NEW SCHEMES FOR COMPUTATIONAL MODELLING OF SIZE-DEPENDENT CONICAL INDENTATION

Y.P. CAO and J. LU
LASMIS, FRE, CNRS 2719, Universite de Technologie de Troyes, 12 rue Marie Curie, BP 2060, 10010 Troyes, France

ABSTRACT

In this report, size-effects observed in conical indentation tests due to the existence of plastically-graded surfaces and the effect of geometrically necessary dislocations (GND) have been discussed. First, a new scheme has been proposed for the computational modelling of conical indentation in plastically-graded materials. Based on the new scheme and the work of Dao et al. [1], an analytical expression to predict the indentation loading curve has been presented which can be further used to establish a reverse algorithm to determine the plastic properties of a plastically-graded surface. Secondly, to characterize the effect of GND, a closed-form expression of the size-dependent sharp indentation loading curve has been proposed based on dimensional analysis and the finite deformation Taylor-based nonlocal theory (TNT) of plasticity (Huang et al. [2]). The key issue is to link the results of FEM based on TNT plasticity with those obtained using conventional FEM by taking as the effective strain gradient that presented in the work of Nix and Gao [3], thus avoiding large-scale finite element computations using strain gradient plasticity theories. Experiments carried out on 316 stainless steel have been used to verify the effectiveness of the present analytical model; the results demonstrate that the present analytical expression of the size-dependent indentation loading curve corresponds very well to the experimental indentation loading curve. The empirical constant, α, in the Taylor model estimated from the experimental data has the correct order of magnitude.

1 INTRODUCTION

During the past two decades, with technological advances and the increasing need to measure the mechanical properties of materials on a small scale, much effort has been devoted to develop systematic methods to extract the mechanical properties of materials using depth-sensing instrumented indentation. In the present report, we mainly refer to the research related to the determination of plastic properties of metal materials from conical indentation loading curves. The dual indenter algorithms devised recently by Bucaille et al. [4] and Chollacoop et al. [5] appear to be effective methods which have been further analyzed by authors [6] from the viewpoint of mathematical theory of inverse problems. With respect to the shape of the loading $p-h$ curve, most work has been based on an important assumption i.e.

$$p = Ch^2 \tag{1}$$

which is a natural outcome of dimensional analysis (Cheng and Cheng [7, 8]). The loading curvature, C, is constant for given material properties and independent of the indentation depth. However, in practice, eqn (1) will not be true in certain cases, which causes the dual indenter algorithms to lose their accuracy. Here two factors frequently encountered in practice have been discussed: (1). Effects of the plastically-graded surface. In many surface preparation methods, a plastically-graded surface can be produced. For conical indentation in plastically-graded materials, eqn (1) is not true. Suresh [9] has pointed out that the theoretical formulations for certain idealized cases were first given by Suresh and Giannakopoulos [10]. More detailed work was published by Giannakopoulos [11]. Recently, a new scheme has been proposed by the authors [12] for the computational modelling of conical indentation in plastically-graded materials. Based on the new scheme and the work of Dao et al. [1], an analytical expression to predict the indentation loading curve has been presented which can be further used to establish a reverse algorithm to determine the plastic properties of a plastically-graded surface. Detail information will be presented in section 2. (2). Effects of geometrically necessary dislocations. Another difficulty in applying dual indenter algorithms to measure the plastic properties of materials is the size-effect caused by geometrically necessary dislocations (GND) (Fleck et al. [13]; Stelmashenko et al. [14]; De Guzman et al. [15]; Ma and Clarke [16]; Nix and Gao [3]). Bucaille et al. [17] proposed a method to modify the indentation loading curve using the model of Nix and Gao [3]. For cold worked aluminum, their method shows good accuracy, but for annealed aluminum, the results are still poor. The authors [18, 19] have proposed a closed-form expression of the size-dependent indentation loading curve in which a strain-independent material length scale was included to express the effect of geometrically necessary dislocations; by applying the analytical results, a reverse algorithm to extract the plastic properties of materials can be further established. Our research for this problem will be presented in detail in section 3.

