NON PLANAR CRACK PROPAGATION IN CONCRETE SPECIMENS

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ABSTRACT

This paper presents numerical results related to the fracture of concrete specimens loaded in mode I and mixed mode conditions (eccentrically notched beams). The cohesive crack model is used within the framework of the finite element method. The fictitious crack is assumed to grow perpendicularly to the principal tensile stress. The cracking process is controlled through the mode I fracture energy, which is assumed as a material property. No other energy dissipation phenomena are considered. In this case the crack trajectory is not known a priori and a remeshing technique has to be used (see Carpinteri et al. (1993)). Theoretical results are in good agreement with the experimental data by Garcia-Alvarez et al. (1998).

1 INTRODUCTION

A realistic description of the behaviour of cracked concrete structures (e.g., concrete dams) requires a non-linear theory that can predict the well-known phenomenon of *size-effect*. Such a theory was first proposed by Hillerborg (*fictitious crack* model) based on earlier works by Barenblatt and Dugdale on metals (*cohesive crack* model). It assumes the existence of a *Fracture Process Zone* (FPZ), where the material undergoes strain-softening, while the material outside this zone behaves linearly and elastically. The FPZ, whose existence is borne out by experimental evidence, especially in materials characterised by a heterogeneous microstructure, makes it possible to avoid the stress singularity at the crack tip. The fictitious crack model, implemented through the finite element method, has been successfully used by the authors to analyse dam models (see, for instance, Barpi et al. (1999) and Barpi & Valente (2000)).

This paper presents a numerical simulation conducted on a set of concrete specimens loaded in mode I and mixed mode conditions (eccentrically notched beams) that had been tested by Garcia-Alvarez et al. (1998). In this case the crack trajectory is not known a priori and a remeshing technique (see Fig. 1, right) has to be used (see Carpinteri et al. (1993)). Specimens of three different sizes are examined to assess the size effect on the response. The numerical results obtained with the fictitious crack model in terms of load vs. crack mouth opening and crack trajectories are in good agreement with the experiments mentioned above.

Height	Span	Length	Eccentricity (case “a” / “b” / “c”)	Notch depth
d	s	L	e	l
80	200	250	50 / 25 / 0	20
160	400	500	100 / 50 / 0	40
320	800	1000	200 / 100 / 0	80

Table 1: Specimen dimensions (mm).

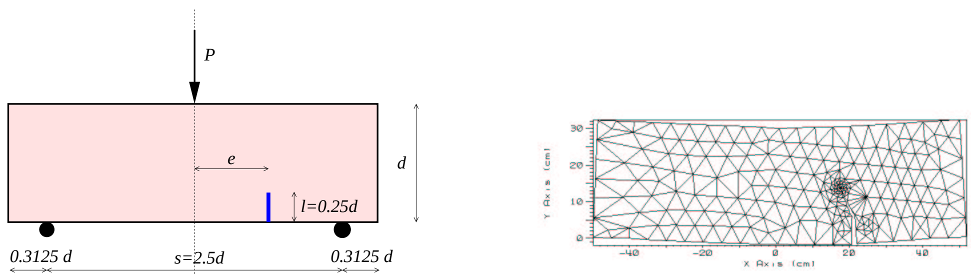

Figure 1: Eccentrically notched beam (left) and finite element mesh (right) for $e = 0.6250d$.

2 EXPERIMENTAL TESTS

The experimental tests are presented in details by Garcia-Alvarez et al. (1998). They basically consist of three eccentrically notched beams (case with eccentricity $e = 0.6250d$ is denoted with “a”, $e = 0.3125d$ with “b” and $e = 0$ with “c”). The load is applied in the middle of the beam: it means that case “a” and “b” are loaded in mixed mode condition while case “c” in mode I (the crack trajectories being symmetrical); see Fig. 1 (left) and Table 1. Height d is taken to be equal to 80, 160 and 320mm while thickness t is 50mm and notch length is $l = 0.25d$.

Garcia-Alvarez et al. (1998) also show that the crack paths are independent of specimen size (of coarse, in the case of non planar crack paths) and that the mode II component of the energy dissipation is negligible, i.e., failure is reached in mode I conditions.

3 THE COHESIVE MODEL: DESCRIPTION AND IMPLEMENTATION

In the theoretical modelling of the behaviour of concrete in tension, strain-softening, i.e., the reduction in bearing capacity brought about by an inelastic deformation process, has to be taken into account. When an initial imperfection (e.g. a notch or a pre-existing crack) is present in a concrete-like material, the non-linear zone localises in a very narrow band, while the material outside this band retains a linear behaviour. The cohesive crack model represents the narrow band as an extension of the real crack called *fictitious crack*. Though it is damaged, the material in this zone is still able to transfer stresses that are decreasing functions of the displacement discontinuity.

