A MODIFIED RELAXATION-MODEL FOR THE PREDICTION OF RESIDUAL STRESSES IN TWO-DIMENSIONAL ROLLING/SLIDING CONTACT

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ABSTRACT
Residual stresses are a crucial matter regarding rolling contact fatigue, either for ratchetting and low cycle fatigue failures as well as for high cycle fatigue. In this paper, an algorithm to compute the residual stresses under two dimensional rolling and sliding contact is proposed, based in some modifications over a previous semi-analytical approach forwarded by Jiang and Sehitoglu [1],[2]. The modified model here proposed, which takes into account a full nonlinear kinematical hardening, is stable and gives accurate solutions for high values of plastic loads. The method was also validate with finite element data extracted from the literature, showing a very good agreement.

1 INTRODUCTION

Elastic-Plastic stress analysis in rolling contact is an important topic in engineering, although it is still at present at highly complicated task. For small contact loads, the material response is full elastic and in these conditions neither residual stresses neither remaining strains after each contact are present. As contact load increases, the material response becomes elastic-plastic and in this way residual stresses and permanent strains could arise. A method to address this problem is by the utilisation of the so-called Shakedown Maps [3],[4], first proposed by Johnson and Jefferis. In this context, depending on normal and traction contact loads, different mechanisms of structural response to cyclic loading can be identified. Excluding full elastic response, these behaviours are: (a) *Elastic shakedown*: where the elastic limit is reached in the first few cycles, but the steady-state is entirely elastic (the maximum load for which elastic shakedown can be achieved is known as the *shakedown limit*); (b) *Cyclic plasticity or plastic shakedown*: where the steady state consists of a closed cycle of plastic deformation; (c) *Ratchetting or incremental collapse*: where the structure accumulates increments of uni-directional plastic strain, leading to collapse.

It is possible, of course, to find situations where *(b)* and *(c)* act simultaneously, but in these cases the material is likely to fail by ratchetting [5]. So, the shakedown map is extremely useful to avoid, by design, the ratchetting region, where the failure is likely to be produced in a small number of cycles. Indeed, very accurate life predictions can be obtained, for ratchetting failures, with the Kapoor and co-workers' model [5],[6], where incremental strain by cycle is assumed to be proportional to the ratchetting load, assumed as the relation between p_0/k (being p_0 the maximum pressure and k the yield shear stress) and p_s/k (being p_s the pressure to achieve shakedown limit). The interested reader is referred at references [5]-[7] for a full description of this model and some of its applications. Regarding cyclic plasticity and, fundamentally, elastic shakedown regions, the shakedown map presents its major drawback. In effect, this approach does not give information about residual stresses in these conditions. So, in this way, it is not possible to address the verification of material against high cycle fatigue, since the real stress path (to be inserted, for example, in a multiaxial criterion) remains unknown. It is worthwhile to note that high cycle fatigue could occur, in some 'high-transit' railways lines, in a number of cycles that can be traduced in terms of just one year. For this reason, high cycle fatigue, and therefore residual stresses, are an important issue to be addressed in rolling contact phenomena.

Basically, there are two ways for computing the residual stresses under rolling contact loads: semi-analytical approaches and finite element modelling. Although significant and constant advances in computing, the task of simulating rolling contact behaviour using finite element approach remains cumbersome, even in the case where simplified material model are used. For this reason, semi-analytical models, which are the premise of obtain residual stresses in a more simple way (in terms of preparation of the algorithm and computing time) are an interesting way to address the problem. Semi-analytical approaches for rolling contact have started with the work of Merwin and Johnson [8], which solved the problem of two-dimensional plastic deformation, assuming that the strain cycle remains identical with the elastic strain cycle. Hearle and Johnson [9] have then proposed another approach assuming that orthogonal shear strain component is the only plastic strain occurring throughout a rolling passage. Bower and co-workers have modified this approach, taking also into account a nonlinear hardening plasticity model [10],[11]. More recently, Yu and co-workers [12] applied Zarka's operator technique [13] to solved the problem, but the shortcoming of this approach is that it is confined to linear hardening material. Another two interesting approaches are those of McDowell-Moyar [14] and Sehitoglu-Jiang [1],[2]. McDowell has proposed also an hybrid-model, take into account the advantages of each of these two models [15].