2 A COMPUTATIONAL MODELLING SCHEME FOR CONICAL INDENTATION IN PLASTICALLY-GRADED MATERIALS

In this section we will present a scheme for analyzing the indentation of plastically-graded materials using FEM, aimed at providing a method to determine the closed-form expression of the indentation loading curve. In terms of plastically-graded materials, elastic-plastic materials are considered which exhibit power-law hardening, and have a yield limit which varies linearly with the depth. All the other material constants are independent of the position. For a Berkovich or Vikers indenter (equivalent to a 70.3^0 cone), based on eqn (1), a closed-form expression of the indentation loading curve for a wide range of metal engineering materials has been proposed by Dao et al. [1], i.e.

$$P = Ch^2 = \sigma_r \Pi_1\left(\frac{E^*}{\sigma_r}\right)h^2 \tag{2}$$

where Π_1 is the dimensionless function, the details of which are given in reference [1].

Further analysis [12] reveals that for the indentation of plastically-graded materials with a linearly-varied yield strength, e.g. $\sigma_y = \sigma_y^0(1+b\xi)$, eqn (2) can be directly used to present the closed-form expression of indentation loading curve by replacing the loading curvature with

$$\overline{C} = \overline{\sigma}_r \Pi_1\left(\frac{E^*}{\overline{\sigma}_r}\right) = \overline{\sigma}_r\left(C_1 \ln^3\left(\frac{E^*}{\overline{\sigma}_r}\right) + C_2 \ln^2\left(\frac{E^*}{\overline{\sigma}_r}\right) + C_3 \ln\left(\frac{E^*}{\overline{\sigma}_r}\right) + C_4\right) \tag{3}$$

where

$$\overline{\sigma}_r = \overline{\sigma}_y\left(1 + \frac{E}{\overline{\sigma}_y}0.033\right)^n \tag{4}$$

and

$$\overline{\sigma}_y = \sigma_y^0\left(1 + b\frac{\kappa a}{s_0}\right) \tag{5}$$

where σ_y^0 is the yield strength of the material on the surface, b is a dimensionless number which measures the variation in the yield strength from the surface to a fixed point s_0 inside the material, a is the indentation radius without considering piling-up and sinking-in and the parameter κ in eqn (5) can be defined as the following dimensionless function

$$\kappa = \Pi_\kappa \left(n, \frac{E}{\sigma_y^0}, b, \frac{h}{s_0} \right) \tag{6}$$

which has been determined with FEM as

$$\kappa = \Pi_\kappa = \phi^+ \left(n, \frac{E}{\sigma_y^0} \right) + \varphi^+ \left(n, \frac{E}{\sigma_y^0} \right) b \qquad (0 \le b \le 12.0) \tag{7a}$$

$$\kappa = \Pi_\kappa = \phi^- \left(n, \frac{E}{\sigma_y^0} \right) + \varphi^- \left(n, \frac{E}{\sigma_y^0} \right) b \qquad (-0.9 \le b \le 0.0) \tag{7b}$$

where ϕ^+, φ^+, ϕ^- and φ^- can be found in reference [12]. Figure 1 shows that the present analytical results correspond very well to that of the finite element computations. The analytical results can be further applied to establish an inverse approach to extract plastic properties of metal materials.

