THERMAL-MECHANICAL BUCKLING FAILURE OF MULTILAYERED BEAM-PLATE WITH ARBITRARY DELAMINATION LOCATION

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ABSTRACT

An extended one-dimensional interface crack model is proposed to analyze the thermal-mechanical buckling behavior of multilayered beam-plate with arbitrary interface crack location under clamped boundary condition. The equilibrium equations, stability equations and characteristic equation governing buckling under thermal and mechanical loads are derived based on the first order shear deformation and state space method (SSM). The extended model was used to study the thermal-mechanical buckling failure of thermal barrier ceramic coatings system (TBCs) with arbitrary delamiantion location. The TBC system consists of superalloy substrate, bond coat, thermal growth oxidation (TGO) and top ceramic coating. It was loaded by a compressive applied mechanical load along the axial direction and a high temperature gradient along the thickness direction. The critical buckling loads such as the mechanical loads and thermal loads were obtained under the combination of different conditions. Finally the influence of the beam-plate aspect ratio, the transverse shear and the temperature gradient on buckling failure difference is all discussed.

Key Words: thermal barrier ceramic coating, thermal-mechanical buckling failure, delamination

1. INTRODUCTION

In recent years, the composites structures exhibit notable mechanical characteristics, such as high strength-to-weight and stiffness-to-weight ratios, as a result, they are widely used in many fields of structural engineering. On the other hand, another functional composites such as thermal barrier coating system (TBCs) have also attracted much interest as heating-shielding materials for aircraft, space vehicles and other engineering applications. However, delamination is one of the most severe problems concerning composites, which is developed due to manufacturing defects or impact loads and thermal loads. When the compressive loads parallel to interface direction are applied under the conditions of the different temperatures, the delamination or interface crack would propagate along the interface. Onset of the seriousness leads to local buckling failure of the sub-beam-plate and growth of the delamination, causing a sudden loss of the load carrying capability or thermal barrier function of the structure. It is extremely important to understand the behavior of the composites materials with delamination.

In these works different approaches were used, leading in some cases to different results. MSRao Parlapalli studied the buckling behavior of a two-layer beam-plate with single delamination under clamped and simply supported boundary conditions. The critical buckling loads were accurately obtained [1]. Wu Lanhe has studied thermal buckling behavior of a simply supported moderately thick rectangular FGM plate under thermal loads. The buckling temperature was derived and discussed [2]. Yeh and Tan [3] studied the buckling of laminated plates with elliptic delamination. Shu and Mai [4] studied the buckling analysis of a delaminated beam-plate with bridging.

The existing analyses in the above literatures can't be applied to the present delaminated multilayers beam-plate made of different materials under mechanics and thermal loads. The intent of the present paper is to investigate the delaination thermal buckling failure in layered beam-plate, as related to the analysis of the minimum critical buckling loads and thermal buckling temperature of these structures. We deal with a homogeneous isotropic beam-plate containing an arbitrary across-the-width delamination. Moreover an extended one-dimensional mathematical model is introduced to obtain the critical buckling loads of a delaminated multiplayer beam-plate under clamped boundary condition. The equilibrium equations, stability equations and characteristic equation governing buckling under thermal and mechanical loads are derived based on the first order shear deformation and state space method (SSM). It should be noted that the method could be easily extended to structures containing multiple asymmetric delaminations.

2. EXTENDED INTERFACE MODEL

2.1 Problem definition

In the present study, the configuration of the delaminated multilayer beam-plate under study is represented in the sketch of Fig. 1. It should be noted that the analytical model for buckling failure in multilayers has been developed recently [1]. It consists of a homogeneous, isotropic beam-plate of thickness of H, of length L and of unit width beam-plate containing an arbitrary, parallel thorough delamination with length a at depth h_3 from the top surface of the beam-plate. The coordinate axes for the sub-beam-plate are shown in Fig. 1. Because of the presence of the thorough delamination, the multiplayer beam-plate is divided into virgin beam-plates 1 and 4, and to sub-beam-plates 2 and 3. The left tip of the delamination is located at length l_1 from the left edge of the multilayered beam-plate. The multilayered beam-plate is clamped at the two edges. Moreover it is

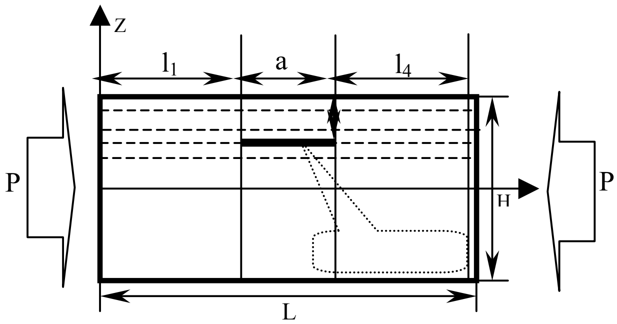

Fig.1. The extended interface crack model of TBCs.