On this basis, the authors, which are working in the application of a new model for the three-dimensional analysis of elastic-plastic problem, will propose in this paper some modifications in the Sehitoglu-Jiang model in order to make the solution accurate for high p_0/k values, because, as McDowell has observed, the algorithm seems to becomes inaccurate for high levels of cyclic plasticity (e.g. $p_0/k > 5.5$). The authors have selected this algorithm because of the drawback (only linear hardening material) of the semi-analytical approach proposed by Yu and co-workers [10] and, on the other hand, the McDowell-Moyar algorithm has some drawback to the future generalisation to three-dimensional problems, basically because in this model some conditions are imposed on each incremental step, as for example, zero strain rate in the rolling direction. In reality, the sum of the elastic- and plastic-strain in this direction need not to be cancelled at each increment, but rather only at the end of the cycle.

METHOD

2.1) General approach

First of all, to simplified the elastic-plastic analysis of rolling contact, stress cycle is assumed to be equal to the elastic solution, i.e.

$$\sigma_{ij} = \sigma_{ij}^{el} \qquad (1)$$

where σ_{ij} is the stress tensor and *el* denotes elastic solution. After each passage of load, the strains are enforced to relax proportionally to meet the stress and strain boundary conditions. The geometry here adopted for the rolling line contact is considered in Figure 1, where the coordinates system moves with the load. The load is assumed to be two-dimensional Hertzian and the tangential tractions, q, in this region is proportional to the normal pressure p, i.e.

$$p = p_0\sqrt{1-\left(\frac{x}{a}\right)^2} \quad ; \quad q = q_0\sqrt{1-\left(\frac{x}{a}\right)^2} \quad ; \quad q_0 = -\mu . p_0 \qquad (2)$$

being p_0 and q_0 the maximum pressure and traction respectively. Stress components due to normal and tangential loads can be computed with the McEwen equations [16].

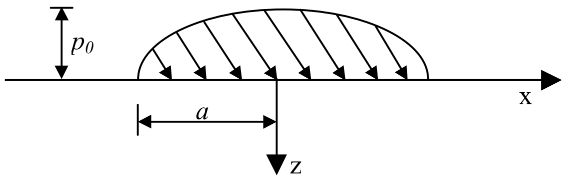


Figure 1: Coordinate system for rolling/sliding contact

For the first passage, when the load is far from the section under analysis, only elastic response is achieved. Then, as load is approaching, an elastic-plastic response could take place. When the load is again far away from the section under analysis, stresses can be neglected (for example, for $x=100.a$, $\sigma_{z,max}$ $(x=100.a)/p_0<0.01$, so it is possible to add a virtual step, enforcing to zero all stresses, without losing precision). On the other hand, as the solution comes from Boussinesq and Cerruti potentials, it is clear that, in order to obtain stresses strictly equal to zero, one should take the solution as x tends to infinitive, but again, all these steps, far from the section under analysis, let say, bigger than $x>100a$, do not add substantial information to the stress-strain history. Thus, after the passage of the first load, stresses are zero but some residual strains, as ε_x and ε_y arise. But these strains are not in agreement with the conditions for line contact, which requires (see, for example, reference [6]):

$$\varepsilon_x = \varepsilon_y = 0 \tag{3}$$

so a relaxation procedure follows in order to enforce these strains to zero. In this process, σ_x and σ_y arise. These stresses are, in this way, the residual stresses after the first passage of load. Relaxation procedure will be explained in detail in section 2.3. The same routine is repeated for the second and subsequent passages of the load, but using the previous residual stresses as the initial stresses for the next passage of the load.

