu_2}{\mu_1-\mu_2}\left(\frac{\mu_2}{z_2}-\frac{\mu_1}{z_1}\right)\right] \quad , \quad B=\operatorname{Re}\left[\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_2^{\,2}}{z_2}-\frac{\mu_1^{\,2}}{z_1}\right)\right]$$

$$C = \mathrm{Re}\left[\frac{1}{\mu_1 - \mu_2}\left(\frac{\mu_1}{z_2} - \frac{\mu_2}{z_1}\right)\right] \quad , \quad D = \mathrm{Re}\left[\frac{1}{\mu_1 - \mu_2}\left(\frac{1}{z_2} - \frac{1}{z_1}\right)\right]$$

$$E = \mathrm{Re}\left[\frac{\mu_1\mu_2}{\mu_1 - \mu_2}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\right] \quad , \quad F = \mathrm{Re}\left[\frac{1}{\mu_1 - \mu_2}\left(\frac{\mu_1}{z_1} - \frac{\mu_2}{z_2}\right)\right] \tag{9}$$

d_{IJ} , μ_1 and μ_2 are defined above and $z_k = (\cos\theta + \mu_k \sin\theta)^{1/2} \quad (k = 1,2)$.

3. RESULTS AND DISCUSSION

For simplicity reasons, one has envisaged only the cases where the crack direction $\vec{i}$ is always confused with one of the material axes $\vec{l}$, $\vec{r}$ or $\vec{t}$ ($\vec{l} \equiv \vec{i}$, $\vec{r} \equiv \vec{i}$ or $\vec{t} \equiv \vec{i}$). In each case, we denote by φ_L , φ_R and φ_T the rotation angle around $\vec{l} \equiv \vec{i}$, $\vec{r} \equiv \vec{i}$ and $\vec{t} \equiv \vec{i}$ ($0° \le \varphi_L , \varphi_R , \varphi_T \le 90°$). The angle of the load with the crack direction is denoted by β ($\beta = 0°$ in mode II, $0° \prec \beta \prec 90°$ in mixed mode, and $\beta = 90°$ in mode I). the study of the modes (I, I+II and II) allows us to deduce the following results. Note that the critical angle θ_C is obtained by the minimum of the curve $S(\theta)$; except the case $\vec{l} \equiv \vec{i}$ (in modes I and I+II), where the crack bifurcation angle is defined by the maximum of strain energy density (Figure 2).

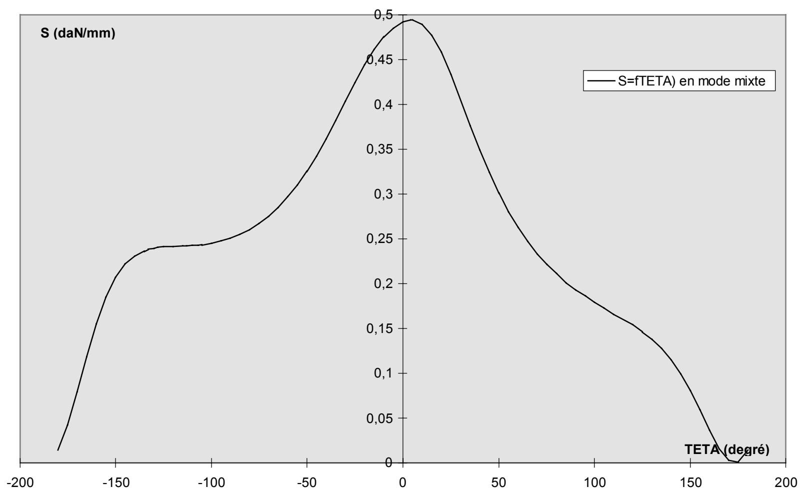

Figure 2 : $S = f(\theta)$ in mixed mode for the orthotropic wood maritime pine

3.1 Variation of the critical angle θ_C in function of the crack legth

For the three studied orientation cases ($\vec{l} \equiv \vec{i}$, $\vec{r} \equiv \vec{i}$ or $\vec{t} \equiv \vec{i}$). we have fixed angles φ_L , φ_R or φ_T ; and we have varied the crack length. We note that the critical angle θ_C tends to decrease with increasing crack length.The figure 3 represents for maritime pine wood, variations of the critical angle θ_C in function of the crack length in the case $\vec{r} \equiv \vec{i}$. Note that in mode I, the crack bifurcation angle $\theta_C = 0°$ for each crack length and angle orientation (φ_L , φ_R or φ_T) tested. this is in agreement with the theory (all fissure submit to the mode I propagates in its own plan).

