CREEP RUPTURE OF FIBER BUNDLES

Ferenc Kun[1], Raul Cruz Hidalgo[2], and Hans J. Herrmann[3]
[1]Department of Theoretical Physics, University of Debrecen, H-4010 Debrecen, Hungary
[2]Department of Theoretical Physics, University of Guelph, Guelph, Ontario, Canada N1G 2W1
[3]Institute for Computational Physics (ICP), University of Stuttgart, Pfaffenwaldring 27, D-Stuttgart

ABSTRACT

We present fiber bundle models of creep rupture of fiber composites considering two different microscopic mechanisms that can lead to time dependent macroscopic behavior: *(i)* the fibers themselves are visco-elastic showing time dependent deformation under a constant load and break when their deformation exceeds a stochastically distributed threshold value. *(ii)* The fibers are linearly elastic until they break in a stochastic manner, however, the load on them does not drop down to zero instantaneously after breaking, due to the creeping matrix, they undergo a slow relaxation process. Assuming global load sharing following fiber failure, we show by analytic calculations and computer simulations in both models that increasing the external load a transition takes place in the system from a partially failed state of infinite lifetime to a state where global failure occurs at a finite time. It was found that irrespective of the details of the two models, a universal behavior emerges in the vicinity of the critical point: the relaxation time and the lifetime of the composite exhibit a power law divergence with an exponent independent of the disorder distribution of fiber strength. Above the critical point the lifetime of the bundle has a universal scaling with the system size. On the micro level the process of fiber breaking is characterized by a power law distribution of waiting times between consecutive fiber breaks below and above the critical load.

1 INTRODUCTION

Under high steady stresses, materials may undergo time dependent deformation resulting in failure called creep rupture which limits their lifetime, and hence, has a high impact on their applicability in construction elements. Creep failure tests are usually performed under uniaxial tensile loading when the specimen is subjected to a constant load σ_o and the time evolution of the damage process is followed by recording the strain ε of the specimen and the acoustic signals emitted by microscopic failure events.

Theoretical studies of creep rupture encounter various challenges: on the one hand, applications of fiber composites require the development of analytical and numerical models which are able to predict the damage histories of loaded composites in terms of the characteristic parameters of the constituents. On the other hand, it is important to reveal universal aspects of creep rupture phenomena, which are independent of specific material properties relevant on the microlevel. In this paper we study the creep rupture of fiber composites by means of fiber bundle models (Daniels [1]) considering two different microscopic mechanisms that can lead to macroscopic creep behavior. Assuming global load sharing among fibers, analytical and numerical calculations show that in both models there exists a critical load that determines the final state of the material. Our detailed study revealed that irrespective of the details of the models a universal behavior of the fiber bundle emerges in the vicinity of the critical point.

2 FIBER BUNDLE MODELS OF CREEP RUPTURE

2.1 Viscoelastic fibers

Our model consists of *N* parallel fibers having viscoelastic constitutive behavior. For simplicity, the pure viscoelastic behavior of fibers is modeled by a Kelvin-Voigt element which consists of a spring and a dashpot in parallel and results in the constitutive equation $\sigma_0 = \beta\dot{\varepsilon} + E\varepsilon$, where σ_0 is the external load, β denotes the damping coefficient, and E the Young modulus of fibers, respectively. In order to capture failure in the model a strain controlled breaking criterion is imposed, *i.e.* a fiber fails during the time evolution of the system when its strain exceeds a breaking threshold ε_i, $i=1,\ldots N$ drawn from a probability distribution $P(\varepsilon)=\int_0^{\varepsilon} p(x)dx$. For the stress transfer between fibers following fiber failure we assume that the excess load is equally shared by all the remaining intact fibers (global load sharing), which provides a satisfactory description of load redistribution in continuous fiber reinforced composites.

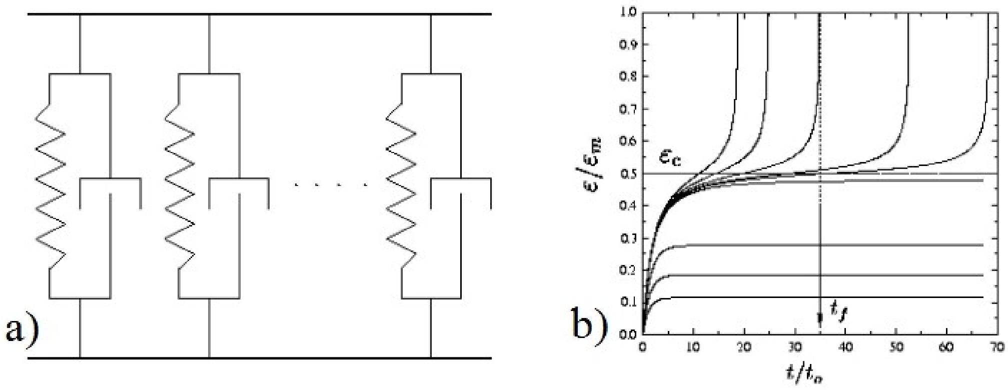

Figure 1: a) The viscoelastic fiber bundle model. Intact fibers are modeled by Kelvin-Voigt elements.b) ε(t) for several different values of the external load σ_o below and above σ_c.

