AN ELASTOPLASTIC SHEAR-FRACTURE MODEL FOR SOIL AND SOFT ROCK

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ABSTRACT

An elastoplastic shear-fracture approach to the constitutive behaviour of soil and soft rock is proposed. The model is an extension of critical state soil mechanics in that it adopts the Hvorslev-modified Cam clay (MCC) idealization as its starting point. However, once shear band localization failure occurs, ductile elastoplastic fracture may follow. When the material is sufficiently brittle, as in the case of hard, partially saturated soil or soft rock, tensile elastic fracture occurs after peak stress. A transitional state can also occur whereby tensile elastic fracture is followed by elastoplastic shear fracture. In this way, the model may, in principle, address the entire range of soil behaviour, from the ‘wet’ side of critical state to that of a hard consistency, including soft rock. In the present discourse, the key features of the proposed model will first be outlined. The example of brittle elastic fracture will then be dealt with, in order to demonstrate the application of the proposed model.

1 INTRODUCTION

In the phenomenological (that is mathematical, rather than physical or particulate) constitutive modeling of soil and soft rock, to solve boundary value problems, scant attention has been paid to the development of discontinuities in the presence of applied loading. Various studies have been conducted on detailed aspects of such discontinuities, notably on shear band localization failure [1,2], but these have been of limited practical application in an actual situation. And yet, the presence of such discontinuities, which are clearly observable in soils and soft rock, is evidently a major feature to be taken into account in any reasonably consistent analytical model of the material behaviour. Moreover, specimens of such soils are commonly subjected to triaxial compression testing, from which smeared properties are obtained that reflect the particular failure conditions of the test specimen. These conditions, more often than not, consist of three-dimensional localized shear banding, and therefore the corresponding properties obtained would not, in principle, be applicable to other boundary conditions.

As shown in Figure 1, after the onset of shear band localization failure or upon attaining the peak stress, whichever occurs first, the Hvorslev-MCC model begins to exhibit an increasing departure from actual behaviour, particularly under undrained conditions. This is due to the local dissipation of pore pressures, which is not reflected by the continuum model of Hvorslev-MCC. Concurrently, a shear band localization failure zone develops and spreads throughout the post peak strain softening phase until the ultimate condition is reached. In the latter condition, only the frictional resistance between two discrete bodies of the material medium sliding over each other, along their common discontinuity, remains. In the light of the preceding considerations, the present proposed model takes up from where the Hvorslev-MCC model leaves off, which is at the onset of localized shear banding. It addresses the behaviour of relatively more brittle soils than the foregoing elastoplastic shear materials, as well as soft rock, from the standpoint of elastoplastic, and thereafter (when the soil or rock is sufficiently brittle) elastic fracture.

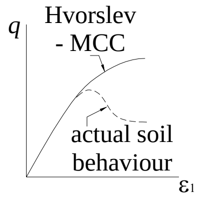

Figure 1

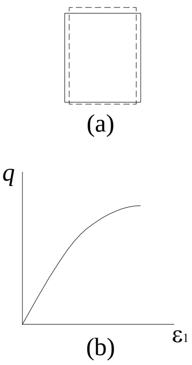

Figure 2

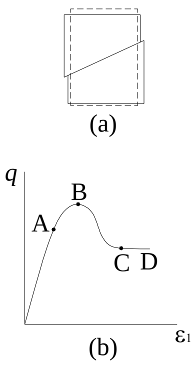

Figure 3

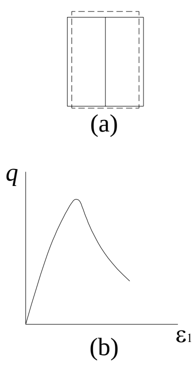

Figure 4

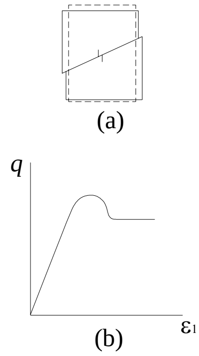

Figure 5

Taking its cue from the Hvorslev-MCC model, the proposed model is construed to be both phenomenological and reasonably tractable, so as to be amenable to both practical and general research usage. In addition, the veracity of the model has been partly established in conjunction with plane strain testing, which does not have the disadvantages of a three-dimensional failure pattern, and is, moreover, representative of conditions which are commonly encountered in the field. By its nature, the proposed elastoplastic shear-fracture model would, in principle, be applicable to the entire range of soils from normally consolidated (nc) to lightly overconsolidated (oc) soil (that is 'wet' soil) like marine clay, to hard soil like partially saturated residual soils, as well as soft rock.

