CRACK PROPAGATION IN RUBBER-LIKE MATERIALS

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ABSTRACT

It is known that the application of fracture mechanics methods to rubbers generates some peculiarities. The first part of this paper deals with the simulation of tear fatigue analyzer (TFA) tests which are used to characterize crack propagation in rubber-like materials. With a combination of a mesoscopic model of self-similar crack propagation and a flow enhanced standard solid with nonlinear elastic and viscous elements it is possible to establish interconnections between mechanical fracturing tests and intrinsic material properties on the assumption of small elongations. In the second part we present a continuum mechanics approach based on the balance of the so-called pseudomomentum as the pull back of the balance of linear momentum onto the material manifold. Configurational forces appearing in this balance take into consideration anelastic behavior, thermal gradients and inhomogeneities. In this way fracture parameters similar to the well-known J-Integral can be established for rubber-like materials.

1 INTRODUCTION

Service life prediction of materials is clearly of high practical and scientific interest and has attracted the attention of chemists, engineers, and physicists. In the case of rubber and other polymer materials, it is reasonable to assume that microcracks from which failure originates are formed via the sequence of several steps: i) Molecular chains attempt to move in tensile direction, causing slip between chains and producing reorientation. ii) Cross-links, entanglements and filler particles hinder the motion of chains. Chains acquire a state of tension and local scission occurs. iii) Scission of one chain transfers stress to the neighbouring chains, chain scission propagates to the surrounding molecules; the cumulative effect produces a microvoid; iv) Microvoids that have grown to a critical size form microcracks which continue to grow irreversibly. However, estimating the fatigue lifetime of elastomeric components is still a difficult task for compound chemists and designers in rubber industry. Therefore, it is essential to use fracture mechanics methods to characterize the toughness of different elastomers under quasi-static and dynamic stress when predicting the life time of highly stressed elastomer components.

With tear fatigue analyzer (TFA) tests the progress of the crack tip is first determined over the period of a pre-cracked sample with fixed stress parameters. The materials toughness

can be characterized as the crack length changes per stress cycle da/dN via different stress intensities for small deformation assumption, N being the number of cycles and a being the crack length. It is important to understand, and to access at least on a semi-microscopic level, the material behavior occuring directly within the tip of the crack - derived from the macroscopic, in our case nonlinear viscoelastic, rubber material. With an analytical semi-microscopic model, which combines the assumption of self-similar crack propagation with a complex viscoelastic rubber model we present the basic principles and demonstrate a comparison between modeling of dynamic crack propagation and TFA test for the simple case of an unfilled synthetic rubber [1].

However as known, the application of linear elastic fracture mechanics (LEFM) methods to rubber-like materials leads to several conceptual problems. So, we present a continuum mechanics framework to characterize fracture in rubber-like materials. Due to dissipation in the bulk so-called configurational forces appear in the local balance of pseudomomentum, the pull back of the local balance of linear momentum onto the material manifold. Parameters based on this balance equation can be used to characterize fracture accounting for dissipative contributions.

2 DYNAMIC SELF-SIMILAR CRACK PROPAGATION

We assume that the singularity under cyclic Mode 1 conditions at the tip of the crack is clearly defined and characterized by a stress intensity factor

$$K_I(t) = K_{min} + \frac{\Delta K}{2}\,(1 + \sin \omega t) \tag{1}$$

with

$$\Delta K = K_{max} - K_{min}. \tag{2}$$

The cohesive stress σ ahead of the crack tip varies as

$$\sigma(t) = \sigma_c \frac{K_I(t)}{K_{Ic}}, \tag{3}$$

depending on the stress intensity factor K_I, where σ_c is the critical cohesive stress and K_{Ic} the critical stress intensity factor.

The variation $\Delta\delta$ of the crack opening displacement (COD) δ during one cycle is proportional to the deformation rate $d\epsilon/dt$ integrated over one cycle τ [2]:

$$\frac{\Delta\delta}{\delta} = \int_0^\tau \frac{d\epsilon}{dt}\,dt \longrightarrow \frac{\Delta s}{s} \tag{4}$$

with s representing the length of the deformation and Δs the crack zone propagation per cycle. As the shape of the deformation zone does not change but becomes self-similar after