compressed by the axial load P at the two edges along the neutral axis of the multiplayer beam-plate and heated up to different high temperature T in the initial state. To simplify the analysis, the following assumptions are made. The compressive load is uniform and uniaxial. Delamination and the high temperature exist prior to applying the compressive load. “Slender” beam-plate is assumed to be easy to buck first among the sub-beam-plate 2 and 3. The sub-beam-plate 2 and 3 are assumed to be not contacting each other in the initial stage of buckling because the sub-beam-plate 3 is more slender and flexible than sub-beam-plate 2. The necessary conditions and equations required to derive the governing buckling equation are discussed in the following sections.

2.2 Basic equations

In this paper, the displacement model of virgin/sub beam-plates are based on the first order shear deformation theory and written as follows,

$$\bar{u}_i(x_i, z_i) = u_i(x_i) + z_i \psi_i(x_i) \quad (1)$$

$$\bar{w}_i(x_i, z_i) = w_i(x_i) \quad (2)$$

Assume that $\bar{u}_i(x_i, z_i)$, $u_i(x_i)$, $\psi_i(x_i)$ in eqn (1) denote, respectively, the displacement model of the i th virgin/sub beam-plate along the x direction, the displacement of the i th virgin/sub beam-plate along the x direction and the rotation of the normal to the beam-plate midplane. $\bar{w}_i(x_i, z_i)$ denotes the small elastic deflection of the i th virgin/sub beam-plate along the z direction. x_i is measured from the left end of the i th virgin/sub beam-plate to the right end of the i th virgin/sub beam-plate. z_i is measured from $-h_i/2$ to $h_i/2$ $(i = 1,2,3,4)$. According to the first order shear deformation theory and Hooke’s law, the strain energy of the i th segment and the potential energy of the compressive load can, respectively, be obtained as follows,

$$U_i = \frac{1}{2}\int \left(D_{0i}u_{i,x}^2 + 2D_{1i}u_{i,x}\varphi_{i,x} + D_{2i}\varphi_{i,x}^2\right)dx_i + \frac{1}{2}\int B_{0i}\left(\varphi_i + w_{i,x}\right)^2 dx_i - \frac{1}{2}\int \left(F_{0i}\left(u_{i,x} + \tfrac{1}{2}w_{i,x}^2\right) + F_{1i}\varphi_{i,x}\right)dx_i \quad (3)$$

$$V_i = -P_i\int \varepsilon_{xi}dx_i = -P_i\int (u_{i,x} + z_i\varphi_{i,x} + 1/2 * w_{i,x}^2)dx_i \quad (4)$$

Here $\overline{Q}_{11i}$ and $\overline{Q}_{55i}$ are proportionality constants. R_i and ΔT_i equal to, respectively, $E_i\alpha_i/(1-v_i)$ and $T_i(z) - T_0(z)$. Moreover D_{ij} and B_{ij} in eq. (3) denotes the stiffness coefficients. It should be noted that P_i denotes the axial load of the i th segment. In order to simplify the analysis, in eq. (4), $P_1 = P_2 = P$ and the local axial force P_i is approximatively regarded as $h_i P/H$. Therefore, the total potential energy of the multilayered beam-plate can be written as,

$$\Pi = \sum_{i=1}^{4}(U_i + V_i) \quad (5)$$

For the purpose of analyzing the multilayered beam-plate by the potential energy method, the unknown displacements must be identified first. According to the principle of virtual displacement, the displacement variables of the i th segment could be divided into two parts,

$$u_i = u_i^0 + u_i^a, \quad \psi_i = \psi_i^0 + \psi_i^a, \quad w_i = w_i^0 + w_i^a \quad (6)$$

where $\{u_i^0 \quad \psi_i^0 \quad w_i^0\}$ is the displacement function of initiate balance state and $\{u_i^a \quad \psi_i^a \quad w_i^a\}$ is a very small variables of the displacement function when the multilayered beam-plate occurs buckling. The balance equations of the i th segment can be deduced by using the potential variation principle,

$$\begin{cases} D_{0i}u_{i,xx}{}^a + D_{1i}\psi_{i,xx}{}^a = 0, \;\; D_{1i}u_{i,xx}{}^a + D_{2i}\psi_{i,xx}{}^a - B_{0i}(\psi_i{}^a + w_{i,x}{}^a) = 0 \\ P_i w_{i,xx}{}^a - B_{0i}(\psi_{i,x}{}^a + w_{i,xx}{}^a) + 1/2 * F_{0i} w_{i,xx}{}^a = 0 \end{cases} \tag{7}$$

