THREE-DIMENSIONAL MODELING OF STRONG DISCONTINUITIES IN ROCK

RICHARD A. REGUEIRO
Materials and Engineering Sciences Center
Sandia National Laboratories
P.O. Box 969, MS 9405
Livermore, California 94551-0969
USA
raregue@sandia.gov

ABSTRACT

This paper presents a post-bifurcation constitutive model and a modified assumed enhanced strain (AES) finite element implementation for simulating strong discontinuities in rock in three-dimensions. A plasticity model appropriate for modeling pre-failure deformation response in rocks is formulated with strong discontinuity (jump is displacement field), leading to bifurcation criteria and a form for post-bifurcation constitutive equations. The finite element implementation follows a modified AES approach, embedding the jump displacement evolution within a bifurcated hexahedral element response [4]. Previously, for geomaterials, this approach was developed for two-dimensional, plane strain problems [1] [3]. Although this approach cannot resolve stress at a crack tip, it is useful for tracking failure in geomechanical scale problems (m-km) involving rock, such as tunneling construction or oil exploration. Numerical examples in three-dimensions will demonstrate the approach.

1 Introduction

Strong discontinuities, or cracks, naturally occur in rock under various loading conditions [7]. We attempt to account for this form of localized deformation within the context of plasticity theory and to track the crack propagation and post-bifurcation constitutive response using a modified AES implementation. Details on plasticity models with strong discontinuity (displacement jumps) and their numerical implementation are given in previous papers [6] [1] [3] and will not be repeated here. The formulation in this paper is restricted to small deformations.

2 Post-bifurcation model

Upon detecting the stress state at which a strong discontinuity mode of deformation is admissible in a body (cf. [2] [5]), a post-bifurcation constitutive model is activated. In general, such a model takes the following form for a discontinuity surface with unit normal $\boldsymbol{n}$ and tangent $\boldsymbol{t}$

$$\text{traction}: \quad \boldsymbol{T} = [T_n \; T_t] \, ; \; T_n = \boldsymbol{n} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} \, , \; T_t = \boldsymbol{t} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} \tag{1}$$

$$\text{displacement}: \quad \dot{\boldsymbol{u}} = \dot{\gamma} \partial G(\boldsymbol{T}, \boldsymbol{q}) / \partial \boldsymbol{T} \tag{2}$$

$$\text{yield function}: \quad F(\boldsymbol{T}, \boldsymbol{q}) = 0 \tag{3}$$

$$\text{evolution equations}: \quad \dot{\boldsymbol{q}} = \dot{\gamma} \boldsymbol{h} \tag{4}$$

where $\boldsymbol{T}$ is the traction vector with normal T_n and tangential T_t components, $\boldsymbol{\sigma}$ is stress, $\dot{\boldsymbol{u}}$ is the jump velocity, $\dot{\gamma}$ is a plasticity consistency parameter, G is the plastic potential function, $\boldsymbol{q}$ is a vector of internal variables, F is the yield function, and $\boldsymbol{h}$ is a vector of hardening/softening functions.

We choose to implement this model using a modified AES approach, although this model is not limited to this numerical implementation (cf. [4]).

3 Modified AES implementation

A modified AES implementation was developed for the bilinear quadrilateral element [1] [3] and for the trilinear hexahedral element [4]. A standard AES implementation was used for a constant strain tetrahedron [8]. In the modified AES implementation, the enhanced strain variation is chosen to represent the post-bifurcation evolution equation in weak form (cf. [1] for details). This leads to a more robust numerical method and a physical account of the traction-displacement relationship along the discontinuity within the enhanced finite element. The stress at time t_{n+1} is written as [1] [3]

$$\boldsymbol{\sigma}_{n+1} \;=\; \boldsymbol{\sigma}_{n+1}^{\text{trial}} \;-\; \boldsymbol{D} \cdot \boldsymbol{G}^e \, \Delta\zeta^e \quad \text{in} \quad \Omega^e_{\text{loc}} / \mathcal{S}^e \tag{5}$$

where $\boldsymbol{D}$ is the elasticity matrix, $\Delta\zeta^e$ is the jump displacement over an increment in time within element e, Ω^e_{loc} denotes a localized region for element e and $\mathcal{S}^e$ its discontinuity surface, and the regular part of the enhanced strain displacement matrix $\boldsymbol{G}^e$ is

$$\boldsymbol{G}^e \;=\; (\boldsymbol{m}^e \otimes \boldsymbol{\nabla} f^e)^s \tag{6}$$

where $\boldsymbol{m}^e$ is the direction of the jump displacement, $\boldsymbol{\nabla}$ is a spatial gradient operator, and $(\bullet)^s$ denotes symmetric part. For the enhanced strain hexahedron, we need to determine how to construct the enhancement function f^e. Details from [4] are repeated here. Figure 1 shows the five different cutting plane conditions for the hexahedron. Figure 2 demonstrates how to determine whether a node is active in terms of constructing f^e, which then may be constructed as

$$f^e(\boldsymbol{x}) \;=\; \sum_{B=1}^{n_{\text{active}}} N^B(\boldsymbol{x}) \, ; \quad \boldsymbol{\nabla} f^e(\boldsymbol{x}) \;=\; \sum_{B=1}^{n_{\text{active}}} \boldsymbol{\nabla} N^B(\boldsymbol{x}) \tag{7}$$

where $N^B(\boldsymbol{x})$ is the trilinear shape function at node B.