2.2) The elastic-plastic response to the load passage

Here the material is modelled trough a Jiang-Sehitoglu [17] plasticity model, which is substantially based on an Armstrong-Frederick hardening rule following the Chaboche [18] decomposition of the backstress. First, an initially isotropic material which is incompressible for plastic deformation is considered. The material follows the elastic stress-strain relation until the yield condition is reached. Here the von Mises yield function is considered, i.e.:

$$f = \left(\underline{\underline{S}} - \underline{\underline{\alpha}}\right) : \left(\underline{\underline{S}} - \underline{\underline{\alpha}}\right) - 2k^2 = 0 \tag{4}$$

being $\underline{\underline{S}}$ the deviatoric stress tensor and $\underline{\underline{\alpha}}$ the backstress tensor. For the strain increment, the normality flow rule is considered:

$$d\underline{\underline{\varepsilon}}^p = \frac{1}{h}\left(d\underline{\underline{S}} : \underline{\underline{n}}\right)\ \underline{\underline{n}} \tag{5}$$

being h the so-called plastic modulus function and

$$\underline{\underline{n}} = \frac{\left(\underline{\underline{S}} - \underline{\underline{\alpha}}\right)}{\left(\left(\underline{\underline{S}} - \underline{\underline{\alpha}}\right) : \left(\underline{\underline{S}} - \underline{\underline{\alpha}}\right)\right)^{1/2}} \tag{6}$$

Here we have the first difference with the original model. In fact, eq (5) is used by Jiang-Sehitoglu [1],[2] and also by McDowell-Moyar [12] with the MacCauley bracket (i.e $<x>=0.5(x+|x|)$ for the inner product of ($\underline{\underline{S}}$:$\underline{\underline{n}}$). Nevertheless, it is important to observe that it is possible to obtain a plastic strain increment although when ($\underline{\underline{S}}$:$\underline{\underline{n}}$) is negative (and of course, simultaneity $\underline{\underline{S}}$ lies outside of the original yield surface). This is of course correlated with high stress increments, which is just the case of rolling contact phenomena, which is characterised by high gradients in stresses, specially near the surface. The interested reader is referred to the reference [19] where additional justification and examples are given. The hardening rule is expressed in the following form:

$$\underline{\underline{\alpha}} = \sum_{i=1}^{M} \underline{\underline{\alpha}}^{i} \tag{7}$$

$$d\underline{\underline{\alpha}}^{i} = c^{i} r^{i} \left(\underline{\underline{n}} - \left(\frac{\left|\underline{\underline{\alpha}}^{i}\right|}{r^{i}} \right)^{\chi^{i}+1} \frac{\underline{\underline{\alpha}}^{i}}{\left|\underline{\underline{\alpha}}^{i}\right|} \right) dp \qquad ; \qquad (i=1,2,....,M) \tag{8}$$

where it is clear that the backstress has been divided into M parts $\underline{\underline{\alpha}}^{i}$ ($i=1,2,...,M$). In turn, c^i, r^i and χ^i are three sets of materials constants. During plastic deformation the stress state should lie on yield surface (consistency condition). If the plastic modulus function, for a material without isotropic hardening, is computed in the following way:

$$h = \frac{\underline{\underline{n}} : d\underline{\underline{\alpha}}}{dp} \tag{9}$$

being the called equivalent plastic strain increment, dp, defined as

$$dp = \sqrt{d\underline{\underline{\varepsilon}}^{p} : d\underline{\underline{\varepsilon}}^{p}} \tag{10}$$

then consistency is violated. In order to keep the consistency, the eq. (9) must be reformulated as follows:

$$h = \frac{1}{dp}\left(d\underline{\underline{\alpha}} : \underline{\underline{n}}\right) - \left(d\underline{\underline{S}} - d\underline{\underline{\alpha}} : d\underline{\underline{S}} - d\underline{\underline{\alpha}}\right) / \left(2\sqrt{2}k\,dp\right) \tag{11}$$

which constitutes another substantial different of the present model compared with the previous ones, and is due to the passage of infinitesimal approach to a numerical, discrete one. In this way, the numerical analysis of plastic increments becomes more stable and accurately.