2.3 Boundary conditions

For clamped boundary conditions, we can obtain as follows,

$$x_1 = 0 \qquad u_1{}^a = \psi_1{}^a = w_1^a = 0 \tag{8}$$

$$x_4 = l_4 \qquad u_4{}^a = \psi_4{}^a = w_4^a = 0 \tag{9}$$

The delaminated multilayered beam-plate model is assumed to be continuous, these different segments have common positions at both delaminated tips, so the continuity of displacement at the two positions must be satisfied and can be written as,

$$u_i{}^a(0) = u_1{}^a(l_1) + d_i\psi_1{}^a(l_1)\,, \quad \psi_i{}^a(0) = \psi_1{}^a(l_1)\,, \quad w_i{}^a(0) = w_1{}^a(l_1) \tag{10}$$

$$u_i{}^a(l_i) = u_4{}^a(0) + d_i\psi_4{}^a(0)\,, \quad \psi_i{}^a(l_i) = \psi_4{}^a(0)\,, \quad w_i{}^a(l_i) = w_4{}^a(0) \tag{11}$$

where $i = 2,3$. In order to simplify the analysis, the generalized forces are defined as follows,

$$X_i = D_{0i}u_{i,x}{}^a + D_{1i}\psi_{i,x}{}^a - 1/2 * F_{0i} \tag{12}$$

$$\phi_i = D_{1i}u_{i,x}{}^a + D_{2i}\psi_{i,x}{}^a - 1/2 * F_{1i} \tag{13}$$

$$Z_i = B_{0i}(\psi_i{}^a + w_{i,x}{}^a) - P_i w_{i,x}{}^a - 1/2 * F_{0i} w_{i,x}^a \tag{14}$$

Furthermore, the balance of generalized forces at both tips of the delamination can be written as,

$$X_j(l_{14}) = \sum_{i=2,3} X_i(l_{23})\,, \quad \phi_j(l_{14}) = \sum_{i=2,3}[d_i X_i(l_{23}) + \phi_i(l_{23})]\,, \quad Z_j(l_{14}) = \sum_{i=2,3} Z_i(l_{23}) \tag{15}$$

2.4 State-space method

The concept of state space method (SSM) is an important in structure mechanics, mathematic, physics and so on. In the following discussion we will describe how the state space method is used in buckling analysis. All variables of the i th segment are expressed as a form of state space method

$$\{\eta_i\} = \{u_i^a \quad \psi_i^a \quad w_i^a \quad u_{i,x}^a \quad \psi_{i,x}^a \quad w_{i,x}^a\}^T \tag{16}$$

In eqn (16), "T" denotes the transposition of matrix. Therefore, the governing balance equation of the i th segment can be written as,

$$d\eta_i / dx_i = S_i \eta_i \tag{17}$$

where the coefficients matrix S_i is defined as,

$$S_i = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ C_{1i}^{-1}C_{2i} & C_{1i}^{-1}C_{3i} \end{bmatrix} \tag{18}$$

In eqn (18), $I_{3\times3}$ and $0_{3\times3}$ are $3-$order unit matrix and zero matrix, respectively. The matrix forms of coefficients $C_{ji}, (j = 1,2,3)$ are also given in the following.

$$C_{1i} = \begin{bmatrix} D_{0i} & D_{1i} & 0 \\ D_{1i} & D_{2i} & 0 \\ 0 & 0 & P_i - B_{0i} + \frac{1}{2}F_{0i} \end{bmatrix}_{3\times3}, C_{2i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_{0i} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}, C_{3i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & B_{0i} \\ 0 & B_{0i} & 0 \end{bmatrix}_{3\times3} \quad (19)$$

The solution of governing balance equation (17) can be obtained as follows:

$$\eta_i = e^{S_i x_i}\eta_i^0 = K_i(x_i)\eta_i^0 \quad (20)$$

where η_i^0 is an integral constant matrix, i.e. $\eta_i^0 = \begin{pmatrix} C_1^i & C_2^i & \cdots & C_6^i \end{pmatrix}^T$. $K_i(x_i)$ equals to $e^{S_i x_i}$ and denotes the coefficient of η_i^0 in equation (20). Finally, all the boundary conditions and continuity conditions are rewritten as the state-space form in the following,

$$\left.\begin{array}{l} A_2K_1(0)\eta_1^0 = 0\,,\quad A_2K_4(l_4)\eta_4^0 = 0\,,\quad A_1^2K_1(l_1)\eta_1^0 - A_2K_2(0)\eta_2^0 = 0\,,\quad A_1^3K_1(l_1)\eta_1^0 - A_2K_2(0)\eta_3^0 = 0 \\ A_1^2K_4(0)\eta_4^0 - A_2K_2(l_2)\eta_2^0 = 0\,,\quad A_1^3K_4(0)\eta_4^0 - A_2K_3(l_3)\eta_3^0 = 0\,, \\ A_3^1K_1(l_1)\eta_1^0 - A_4^2K_2(0)\eta_2^0 - A_4^3K_3(0)\eta_3^0 = 0\,,\quad A_3^4K_4(0)\eta_4^0 - A_4^2K_2(l_2)\eta_2^0 - A_4^3K_3(l_3)\eta_3^0 = 0 \end{array}\right\} \quad (21)$$