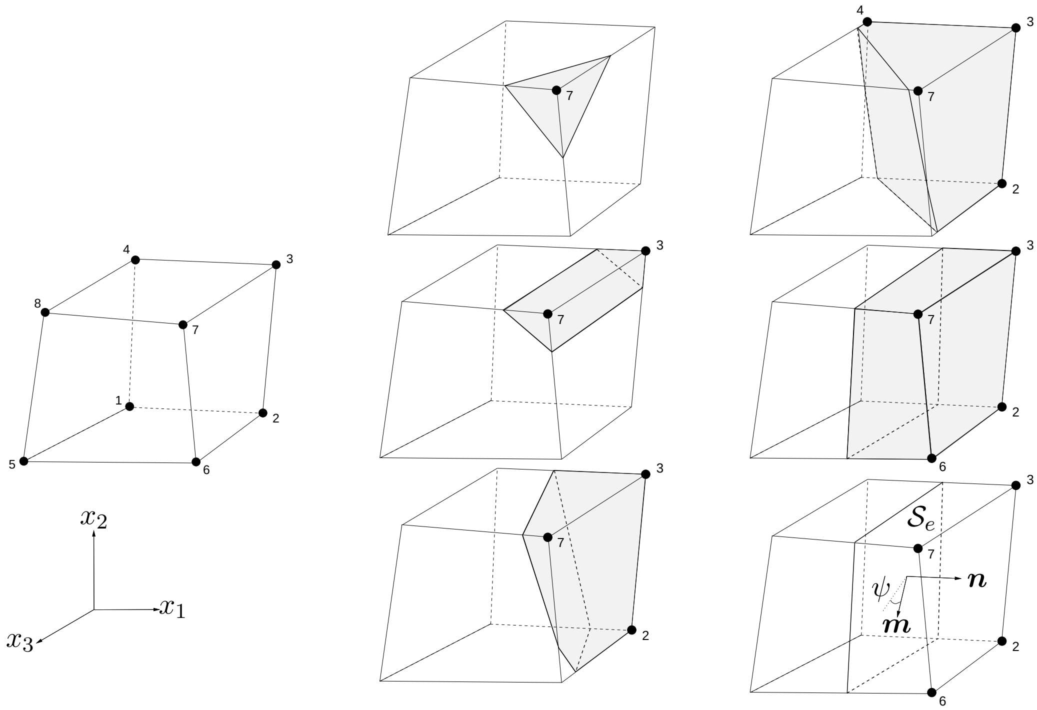

Figure 1: Enhanced strain hexahedron with slip plane showing five possible slip-plane cutting conditions [4].

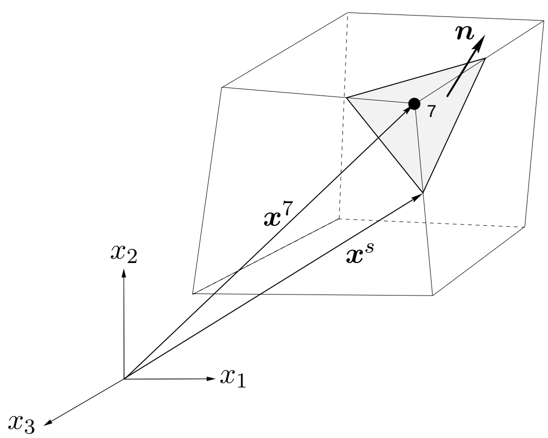

Figure 2: Determining active nodes: if $\boldsymbol{n} \cdot (\boldsymbol{x}^A - \boldsymbol{x}^s) > 0$ then node A is active where $\boldsymbol{x}^A$ is the location of node A and $\boldsymbol{x}^s$ is the location of a point on the slip plane with unit normal $\boldsymbol{n}$ [4].

4 Summary

Future numerical examples will demonstrate the capability of the embedded discontinuity, modified AES hexahedron to model post-bifurcation constitutive response in rock in three-dimensions.

5 Acknowledgements

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-ACO4-94AL85000.

REFERENCES

[1] Borja, R.I. and Regueiro, R.A. Strain localization in frictional materials exhibiting displacement jumps, *Comput. Meth. Appl. Mech. Eng.*, (2001) 190: 2555−2580.

[2] Borja, R.I. and Aydin, A. Computational modeling of deformation bands in granular media, I: Geological and mathematical framework, *Comput. Meth. Appl. Mech. Eng.*, in press.

[3] Regueiro, R.A. and Borja, R.I. Plane strain finite element analysis of pressure-sensitive plasticity with strong discontinuity, *Int. J. Solids Struct.*, (2001) 38: 3647−3672.

[4] Regueiro, R.A., Foster, C.D., and Borja, R.I. Three dimensional modeling of slip surfaces in geomaterials, *Proceedings of the 15th ASCE Engineering Mechanics Conference*, Columbia University, New York, NY, (2002) CD-ROM.

[5] Regueiro, R.A., Foster, C.D., Fossum, A.F., and Borja, R.I. Bifurcation analysis of a three-invariant, isotropic/kinematic hardening cap plasticity model for geomaterials, *GulfRocks 2004*, Houston, TX, (2004) CD-ROM.

[6] Simo, J.C., Oliver, J., and Armero, F. An analysis of strong discontinuities induced by strain-softening in rate-independent inelastic solids, *Comput. Mech.*, (1993) 12: 277-296.

[7] Wawersik, W.R., Rudnicki, J.W., Olsson, W.A., Holcomb, D.J. and Chau, K.T. Localization of deformation in brittle rock: theoretical and laboratory investigations, In *Micromechanics of Failure of Quasi-Brittle Materials*, Elsevier, New York, Eds. Shah, S.P. and Swartz, S.E. and Wang, M.L., (1990), 115-124.

[8] Wells, G.N., and Sluys, L.J. Three-dimensional embedded discontinuity model for brittle fracture, *Int. J. Solids Struct.*, (2001) 38: 897−913.