2 ANALYSIS OF ELASTOPLASTIC SHEAR-FRACTURE

According to the proposed model, there are four distinct states of soil failure and their related stress-strain behaviour, as shown in Figures 2 to 5.

2.1 State I: Adapted Hvorslev-MCC (modified Cam clay) Model

The starting point of the proposed model is the Hvorslev-MCC model of critical state theory, for 'wet' as well as heavily oc soil, up to the onset of shear band localization failure or peak stress, whichever takes place first. Thus, as shown in Figure 2, State I consists of two phases. Phase 1 follows the behaviour of MCC, which is applicable to normally to lightly oc soils. According to critical state theory, a plane strain test specimen would bulge homogeneously as a result of elastoplastic shear failure. Phase 2 behaviour simulates state A to B of following Figure 3.

Although there is a variety of refinements to the Hvorslev-MCC model, for example those of Whittle [3], Stallebrass and Taylor [4] and Mroz and Norris [5], for practical and general research purposes, the model still provides the best of both worlds from the viewpoint of a relatively simple model, which is at the same time consistent, hence its common usage in both areas of application. The details of the model is well-documented by Potts and Zdravkovic [6], amongst others, and need not, therefore, be further elucidated herein.

2.2 State II: Elastoplastic Shear Fracture Model

State II is depicted in Figure 3. As shown in Figure 3(a), an elastoplastic shear, or J_{IIC} fracture of the test specimen follows the Hvorslev-MCC model behaviour, either pre- or post-peak stress, that is at the onset of shear band localization failure. This behaviour is attributable to soils on the 'dry' side of critical state, such as heavily oc clays, where failure is due to the development of an elastoplastic shear fracture running diagonally across the entire test specimen. The growth of the plastic zone around the crack tip may be correlated to the typical trend of the stress-strain curve of a plane strain test shown in Figure 3(b), as follows:

The trace of deviator stress versus axial strain follows the elastic curve of critical state theory up to point ‘A’, which is at the onset of shear band localization failure. The curve moves on to the peak at point ‘B’, during which the plastic zone surrounding the crack tip of the initial flaw or defect grows in the specimen, such that the corresponding increase in toughness of the specimen would have a greater influence than the reduction in its ligament size, as the crack extends. Hence, the net effect would be an increase in resistance of the material (although at a decreasing rate), in going from point ‘A’ to ‘B’. After point ‘B’, however, the plastic zone starts to grow more slowly, so that the specimen strain softens. This trend continues until point ‘C’ is reached, at which stage the crack would have cut through the specimen. The upper portion of soil then slides down the lower portion along the discontinuity so formed, and only frictional resistance would be offered at their interface as the test curve follows the ultimate path CD.

2.3 State III: Tensile Elastic Fracture Model

In the third state of failure, which is shown in Figure 4, the test specimen is sufficiently brittle to exhibit elastic-only fracture. This is the case of hard, partially saturated soils and soft rock. Accordingly, a vertical crack would develop from an initial flaw or defect of the test specimen under biaxial compression in plane strain. In such an instance, linear elastic fracture mechanics (LEFM) would be applicable.

2.4 State II/III: Elastic-Elastoplastic Shear Fracture Model

Finally, in transitional State II/III of Figure 5, which lies between States II and III depicted in Figures 3 and 4 respectively, a vertical crack firstly develops from an initial flaw or defect in the specimen, according to LEFM, similarly as for State III. However, this tendency only progresses until such a stage is reached when a diagonal elastoplastic shear fracture develops preferentially. The remaining trend would then be that of elastoplastic shear fracture, whose development may be dealt with based on the J-integral method, as in the case of State II.