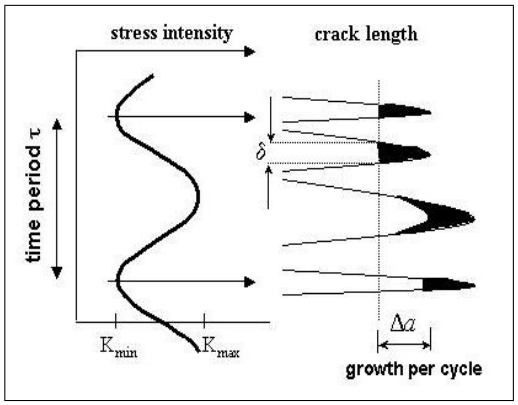

Figure 1: Model of self-similar crack propagation

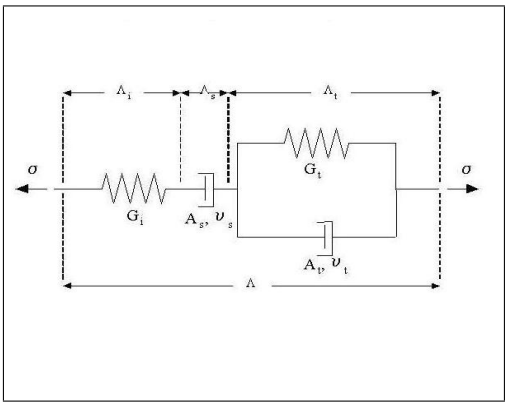

Figure 2: Schematic diagram of the nonlinear mechanical model

each cycle we assume that the deformation zone continues to creep by Δs during every cycle, see figure 1. This value corresponds to the known crack length change per cycle da/dN. The COD values can be calculated via the the stress intensity factors in the framework of LEFM.

We use a standard solid model expanded with a series dashpot [3], as shown in figure 2, consisting of nonlinear elastic and viscous elements. The following model assumption were made:

- Neo Hooke's law is assumed in spring elements between the nominal tension and deformation according to the theory of rubber elasticity:

$$\sigma = G(\lambda - \lambda^{-2}) = G\,\Lambda \tag{5}$$

  with G the initial shear modulus and $\Lambda = \lambda - \lambda^{-2}$ a nonlinear elongation ratio.

- The temporal change of the elongation parameter Λ is simulated using a reaction kinetics law for activated non-Newton viscose materials:

$$\dot{\Lambda} = A \sinh \frac{\nu\sigma}{RT} \tag{6}$$

  where A is associated with an activation energy, ν is an activation volume, R the gas constant and T the absolut temperature.

- For small elongations the simplified interconnection between strain $\epsilon = \lambda - 1$ and the elongation parameter Λ applies:

$$\Lambda = \lambda - \lambda^{-2} = 3\epsilon + O(\epsilon^2) \approx 3\epsilon. \tag{7}$$

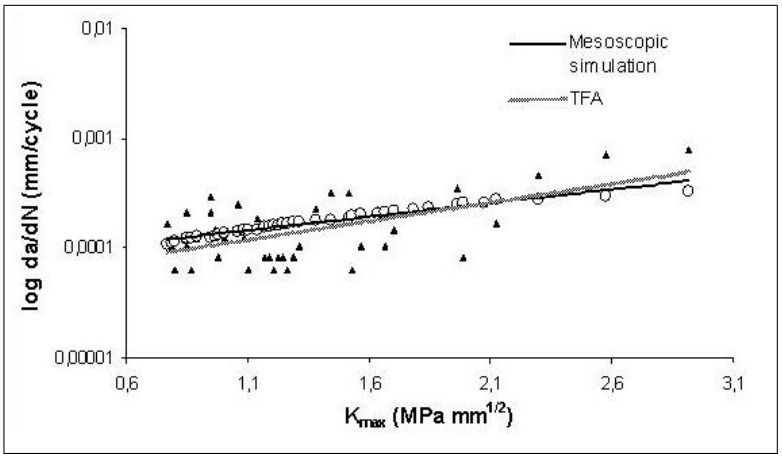

Figure 3: Compairison between simulation and TFA results

In the case of filler loading the strain ϵ is replaced by a local intrinsic elongation ϵ_{int}. New intrinsic material parameters ν_s, ν_t, A_s and A_t are introduced which can be estimated from simple (non-destructing) stress-strain and creep experiments. The parameters $\tau_s^{-1} = A_s \nu_s G_t$ and $\tau_t^{-1} = A_t \nu_t G_t$ represent characteristic times of the material. A simplified model with $\tau_t^{-1} = 0$ yields an analytical solution for the interconnection between the crack growth rate da/dN and the stress intensity factor K_I in form of the Paris law. This can be utilized, for example, to simulate very well the dynamic crack propagation in different highly cross-linked polymer networks.

