FATIGUE FRACTURE IN CONCRETE STRUCTURES

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ABSTRACT

The safety of cracked concrete dams is fundamentally affected by the mechanical behaviour of the material under seismic excitation. In this case the crack can grow at a lower load level compared to the monotonic case. This phenomenon is called *subcritical* crack propagation and depends on the mechanical behaviour of the *fracture process zone* (FPZ), where the material, albeit damaged, is still able to transfer stresses. In this zone, which appears at the tip of the macroscopic crack, many non-linear phenomena occur: microcrack interaction with aggregate, porosity and other microcracks. These phenomena are analysed with the Huang-Li *micro-mechanical* model (see Huang & Li (1989)).

In this paper, the FPZ is modelled as a *fictitious extension* of the real crack (stress free crack), where tensile stresses are a decreasing function (*softening*) of the displacement discontinuity (see Hillerborg et al. (1976)). Through this approach, by assuming the fracture energy as a material property, it is possible to predict many *size effects*, which become evident in laboratory tests. The local behaviour under *unloading and reloading* conditions is modelled through Hordijk's *continous function model* (CFM, see Hordijk (1991)). At any point of the FPZ, the stress path vs. crack opening displacement (COD) is not fixed a-priori, but depends on the interaction between hysteretic cycle and crack growth (Barpi & Valente (2004)).

The loading process is based on a first monotonic step stopped before reaching the peak load. Afterwards a series of cyclic loading phases is applied, at increasing load levels, until collapse occurs. In numerical simulations the collapse condition is achieved when the effective tangent stiffness matrix loses the positiveness condition. The results of numerical analyses appear in good agreement with the experimental data obtained by Slowik et al. (1996) in the case of wedge splitting tests.

1 INTRODUCTION

The safety of cracked concrete dams is fundamentally affected by their mechanical behaviour under seismic excitation. It is well known that concrete presents a diffused damage zone within which micro-cracking increases and stresses decrease as the overall deformation increases. This results in the softening of the material in the so-called *fracture process zone* (FPZ), whose size is comparable with that of a characteristic dimension of the structure. This dimension is not constant and may vary during the evolutionary process. In this context, a numerical method has to be used together with the *cohesive* or *fictitious* crack model as shown by Hillerborg et al. (1976) The interaction between strain-softening and fatigue behaviour is analysed locally along th FPZ by modelling the hysteresis loop under unloading-reloading conditions.

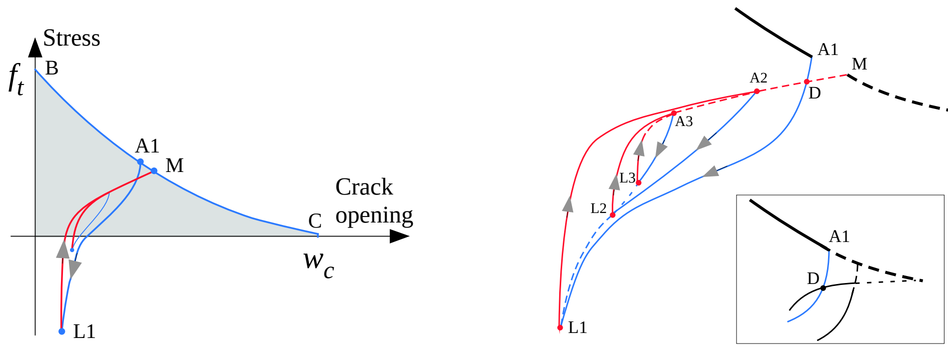

Figure 1: Cohesive stress-COD law (left) and hysteretic unloading and reloading loops according to the CFM (right).

2 FINITE ELEMENT ANALYSIS

In this work, the continuum surrounding the process zone is taken to be *linear elastic*. All non-linear phenomena are assumed to occur in the *process zone*. When the fictitious crack tip advances by a pre-determined length, each point located along the crack trajectory is split into two points. The virtual mechanical entity, acting on these two points only, is called *cohesive element*: the local behaviour of such an element follows the rules described in the previous section. Each cohesive element interacts with the others only through the undamaged continuum, external to the process zone.