This model was initially proposed by Barenblatt (1959) and, independently, by Dugdale (1960). More recently, it was used by Hillerborg et al. (1976) with the name of Fictitious

Crack Model and applied primarily to concrete-like materials. It was numerically implemented through a finite element program.

Subsequently, the cohesive model was applied to mode I and mixed-mode problems by Carpinteri & Valente (1988), Bocca et al. (1990), Valente (1992) and Valente (1993) (among others). They were able to account for the transition from *ductile* to *brittle* behaviour as a function of varying specimen size alone, the material and geometric ratios being the same[1].

The cohesive model rests on the assumption that, as an extension of the *real crack*, a fictitious crack (also referred to as *process zone*) is formed, where the material, albeit damaged, is still able to transfer stresses which are decreasing functions of the relative displacement discontinuity. The fictitious crack grows perpendicularly to the principal tensile stress, at the point where the latter reaches σ_u. This point, called fictitious crack tip (FCT), marks the boundary between the uncracked and the damaged parts of the material. The total (real and fictitious) length of the crack, as measured from the notch to this point, is denoted by ℓ_f. Finally, there is a point where the opening displacement reaches a limit value, w_{nc}, beyond which stresses are no longer transferred: this is known as the real crack tip (RCT). The length of the real crack (stress free) is denoted by ℓ_r. In other words, it can be stated that the cohesive model is based on two constitutive laws: one that governs the undamaged material, and corresponds to the classical elastic linear relationship:

$$\boldsymbol{\sigma} = \boldsymbol{D}\,\boldsymbol{\varepsilon}, \tag{1}$$

and another, referring to cohesive stresses, which, in its simplest form ($\tau_c = 0, \dot{w}_n > 0$), and for a linear σ_c-w_n relationship, can be written as follows:

$$\sigma_c = \sigma_u \left(1 - \frac{w_n}{w_{nc}}\right) \quad \text{for } 0 < w_n < w_{nc}, \tag{2a}$$

$$\sigma_c = 0 \quad \text{for } w_n > w_{nc}, \tag{2b}$$

where σ_c and τ_c stand for the normal and tangential stresses acting on the edges of the fictitious crack, respectively, w_n is the crack opening displacement. The area underneath the σ_c-w_n diagram represents the fracture energy $\mathcal{G}_F$ dissipated per unit area during the entire crack growth process. It should be noted that the presence of cohesive forces eliminates the singularity in the stress field. The condition $\dot{w}_n > 0$ is verified a *posteriori* during the numerical simulation.

According to the finite element method, keeping in mind that both constitutive laws are of the linear type, by taking the unknowns to be the n nodal displacements, $\boldsymbol{u}$, and assuming that compatibility and equilibrium conditions are satisfied at all points in the solid, we get the following system of n equations with $n+1$ unknowns ($\boldsymbol{u}$, λ) (Carpinteri & Valente (1988), Bocca et al. (1991)):

$$\boldsymbol{L}\boldsymbol{u} = \boldsymbol{F}_1 + \lambda \boldsymbol{F}_2. \tag{3}$$

where:

[1] This phenomenon, called size-scale transition from ductile to brittle behaviour, is governed by a non-dimensional *brittleness number*, $s_E = \sigma_u/(\mathcal{G}_F\, d)$, which is a function of material properties σ_u (material's tensile strength), $\mathcal{G}_F$ (fracture energy) and structural size d.

Young modulus	Poisson ratio	Fracture energy	Tensile strength	Density
E	ν	$\mathcal{G}_F$	σ_u	ρ
GPa	–	N/m	MPa	kg/m^3
33.8	0.2	80	3.5	2350

Table 2: Parameters used in the numerical simulations.

- $\boldsymbol{L}$: symmetrical $n \times n$ matrix assembled by bringing together contributions from both constitutive laws,
- $\boldsymbol{F}_1$: vector depending on σ_u, ℓ_f, ℓ_r and all constant loads acting during one step of crack propagation (dead-weight),
- $\boldsymbol{F}_2$: vector depending on the external loads,
- λ: external load multiplier.

The crack is assumed to propagate perpendicularly to the maximum principal stress over a predetermined length, i.e.:

$$\vartheta = \frac{1}{2}\arctan\left(\frac{2\,\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}\right). \tag{4}$$

4 NUMERICAL SIMULATIONS

This Section presents the results obtained with the cohesive model by using the properties listed in Table 2. The stress-displacement relationship has been assumed to be bilinear with knee point coordinates equal to $(\frac{2}{9}w_{nc}, \frac{1}{3}\sigma_u)$, so that $\mathcal{G}_F = \frac{5}{18}\,\sigma_u\,w_{nc}$.