2.3) The relaxation procedure

Finally, the relaxation procedure is performed in the following way. First of all, the residual stress and strain conditions in line contact are [6]:

$$\begin{array}{ll} \varepsilon_x^{\,r}=0 & \sigma_x^{\,r}=f_1(z) \\ \varepsilon_y^{\,r}=0 & \sigma_y^{\,r}=f_2(z) \\ \varepsilon_z^{\,r}=0 & \sigma_z^{\,r}=0 \\ \gamma_{xz}^{\,r}=f_3(z) & \tau_{xz}^{\,r}=0 \end{array} \tag{12}$$

where $f(z)$ means that f is only a function of z. Thus, in order to achieve these conditions, ε_x, ε_y and ε_z should be enforced to zero in N steps, in which:

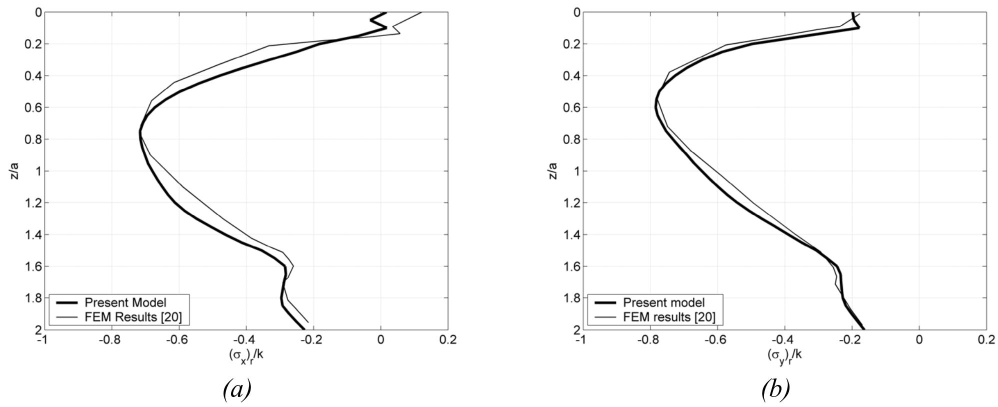
$$\Delta\varepsilon_x=-\varepsilon_x/N \qquad \Delta\varepsilon_y=-\varepsilon_y/N \qquad \Delta\varepsilon_z=-\varepsilon_z/N \tag{13}$$

With these inputs, i.e., knowing $\Delta\varepsilon_x$, $\Delta\varepsilon_y$, $\Delta\varepsilon_z$ and imposing $\Delta\sigma_z=\Delta\tau_{xz}=0$, a system of four equations with three unknowns ($\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\gamma_{xz}$) can be obtained for each relaxation step. After that, this system, clearly over-constrained, is solved by least square method. Further details are given in [19]. The number of relaxation steps, N, typically is chosen between 100 and 500 steps.

3 RESULTS AND CONCLUSIONS

Some results are shown in Figure 2 and 3. In Figure 2, the comparison for the rolling contact residual stresses between the method here proposed and a Finite Element Solution, for a SAE 1070 steel, fully described with a non-linear kinematical hardening is presented [20]. As it can be observed, the results provided for this high value of p_o/k show a very high correlation with the FEM results. The prediction of the model for p_o/k=7 is presented in Figure 3.

As conclusion, it can be said that the proposed method seems to constitute an attractive and efficient tool to compute the residual stresses and strain under two-dimensional, line, rolling and sliding contact.



(a) *(b)*

Figure 2: Comparison with FEM results provided by Jiang et al [20]; a)residual stresses in the rolling direction; b)residual stresses in the transversal direction. (p_0/k=6, full rolling contact, material SAE 1070).

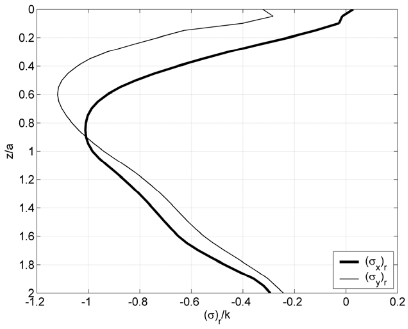


Figure 3: Predicted residual stresses for a full rolling contact, at $p_0/k=7$.

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