An analysis of the State III brittle elastic fracture of Figure 4 will now be undertaken, which has been found to be applicable to the case of a hard, carbonaceous sand test specimen [7]. A similar consideration of the other states of material behaviour is beyond the present scope of study, and will be relegated elsewhere.

3 EXAMPLE OF BRITTLE SOIL FRACTURE

Partially saturated soil is a three-phase medium comprising air, water and solid. As such, the degree of saturation *S* of the soil, and hence its matric suction (u_a - u_w) can vary as the soil is loaded. Thus, it would be necessary to keep track of these changes at all stages of loading. For instance, when brittle fracture takes place, the fracture toughness would depend on their ambient values. Consequently, unlike the usual single phase material behaviour of fracture mechanics, during crack development in soils, the applied loading would not only raise the level of total stresses required to cause further crack extension, but also influence the properties of the medium which determine whether a crack would extend. In the following discourse, a model will be proposed for the elastic tensile fracture of brittle soil, which is based on the above considerations. The model has been confirmed experimentally elsewhere [7].

3.1 Analysis of Tensile Elastic Fracture

In the analysis of the tensile elastic fracture of brittle, partially saturated soils, the pore pressure parameters B_a and B_w would be required, in order to determine the pore pressures increments Δu_a and Δu_w, and hence too the matric suction (u_a - u_w). Following the procedure proposed by Fredlund and Rahardjo [8], although adapted to plane strain conditions, the pore pressure parameters may be deduced from the volumetric deformation coefficients C_t, C_m, D_t and D_m, which may, in turn, be obtained by laboratory testing. In addition, the mode I fracture toughness would need to be obtained experimentally [7].

3.1.1 Pore Pressure Parameters B_a and B_w

In determining the pore pressure parameters, the value of the degree of saturation S, at any given net normal stress and matric suction, would first have to be determined from the constitutive surfaces of void ratio e and water content w, which are in turn obtained experimentally, as above. The pore pressure parameters may then be determined as follows:

The compression indices C_t and C_m, from which the parameters B_a and B_w are derived, would have to be determined under plane strain loading conditions. The pore pressure parameters D_a and D_w would likewise have to be determined in plane strain, in principle, although with an applied σ_1 and σ_3 = 0. However, it may be shown that the parameters D_a and D_w are dependent on the parameters B_a and B_w. Therefore B_a and B_w would, together, be sufficient to obtain the changes in pore pressures under generalised plane strain loading. Accordingly,

$$B_a = \frac{R_2 R_3 - R_4}{1 - R_1 R_3} \tag{1}$$

and

$$B_w = \frac{R_2 - R_1 R_4}{1 - R_1 R_3} \quad , \tag{2}$$

in which

$$R_1 = \frac{R_s - 1 - \left[(1 - S + hS)n / \left(\bar{u}_a m_{1p}^s\right)\right]}{R_s + \left(SnC_w / m_{1p}^s\right)} \quad , \tag{3}$$

$$R_2 = \frac{1}{R_s + \left(SnC_w / m_{1p}^s\right)} \, , \tag{4}$$

$$R_3 = \frac{R_a}{R_a - 1 - \left[(1 - S + hS)n / \left(\bar{u}_a m_{1p}^a\right)\right]} \tag{5}$$

and

$$R_4 = \frac{1}{R_a - 1 - \left[(1 - S + hS)n / \left(\bar{u}_a m_{1p}^a\right)\right]} \, , \tag{6}$$

where

$$R_s = \frac{m_2^s}{m_{1p}^s} \tag{7}$$

and

$$R_a = \frac{m_2^a}{m_{1p}^a} \quad , \tag{8}$$

C_w is the water compressibility, h the proportion of dissolved air in the water, $\bar{u}_a$ the absolute air pressure, n the porosity and $m_{1p}^s, m_2^s, m_{1p}^w$ *and* m_2^w the volumetric deformation coefficients. The latter coefficients may be evaluated from the volumetric deformation coefficients C_t, C_m, D_t and D_m, which may be determined experimentally, as follows:

$$m_{1p}^s = \frac{0.435C_t}{(1+e_0)(\sigma_{ave} - u_a)_{mean}} , \tag{9}$$

$$m_2^s = \frac{0.435C_m}{(1+e_0)(u_a - u_w)_{mean}} , \tag{10}$$

$$m_{1p}^w = \frac{0.435D_t G_S}{(1+e_0)(\sigma_{ave} - u_a)_{mean}} \tag{11}$$

and

$$m_2^w = \frac{0.435D_m G_S}{(1+e_0)(u_a - u_w)_{mean}} , \tag{12}$$

where $(\sigma_{ave}$ - $u_a)_{mean}$ and $(u_a$ - $u_w)_{mean}$ are the averages of the initial and final net normal stresses and matric suctions, in an increment of loading.