According to the finite element method, by taking the unknowns to be the n nodal displacement increments, $\Delta\boldsymbol{u}$, and assuming that compatibility and equilibrium conditions are satisfied at all points in the solid, we get the following system of n equations with $n+1$ unknowns ($\Delta\boldsymbol{u}$, $\Delta\lambda$):

$$(\boldsymbol{K}_T + \boldsymbol{C}_T)\,\Delta\boldsymbol{u} = \Delta\lambda\,\boldsymbol{P} \tag{1}$$

where:

- $\boldsymbol{K}_T$: positive definite tangential stiffness matrix, containing contributions from linear elastic (undamaged) elements and possible contributions from cohesive elements having (σ, w) below the curve of Fig. 1;
- $\boldsymbol{C}_T$: negative definite tangential stiffness matrix, containing contributions from cohesive elements with (σ, w) on the curve of Fig. 1;
- $\boldsymbol{P}$: external load vector;
- $\Delta\lambda$: load multiplier increment. During the numerical analysis the stresses follow a piece-wise linear path. To obtain a good approximation of the non linear curves shown in Fig. 1, $\Delta\lambda$ increments have to be small enough.

The above mentioned model was also succesfully used in the numerical simulation of mixed-mode crack propagation, see Barpi & Valente (1998), Barpi & Valente (2000) as well

Table 1: Geometrical and quasi-static material parameters.

W	a_0	ΔW	ν	β	σ_u	G_f	E
mm	mm	mm	-	-	MPa	N/m	GPa
300	150	1.5	0.15	0.06	2.51	158	16
900	380	2.0	0.15	0.06	2.62	206	17

as in crack propagation under constant load, see Barpi & Valente (2002) and Barpi & Valente (2003).

During the loading phase the stress paths of the cohesive elements are forced to stay on the tension softening curve (Huang & Li (1989)) of Fig. 1 (left), whereas during the cyclic loading phase they are forced to stay on the curves shown in Fig. 1 (right). The $A1-L1-A2$ stress path is called external loop and the $A2-L2$ and $A3-L3$ paths inner loops (Hordijk (1991)).

Fatigue rupture is reached when the smallest eigenvalue of the tangential stiffness matrix becomes negative: this condition means that the external load cannot reach the upper value P_{upper} any longer.

3 WEDGE SPLITTING TESTS

Since dam concrete contains large aggregates, to obtain meaningful results large process zones have to be induced during laboratory tests. In order to keep the specimen size into acceptable values the most effective experimental setup is the so called wedge splitting test. For this reason, as part of an extensive investigation into the safety of cracked concrete dams under seismic excitation, a large serie of experimental fatigue tests was executed by Slowik et al. (1996) using the above mentioned procedure.

4 LOADING PROCEDURE AND MATERIALS PROPERTIES

The loading procedure analysed is based on two phases. In the first, the external load grows monotonically from zero to the fatigue upper level (P_{upper}), a fraction of the quasi-static peak load (P_{peak}). In the second, a cyclic loading condition is applied, from P_{upper} to P_{lower} and vice versa. As the fictitious crack grows, the undamaged ligament reduces and structural compliance increases. If the loading condition is below the endurance limit the rate of compliance growth tends to zero. In both experimental and numerical tests, when this condition is achieved the load level is increased and so on until collapse occurs.

The results shown in Fig. 2 have been obtained for the parameters shown in Table 1 where W is the specimen depth, a_0 the notch depth, W-a_0 the initial ligament, ΔW the mesh size in the FPZ, ν Poisson's ratio, β the Huang-Li tension-softening constant and σ_u the ultimate tensile strength, E the Young modulus, G_f the specific fracture energy,

In WS300 tests, the wedge angle was 15 degrees so that the angle between the applied force and the horizontal line was 15 degrees too. In WS900 tests, the angle between the applied force and the horizontal line was 45 degrees. This condition is due to the upper articulated loading clavis. Since the parameters σ_u and β were not mentioned by Slowik et al. (1996), they were calibrated on the experimental quasi-static peak value of the vertical load measured by the loading cell: 2100N for WS300 tests and 97000N for WS900 tests.

5 VARIABLE AMPLITUDE FATIGUE LOADING INDUCED BY A SEISMIC EXCITATION

It is commonly accepted that under seismic excitation the frequency of the induced stresses in a concrete gravity dam ranges from 3 to 10 Hz. Due to the limited duration of each seismic event, the low-cycle fatigue is of particular interest. This load is far from being harmonic and is characterized by intermittent *spikes*. For these reasons the experimental tests by Slowik et al. (1996) were based on a cyclic load, with pre-defined spike amplitude and frequency.

In order to understand the impact of spikes on the cyclic responce of the FPZ, a deep analysis of the Hordijk (1991) Continuous Function Model (CFM) is necessary. In this direction the position of point $A2$ (beginning of an unloading step) relative to point D in Fig. 1 (right) plays a crucial role:

- if point $A2$ is on the right of point D, point $A1$ moves in such a way that the related unloading path passes through the current point (dotted line in Fig. 1, right). Therefore point $L1$ and M move too, the damage grows, and the current cycle can be locally considered as a *fatigue cycle*;
- if the stress path is on the left of point D, points $A1$, $L1$, M do not move, and an inner loop without damage occurs. The current cycle can be locally considered as a mere *loading cycle* and not as a fatigue cycle.