For the breaking thresholds of fibers a uniform distribution between 0 and 1, and a Weibull distribution of the form $P(\varepsilon)=1-\exp\left[-(\varepsilon/\lambda)^{\rho}\right]$ were considered. The construction of the model is illustrated in Fig. 1a). In the framework of global load sharing many of the quantities describing the behavior of the fiber bundle can be obtained analytically. In this case the time evolution of the system under a steady external load σ_o is described by the differential equation

$$\frac{\sigma_o}{1-P(\varepsilon)}=\beta\dot{\varepsilon}+E\varepsilon \quad , \tag{1}$$

where the viscoelastic behavior is coupled to the failure of fibers. The viscoelastic fiber bundle model with the equation of motion eqn. (1) can provide an adequate description of natural fiber composites like wood subjected to a constant load (Gerhards [2]). For the behavior of the solutions ε(t) of eqn. (1) two distinct regimes can be distinguished depending on the value of the external load σ_o: When σ_o falls below a critical value σ_c eqn. (1) has a stationary solution ε_s, which can be obtained by setting $\dot{\varepsilon}=0$, *i.e.* $\sigma_o = E\varepsilon_s[1-P(\varepsilon_s)]$. It means that until this equation can be solved for ε_s at a given external load σ_o, the solution ε(t) of eqn. (1) converges to ε_s when $t \to \infty$, and the system suffers only a partial failure. However, when σ_o exceeds the critical value σ_c no stationary solution exists, furthermore, $\dot{\varepsilon}$ remains always positive, which implies that for $\sigma_o > \sigma_c$ the strain of the system ε(t) monotonically increases until the system fails globally at a finite time t_f (Hidalgo [3], Hidalgo [4], Hidalgo [5]). The behavior of ε(t) is illustrated in Fig. 1b) for several values of σ_o below and above σ_c with uniformly distributed breaking thresholds. It follows from the above argument that the critical value of the load σ_c is the static fracture strength of the bundle. The creep rupture of the viscoelastic bundle can be interpreted so that for $\sigma_o \leq \sigma_c$ the bundle is partially damaged implying an infinite lifetime $t_f = \infty$ and the emergence of a stationary macroscopic state, while above the critical load $\sigma_o > \sigma_c$ global failure occurs at a finite time t_f, but in the vicinity of σ_c the global failure is preceded by a long lived stationary state. The nature of the transition occurring at σ_c can be characterized by analyzing how the creeping system behaves when approaching the critical load both from below and above.

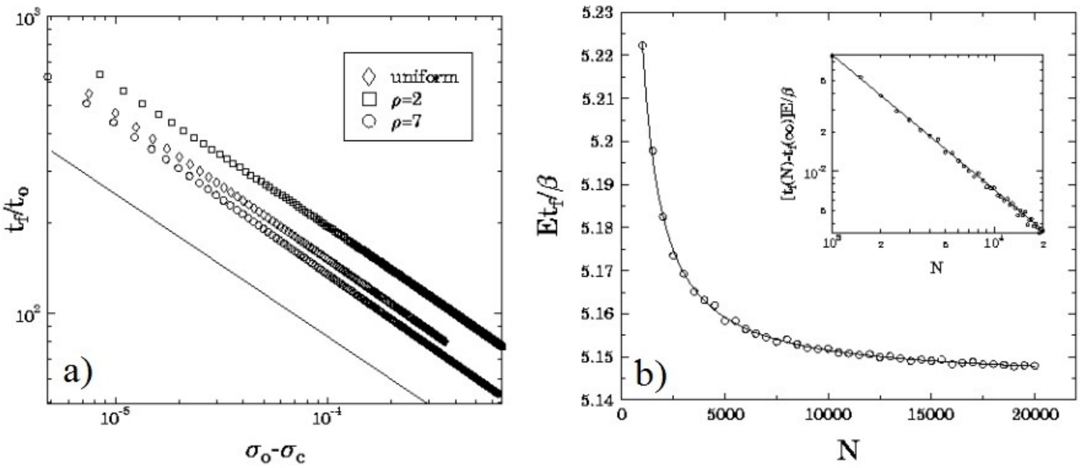

Figure 2: a) Lifetime t_f of the bundle as a function of the distance from the critical point σ_0-σ_c for $\sigma_0 > \sigma_c$. b) t_f as a function of the number of fibers at a fixed value of the external load σ_0. Results of computer simulations (symbols) are in a good agreement with the analytic predictions (solid lines).