The results in terms of load vs. CMOD are presented below in Figures 2, 3 and 4, left. Each figure presents three numerical results (thick line) and three experimental ones (squares) corresponding to three different sizes.

On the other hand (Figures 2, 3 and 4, right), it is possible to notice that, after the peak value, the ratio between the principal stresses at the FCT reaches the value of 1 (isotropic state, i.e., $\sigma_{xx} - \sigma_{yy} \approx 0$). It means that the assumed criterion is not longer applicable.

5 CONCLUSIONS

1. The comparisons of experimental and numerical results show a fairly good agreement between load vs. CMOD and crack trajectories (not presented here for brevity).
2. As already shown (see, for instance Bocca et al. (1991)), the cracking process is controlled through the mode I fracture energy (no other energy dissipation phenomena are taken into account). This gives a good representation of the phenomenon.

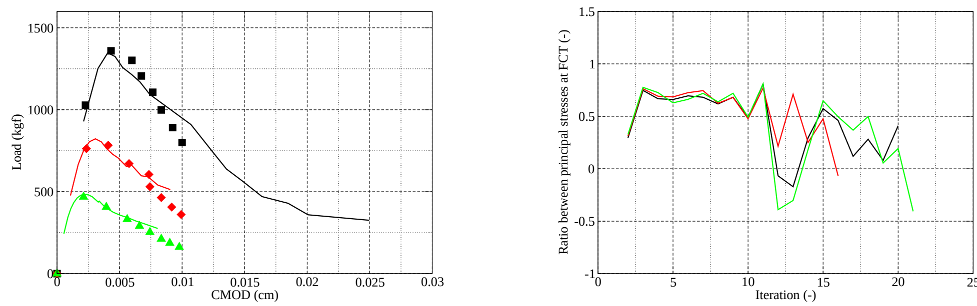

Figure 2: Load vs. CMOD (left) and ratio between principal stresses at FCT (right) for case "a" ($e = 0.6250d$).

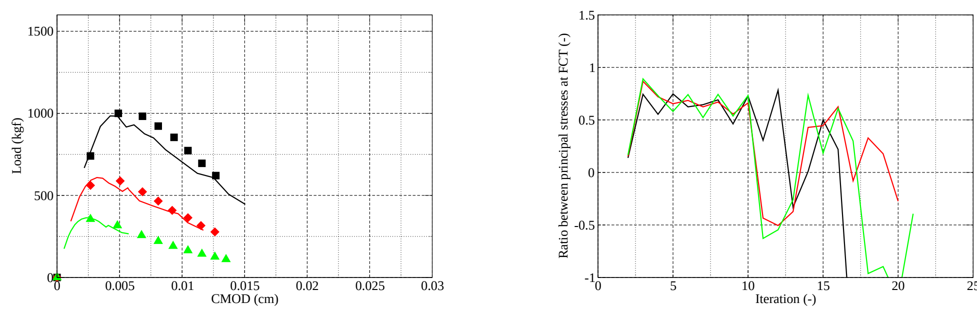

Figure 3: Load vs. CMOD (left) and ratio between principal stresses at FCT (right) for case "b" ($e = 0.3125d$).

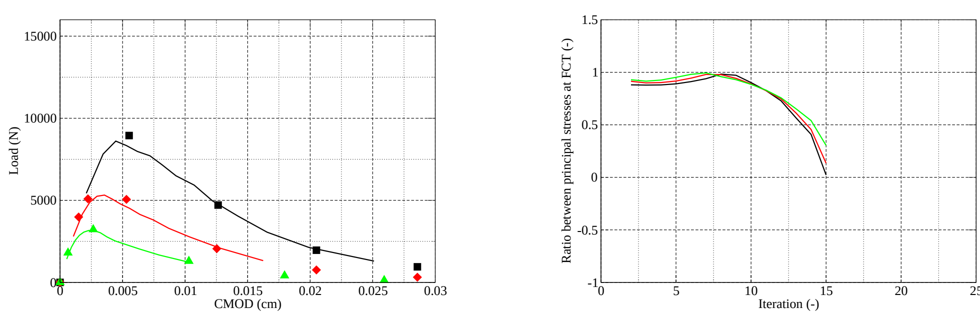

Figure 4: Load vs. CMOD (left) and ratio between principal stresses at FCT (right) for case "c" ($e = 0$).

6 ACKNOWLEDGMENTS

The financial support provided by the Italian Department of Education, University and Scientific Research (MIUR) to the research project on *"Dam-reservoir-foundation systems: dynamic, diagnostic and safety analyses"* (grant number 2002087915_006) is gratefully acknowledged.

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