For elastomer materials far exceeding glass temperature with $\nu_s \sigma \ll 1$, $\nu_t \sigma \ll 1$ and $\tau_t^{-1} \neq 0$ we obtain the result shown in figure 3 for an unfilled rubber consisting of styrene butadien copolymer SBR 1500 and cross-linked with 1.2 phr (parts of weight per hundred parts of polymer) of sulfur and 1.2 phr N-cyclohexylbenzothiazole-2-sulfenamide (CBS) accelerator.

3 CONTINUUM MECHANICS APPROACH

Because large deformations violate the small deformation assumption and material behavior of rubber is non-linear with dissipative contributions LEFM methods are not suitable for crack propagation in such materials. Hence we present in the second part a general continuum mechanics framework for static and dynamic simulation of crack propagation in rubber-like materials.

The tearing energy [4], defined by

$$T = -\frac{dU}{dA}, \tag{8}$$

as a extension of Griffith's theory is widely used to characterize fracture in such materials. The decrease of the stored elastic energy dU is balanced by the energy to form new surface areas dA and the energy dissipated in the process zone ahead of the crack tip. But in rubber-like materials energy dissipation in the bulk due to material behavior is present, too. Note that eqn (8) is formally identical with the definition of the energy release rate in the irreversible progress of the crack tip.

The material behavior of rubber is described in the framework of thermodynamics with internal variables [6] that account for microscopic phenomena responsible for macroscopic dissipation at finite strains. We asuume that the thermodynamic potential

$$W = W(F, \theta, \alpha; X) \tag{9}$$

depends on the deformation gradient F, the absolute temperature θ and internal variables α and is an explicite function of the material particle X in the case of an inhomogeneous material.

In addition to the known balance equations of continuum mechanics the pull back of the local balance of physical momentum onto the material manifold [7] results in the local balance of pseudomomentum

$$\frac{dP}{dt} - div_R\, b = f^{inh} + f^{\theta} + f^{\alpha} \tag{10}$$

at all regular material points with the pseudomomentum P accounting for dynamic contributions, the material divergence div_R of the Eshelby material stress

$$b = -(LI_R + T \cdot F) \tag{11}$$

and the configurational forces f^{inh}, f^{θ} and f^{α} capturing true material inhomogeneities and dissipation due to thermal gradients and anelastic material behavior. In eqn (11) L is the effective "Lagrangian" density, I_R the unit dyadic on the material manifold and T the first Piola-Kirchhoff stress.

In inhomogeneous materials accounting for thermal and anelastic behavior and dynamic contributions the crack tip can be characterized using a global quantity based on eqn (10), for example an integral over a material region surrounding the tip. Note that the well-known J-Integral [5] can be deduced as special case in LEFM.

4 CONCLUSIONS AND FINAL REMARKS

We have shown that the dynamic crack propagation in rubbers can be modeled via the combination of three physical concepts: i) self-similar crack propagation, ii) stress intensity factor (crack tip opening displacement), and iii) Eyring-like flow-enhanced neo-Hookean

standard solid as a representative of the rubber solid. The effect of viscous flow on the creep stretch will not occur in the second creep stage if one removes the dashpot connected in series with the spring. Then, the (linear standard solid) model describes only the viscoelastic behavior of a non-damaged rubber sample. Intrinsic viscous material parameters are related to activation volumes and activation energies, respectively, at the crack tip material which is described with the generalized standard solid model. A reasonable interpretation follows from the inverse parameters $\nu_{s,t}^{-1}$ and $A_{s,t}^{-1}$ having the meaning of intrinsic moduli and times, respectively. The combinations $\tau_s^{-1} = A_s \nu_s G_t$ and $\tau_t^{-1} = A_t \nu_t G_t$ define two characteristic relaxation times of the rubber material. They allow interpretations concerning critical frequencies of dynamic loads. Note that this approach is restricted to Mode I crack propagation and small deformations. The neo-Hooke's law leads to further restriction to amorphous rubbers and excludes strain-crystallizing materials.

To avoid these restrictions we presented a continuum mechanics approach that is related to the notion of configurational forces. The description of local dissipation due to inelastic material behavior with internal variables leads to dissipative contributions in the balance equation of pseudomomentum apart from those due to true material inhomogeneities. Global quantities can be defined accounting for these contibutions to characterize fracture in rubber-like materials.

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