The above mentioned condition may vary from point to point along the FPZ.

At the peak value of a spike all stress paths in the FPZ are on the right of point D, or even on the right of point M. The next two steps will be characterized by a large unloading step (second part of the spike cycle) followed by a small reloading step (first part of an ordinary cycle). As a consequence, all stress paths are on the left of point D and all ordinary cycles will be inner loops without damage until a new spike occurs. In other words, the inner loops are mere loading loops and not fatigue loops.

For these reasons the numerical simulation of experimental tests executed with pre-defined spike amplitude and frequency were based on the spikes only, neglecting the intermediate loading loops.

6 COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

The damage level can be summarized in a parameter: the length of a stress-free crack that induces the same unloading-reloading compliance on a numerical model of the specimen. This length is called *equivalent crack length*.

With reference to the experimental results a first observation is useful: in many cases, during the last loading phase (just before the collapse) (P_{upper}) is larger than the quasi-static peak load (P_{peak}). Some specimens in the WS300 serie (see K08, K12, K13, K14,

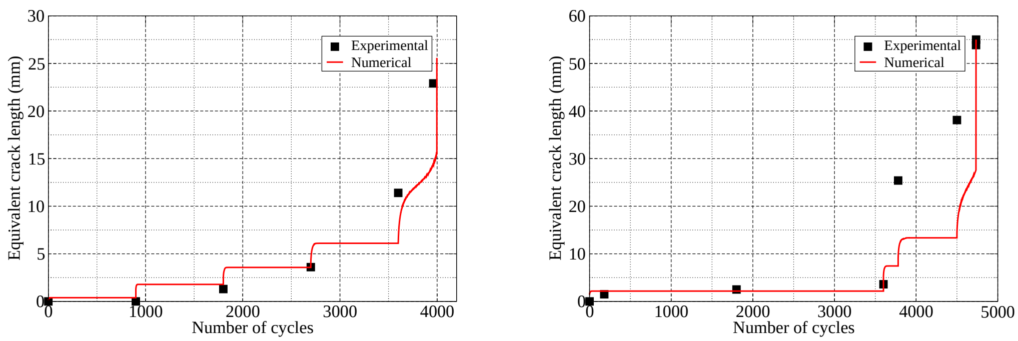

Figure 2: Equivalent crack length vs. number of cycles (WS300/K12: left, WS900/G10: right).

K15, K18 in Slowik et al. (1996)) and some specimens in the WS900 serie (see G06, G07, G08, G10, G12, G16, G18, G19, G20) were in this situation. This phenomenon is due to a rate-dependent material behaviour. In order to compare experimental and numerical results, as suggested by Brühwiler (1988), both values of σ_u and G_f were increased by the same factor . In this paper a factor equal to 1.45 was chosen.

Figure 2 (left) and Fig. 2 (right) show the *equivalent crack length* vs. number of cycle respectively in the case of specimen K12 (WS300 serie) and G10 (WS900 serie). After each load increment, the numerical simulation shows immediatly a compliance increment. If the load level is below the endurance limit, after the load increment the *equivalent crack length* tends to become constant with increasing number of cycles.

During the experimental tests, the decision when to increase the load level was up to the operator. For this reason the number of cycles at failure is not a pure mechanical response. In WS300 tests, since each specimen was subjected to a different loading sequence, the number of cycle at failure varies from 101 (specimen K08) to 11876 (specimen K17). Similarly in WS900 tests it varies from 205 (specimen G12) to 4735 (specimen G10).

A smaller scattering of results is found looking at the last (before failure) *equivalent crack length* measured during the experimental tests: for WS300 tests, it varies from 17 mm (specimen K14) to 75.9mm (specimen K17), and for WS900 tests, it varies from 15.2mm (specimen G17 and G19) to 134.1mm (specimen G11).

Figure 2 (left and right) shows a good agreement between experimental and numerical results.

7 CONCLUSIONS

- The interaction between strain-softening and fatigue behaviour can be analysed locally along th FPZ by modelling the hysteresis loop under unloading-reloading conditions.
- Using the numerical method described above, a unique set of material properties can

explain the subcritical crack propagation in cyclic conditions on two different scales. It is therefore able to predict the size effect on the above mentioned phenomenon.

- The scattering of the experimental and numerical results in terms of *equivalent crack length* is smaller than the scattering in terms of number of cycle at failure.

8 ACKNOWLEDGMENTS

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