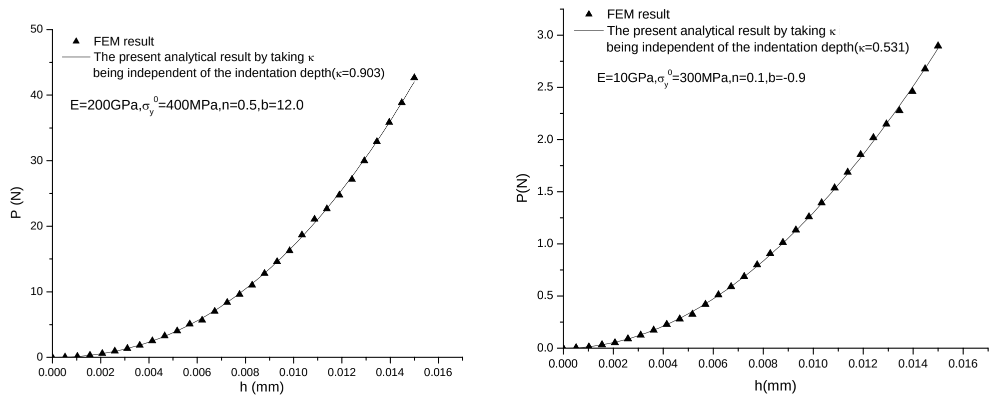

Figure 1: A plot of the present analytical indentation loading curves and the indentation loading curves given by FEM

3 EFFECTS OF GEOMETRICALLY NECESSARY DISLOCATIONS ON INDENTATION LOADING CURVES

In this section, a closed-form expression of the indentation loading curve with a size effect induced by GND has been proposed in this section. The key notion is that the finite element computations using strain gradient plasticity theories have been avoided and the previous analytical results (Dao et al. [1]; Chollacoop et al. [5]) can be directly used to present the closed-form expression of the size-dependent indentation loading curve, i.e.

$$p = \sigma_r h^2 \Pi_1 = \sigma_r h^2 \left(\phi_3(\theta) \ln^3 \left(\frac{E^*}{\sigma_r} \right) + \phi_2(\theta) \ln^2 \left(\frac{E^*}{\sigma_r} \right) + \phi_1(\theta) \ln \left(\frac{E^*}{\sigma_r} \right) + \phi_0(\theta) \right) \quad (8)$$

where the representative stress is given as follows in which the material length scale has been included.

$$\sigma_r = \sigma'_{ref} \left(\varepsilon'_y + \varepsilon'_r \right)^{n'} \approx \sigma_{ref} \sqrt{\left(\varepsilon_y + \varepsilon_r \right)^{2n} + \frac{l}{h \tan^2(\theta)}} \quad (9)$$

where l is the material length scale.

Below is a summary to explain the key idea behind the present method.

- The finite deformation theory of TNT plasticity (Huang et al. [2]) obtained by extending the work of Gao and Huang [20] has been applied in order to analyze the present indentation problem.
- At a given indentation depth, taking as the effective strain gradient that proposed by Nix and Gao [3], the finite deformation theory of TNT plasticity (given in step I) degenerates to the classical finite deformation theory of plasticity.
- At a given indentation depth, FEM based on the classical flow theory of plasticity and the large deformation assumption has been used to evaluate the results of the classical finite deformation theory of plasticity (presented in step II).
- According to steps I, II and III, the computational results proposed by Chollacoop et al. [5] have been used here to present the explicit form of the size-dependent indentation loading curve by replacing the representative stress with that defined in the present work (eqn (9)).