Here, the total integral constant matrix of the multilayerd beam-plate is defined as,

$$\eta^0 = \left\{ (\eta_1^0)^T \quad (\eta_2^0)^T \quad (\eta_3^0)^T \quad (\eta_4^0)^T \right\}^T \quad (22)$$

where "T" expresses the transposition of matrix. Therefore, the equations (10-15) can be written as linear algebraic equations,

$$M\eta^0 = 0 \quad (23)$$

where,

$$M = \begin{bmatrix} A_2K_1(0) & 0 & 0 & 0 \\ 0 & 0 & 0 & A_2K_4(l_4) \\ A_1^2K_1(l_1) & -A_2K_2(0) & 0 & 0 \\ A_1^3K_1(l_1) & 0 & -A_2K_3(0) & 0 \\ 0 & -A_2K_2(l_2) & 0 & A_1^2K_4(0) \\ 0 & 0 & -A_2K_3(l_3) & A_1^3K_4(0) \\ A_3^1K_1(l_1) & -A_4^2K_2(0) & -A_4^3K_3(0) & 0 \\ 0 & -A_4^2K_2(l_2) & -A_4^3K_3(l_3) & A_3^4K_4(0) \end{bmatrix} \quad (24)$$

The lowest eigen value of the determinant of Eq. (24) is the critical buckling load under the different of high temperature.

3. DISCUSSIONS AND CONCLUSIONS

The thermal and mechanical properties used in the calculation were taken from the literatures [5]. Thermal expansion coefficient, Young's modulus and Poisson's ratio in TBCs are all dependent on the temperature and thickness. The thickness of the substrate, bond coat, TGO and ceramic coating are, respectively, assumed to be 2.1mm, 0.1mm, 0.01mm and 0.35mm. Fig.2 shows the relationship of non-dimensional parameter P_{cr}/P_u with respect to the temperature and the delamination length, where P_u denotes the critical buckling loading of the intact two-layer beam. It can be seen that the non-dimensional parameter P_{cr}/P_u decreases with the temperature of TBCs increase under the condition of the same delamination length. For the same temperature, it decreases rapidly when the delamination length increases. It is interested that the temperature ranges, i.e., $1300^0C \sim 1400^0C$, may be a crucial turning temperature ranges. The reason is that the materials strength may severely change under the condition of the temperature ranges. Moreover, for the multiple asymmetric delaminations, the relationship of the non-dimensional parameter P_{cr}/P_u and X_{center}/L with respect to temperature is shown in Fig. 3. where X_{center} is defined as the distance from the left end of multilayered beam-plate to the left tip of the delamination. Obviously, it can obviously see that the non-dimensional parameter P_{cr}/P_u decreases with the increase of the non-dimensional parameter X_{center}/L and temperature. When the temperature is small, the non-dimensional parameter P_{cr}/P_u is severely independent on the increase of the non-dimensional parameter X_{center}/L . However, when the temperature becomes high, the non-dimensional parameter X_{center}/L has slightly influenced on the non-dimensional parameter P_{cr}/P_u . So the increasement of temperature and delamination length have strong influenced on the critical buckling loads of thermal barrier ceramic coatings.

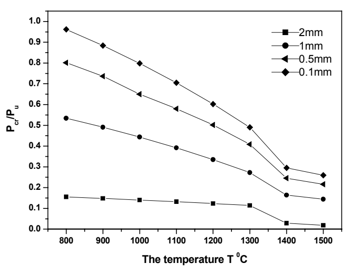

Fig. 2 The relationship of non-dimensional parameter P_{cr}/P_u and temperature with respect to the delamination length

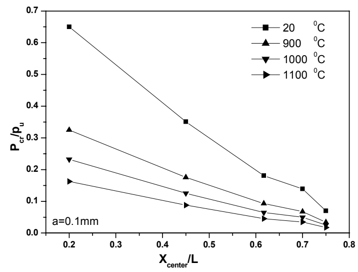

Fig. 3 The relationship of non-dimensional parameter P_{cr}/P_u and X_{center}/L with respect to the different temperature

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