In the mode II, for the case $\vec{l} \equiv \vec{i}$; the critical angle θ_C is independant on the the orientation angle φ_L and the crack length. Its value is equal to $\mp 77°$. For the cases $\vec{r} \equiv \vec{i}$ and $\vec{t} \equiv \vec{i}$, the critical angle θ_C is independant on the crack legth. But its value varies weackly between $79°$ and $84°$ when the orientation angles φ_R or φ_T cover $[0°, 90°]$.

3.2 Variation of the critical angle θ_C in function of the orientation angle φ

In this case, we have fixed the crack length and we have varied the angle φ (φ_L , φ_R or φ_T).
The figure 4 represents for the crack length a=120 mm (the behaviour of curves obtained with other crack lengths is the same); the variations of critical angle θ_C in function of φ in mixed mode ($\beta = 10°$) . Theses curves show that variations of $\theta_C(\varphi_L)$ when $\vec{l} = \vec{i}$ are weak; It's not true for the two other cases ($\vec{r} \equiv \vec{i}$ or $\vec{t} \equiv \vec{i}$), where variations of θ_C depend strongly on the angle φ (φ_R or φ_T). The results obtained for other modes are similar.

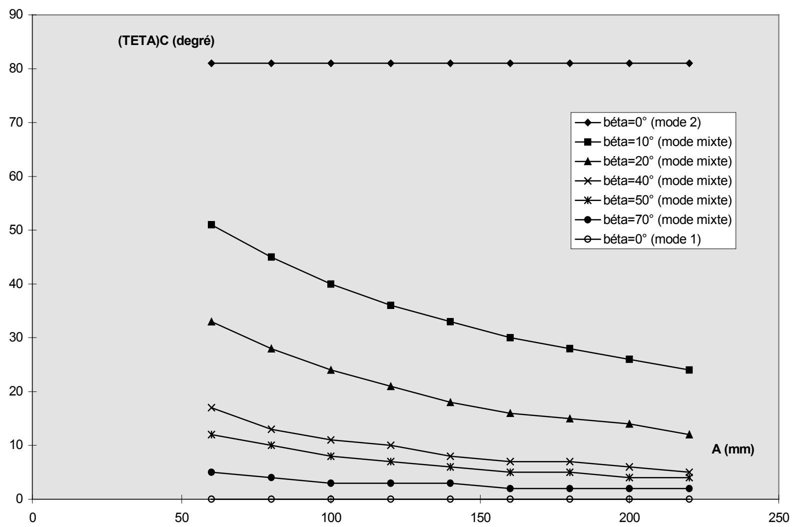

Figure 3 : $\theta_C = f(A)$ in the case $\vec{r} = \vec{i}$, for the orthotropic wood maritime pine

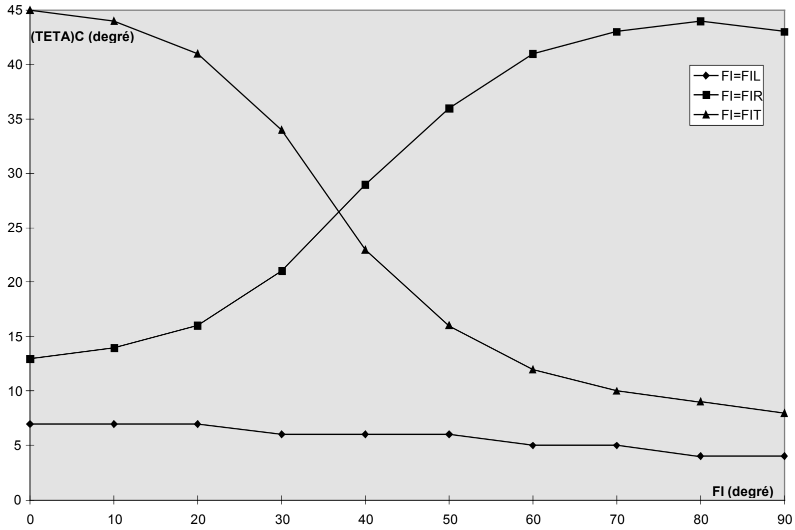

Figure 4. : $\theta_C = f(\varphi_L, \varphi_R, \varphi_T)$ in mixed mode ($\beta = 10°$) for $A = 120mm$

4. CONCLUSION

Analytical expressions of mode I and II stress intensity factors K_I and K_{II} are developed for specimen DCB composed by anisotropic material. These expressions are used for application of the strain energy density in the case where orthotropic axes are not confused with specimen geometrical axes. The study shows us that, for imposed material axes orientation, θ_C decreases when crack length increases. For fixed crack length, values of θ_C depend strongly on the anisotropy introduced by the material axes orientation.

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