3.1.2 Matric Suction

The matric suction, $(u_a$ - $u_w)$, which is defined as the difference between the pore air pressure u_a and pore water pressure u_w,, that are applied individually at the beginning of each loading stage, is required in order to determine the fracture toughness of the soil specimen. The pore pressures may, in turn, be deduced from their respective pore pressure parameters B_a, and B_w, based on the following relationships:

$$du_a = B_a d\sigma_{ave} \tag{13}$$

and

$$du_w = B_w d\sigma_{ave} , \tag{14}$$

where the pore pressure parameters are determined according to the preceding text, and

$$\sigma_{ave} = \frac{\sigma_1 + \sigma_3}{2} . \tag{15}$$

Thus, based on the foregoing discussion, the following considerations would apply:

In the initial state of the soil specimen, the degree of saturation S_0, and hence porosity n_0, may be determined from the constitutive surfaces. The pore air pressure u_{a0} and pore water pressure u_{w0} would be the pre-settings of the test and the loading would be set to zero net normal stress, that is $\sigma_1 = \sigma_3 = u_{a0}$. Subsequently, under the incremental loading of $\Delta\sigma_1$ and $\Delta\sigma_3$, corresponding values of u_a and u_w may be determined.

3.1.3 Fracture Toughness K_C

The fracture toughness, K_c, would have to be obtained too. Moreover, at any given stage of crack development, it would be necessary to obtain an update on the value of the fracture toughness, which is generally dependent on the matric suction. This dependency may be established fundamentally on the basis of Griffith's analogy of the critical rate of energy release G_c and the surface tension γ for glass. Accordingly, a relationship exists between G_c, the matric suction $(u_a$ - $u_w)$ and a characteristic pore size, D_p, given by

$$G_C = k\frac{(u_a - u_w)D_p}{4} , \tag{16}$$

where k is a parameter which reflects the mode of fracture. The pore size is, in turn, related to the pore distribution index, which depends on the effective degree of saturation S_e and matric suction. Accordingly, it may be shown that K_c depends directly on (u_a - u_w). Hence, a relationship exists between the mode I fracture toughness, K_{IC}, and the matric suction.

In view of the preceding considerations, in specifying the basic properties of the proposed tensile elastic model, the fracture toughness K_{IC} would have to be determined by laboratory testing at various values of the matric suction (u_a - u_w) of specimens of the soil being considered.

4 CONCLUSIONS

As a result of the foregoing findings, the following conclusions may be drawn:

Present-day geotechnical models in common usage tend to view the stress-strain behaviour of soils in terms of the uniform point-to-point response of the material medium. This is reflected by the use of continuum models of elastoplasticity coupled with the measurement of material parameters of soil specimens, when loaded, effectively as smeared values. In an alternative approach [8], an empirical fit has been applied to the experimental data, although the problems associated with uniform behaviour and smeared values still persist.

However, it is a well-observed phenomenon that discontinuities, and hence the departure from uniform behaviour, do develop in most soils (that is, apart from highly plastic soils which exist on the “wet” side of critical state) when subject to loading, and may be expected to influence their stress-strain behaviour significantly. The mode of failure of brittle soils such as hard unsaturated clays and soft rocks is an important case in point. Accordingly, a model has been presented to deal with such materials, which is based on LEFM, and the fracture toughness is related to the matric suction of the air-water-solid medium. As such, there is a departure from conventional fracture mechanics in that the fracture toughness is state- and hence load-dependent. The proposed model has been applied to a laboratory test specimen which was subjected to biaxial compression, in which reasonable agreement was obtained between the analytical and experimental predictions [7].

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