The present computational model is based on the finite deformation plasticity theory, while for the analysis of indentation problems, FEM based on the flow theory of plasticity and the large deformation assumption is most appropriate selection. Although the hardness predicted using the present model is expected to be consistent with that produced by the Nix and Gao's model [3], further indentation experiments need to be carried out to verify the effectiveness of the explicit form of the indentation loading curve given in eqn (8). First, tension tests using a UTS machine were performed on 316 stainless steel, The values of σ_{ref} and ε_y and n in equation (9) can be determined from the tension curve, they are $\sigma_{ref}^{steel} = 813.9MPa$, $\varepsilon_y^{steel} = 0.00147$, $n_{steel} = 0.2$. Secondly, nanoindentation tests were then carried out on an MTS XP nano-indenter to obtain the indentation loading curves. A standard diamond Berkovich indenter with an equivalent tip apex angel of $\theta = 70.3^0$ was used. At last, the empirical constant, α , in the Taylor model was determined for 316 stainless steel (see Figure 2) by fitting the experimental indentation loading curves with eqn (8) using the least-square method and taking a Burgers vector of $b = 0.25\text{nm}$. The average values of parameter is $\alpha_{\text{steel}} = 0.23$, which have correct order of magnitude (between 0.1 and 0.5). Moreover, Figure 2 also shows that the proposed closed-form expression of the indentation loading curves correspond very well to the experimental curves. The results obtained by fitting the experimental indentation curve with a function in the form of eqn (1) and the results predicted using eqn (B1) in the work of Dao et al. [1] are also given in Figure 2. It can be seen that the deviation of the size-dependent indentation loading curve from the assumption made in equation (1) is significant. At the same time, the results predicted using the work of Dao et al. [1] show that taking the size effect into consideration is both necessary and important for the maximum indentation depths discussed here.

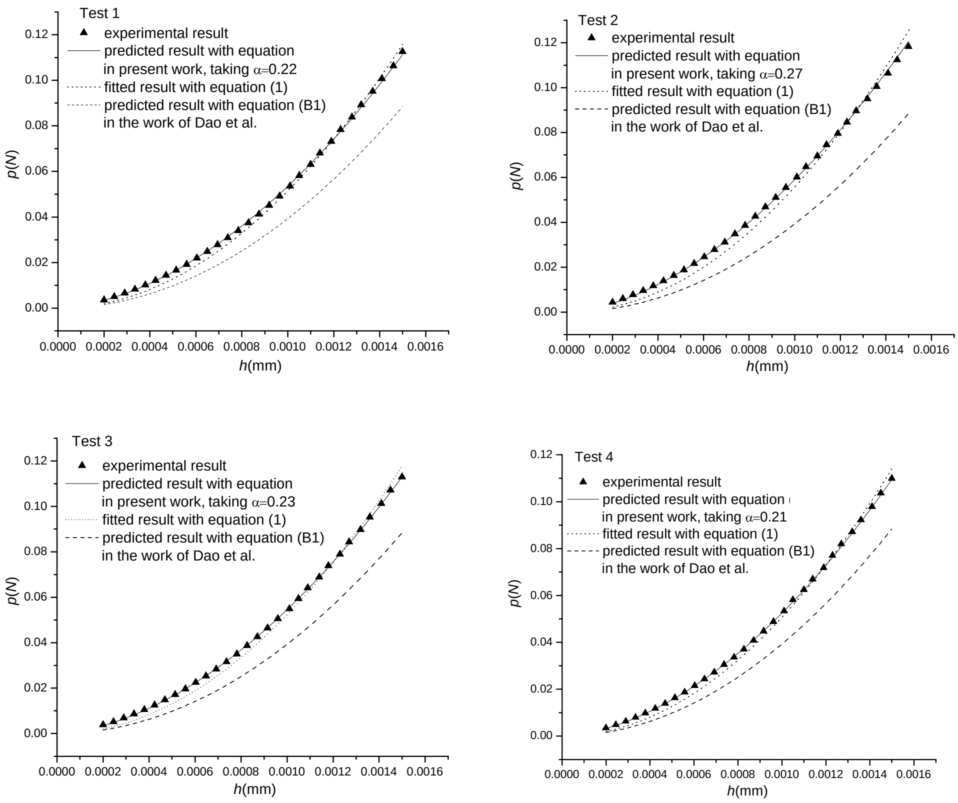

Figure 2: Plot of the experimental $p-h$ response and corresponding fitted results with the present closed-form expression of indentation loading curves (316 stainless steel)

4 CONCLUSIONS

Size-effects are frequently observed in micro or nano-indentation tests. In this report, we propose two computational modeling schemes for characterizing size effects induced by the existence of plastically-graded surface and the effect of geometrically necessary dislocations. The analytical indentation loading curves correspond well with the practical ones and can be further used to establish inverse approaches to extract plastic properties of metal materials from the indentation loading curves.

REFERENCES

1. Dao, M., Chollacoop, N., Van Vliet, K.J., Venkatesh, T.A., Suresh, S., Computational modelling of the forward and reverse problems in instrumented sharp indentation. Acta mater. 49, 3899-3918, 2001.
2. Hwang, K. C., Y. Guo, H. Jiang, Y. Huang, Z. Zhuang., The finite deformation theory of Taylor-based nonlocal plasticity. Int. J. Plasticity 20, 831-839, 2004.

3. Nix, W.D., Gao, H. Indentation size effects in crystalline materials: A law for strain gradient plasticity. J. Mech. Phys. Solids 46, 411-425, 1998.
4. Bucaille, J.L., Stauss, S., Felder, E., Michler, J., Determination of plastic properties of metals by instrumented indentation using different sharp indenters. Acta mater. 51, 1663-1678, 2003.
5. Chollacoop, N., Dao, M., Suresh, S., Depth-sensing instrumented indentation with dual sharp indenters, Acta mater. 51, 3713-3729, 2003.
6. Cao, Y.P., Lu, J., Depth-sensing instrumented indentation with dual sharp indenters: Stability analysis and corresponding regularization schemes, Acta Materialia 52, 1143-1153, 2004.
7. Cheng, Y.T., Cheng, C.M., Scaling approach to conical indentation in elastic-plastic solids with work hardening. J. Appl. Phys. 84, 1284-1291, 1998.
8. Cheng, Y.T., Cheng, C.M., Relationships between hardness, elastic modulus, and the work of indentation. Appl. Phys. Lett. 73, 614-616, 1998.
9. Suresh S., Science 292, 2447, 2001.
10. Suresh S. and Giannakopoulos A.E., Report No: 99-1-IND (Laboratory for Experimental and Computational Micromechanics, Massuchusetts Institute of Technology, Cambrige, MA) 3, 1999.
11. Giannakopoulos A E., International Journal of Solids and Structures 39, 2495, 2002.
12. Cao, Y.P., Lu, J., A new scheme for computational modelling of conical indentation in plastically-graded materials, J. Mater. Res. 19, 1703-1716, 2004.
13. Fleck, N. A., Muller, G. M., Ashby, M. F., Hutchinson, J. W., Strain gradient plasticity: Theory and experiment. Acta. Metallurgica et Materialia 42, 475-487, 1994.
14. Stelmashenko, N. A., Walls, M.G., Brown, L. M., Milman, Y. V., Micro-indentation on W and Mo oriented single crystals: an STM study. Acta. Metallurgica et Materialia 41, 2855-2865, 1993.
15. De Guzman, M. S., Neubauer, G., Flinn, P., Nix, W. D., The role of indentation depth on the measured hardness of materials. Mater. Res. Symposium. Proceedings. 308, 613-618, 1993.
16. Ma, Q., Clarke, D. R., Size dependent hardness in silver single crystals. J. Mater. Res. 10, 853-863, 1995.
17. Bucaille J.L., Rossoll A., Moser B., Stauss S., J. Michler., Determination of the matrix in situ flow stress of a continuous fibre reinforced metal matrix composite using instrumented indentation, Materials Science and Engineering A 369, 82–92, 2004.
18. Cao Y. P., Lu J., Size-dependent sharp indentation-I: A closed-form expression of the indentation loading curve, J. Mech. Phys Solids, accepted for publication.
19. Cao Y. P., Lu J., Size-dependent sharp indentation-II: A reverse algorithm to identify plastic properties of metallic materials, J. Mech. Phys Solids, accepted for publication.
20. Gao, H., Huang, Y., Taylor-based nonlocal theory of plasticity. Int. J. Solids Struct. 38, 2615-2637, 2001.