For $\sigma_o \leq \sigma_c$ the fiber bundle relaxes to the stationary deformation ε_s through a gradually decreasing breaking activity. It can be shown analytically that $\varepsilon(t)$ has an exponential relaxation to ε_s with a characteristic time scale τ that depends on the external load σ_0 as $\tau \propto (\sigma_c - \sigma_o)^{-1/2}$ for $\sigma_0 < \sigma_c$, *i.e.*, when approaching the critical point from below the characteristic time of the relaxation to the stationary state diverges according to a universal power law with an exponent -1/2 independent on the form of disorder distribution *P*. Above the critical point the lifetime t_f defines the characteristic time scale of the system which can be cast in the form $t_f \propto (\sigma_o - \sigma_c)^{-1/2}$ for $\sigma_0 > \sigma_c$ so that t_f also has a power law divergence at σ_c with a universal exponent -1/2 like τ below the critical point, see Fig. 2a). Hence, for global load sharing the system exhibits scaling behavior on both sides of the critical point indicating a continuous transition at the critical load σ_c. It can also be shown analytically that fixing the external load above the critical point, the lifetime t_f of the system exhibits a universal scaling $t_f(N) - t_f(\infty) \propto 1/N$ with respect to the number *N* of fibers of the bundle (Fig. 2b) (Hidalgo [3], Hidalgo [4], Hidalgo [5]).

2. Slowly relaxing fibers

Another important microscopic mechanism which can lead to macroscopic creep is the slow relaxation following fiber failure. In this case, the components of the solid are linearly elastic until they break, however, after breaking they undergo a slow relaxation process, which can be caused, for instance, by the sliding of broken fibers with respect to the matrix material or by the creeeping matrix, which is a typical mechanism for metal matrix composites reinforced by long brittle fibers. To take into account this effect, our approach is based on the model introduced in (Ibnabdeljalil [6], Du [7], Fabeny [8]), where the response of a viscoelastic-plastic matrix reinforced with elastic and also viscoelastic fibers have been studied. The model consists of *N* parallel fibers, which break in a stress controlled way, *i.e.* subjecting a bundle to a constant external load fibers break during the time evolution of the system when the local load on them exceeds a stochastically distributed breaking threshold σ_i, $i=1,\ldots,N$. Intact fibers are assumed to be linearly elastic *i.e.* $\sigma=E_f e_f$ holds until they break, and hence, for the deformation rate it applies $\dot{\varepsilon}_f = \dot{\sigma} / E_f$ Here ε_f denotes the strain and E_f is the Young modulus of intact fibers, respectively.

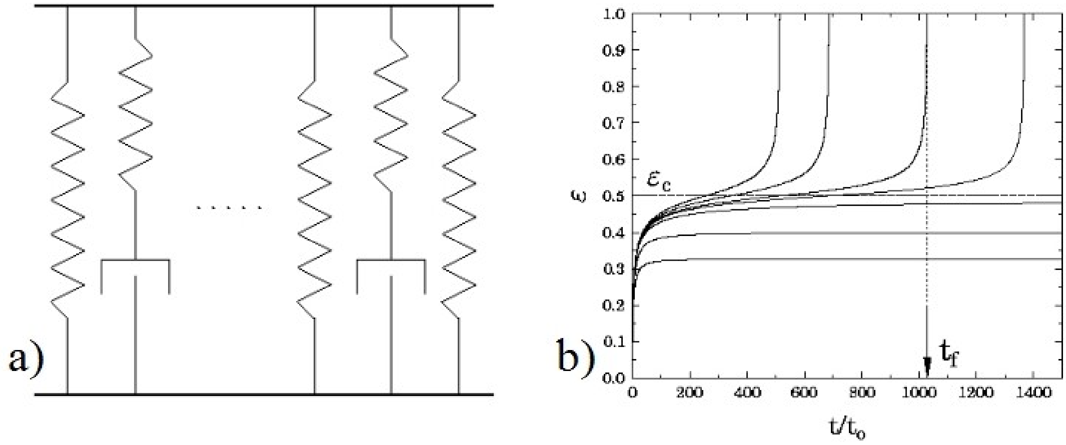

Figure 3: a) The bundle of slowly relaxing fibers. Broken fibers are modeled by Maxwell elements. b) $\varepsilon(t)$ as a function of time for several different values of σ_0 below and above the critical load σ_c.

The main assumption of the model is that when a fiber breaks its load does not drop to zero instantaneously, instead it undergoes a slow relaxation process introducing a time scale into the system. In order to capture this effect, the broken fibers with the surrounding matrix material are modeled by Maxwell elements as illustrated in Fig. 3a), *i.e.* they are conceived as a serial coupling of a spring and a dashpot which results in a non-linear response $\dot{\varepsilon}_b = \dot{\sigma}_b / E_b + B\sigma_b{}^m$, where σ_b and ε_b denote the time dependent load and deformation of a broken fiber, respectively. The relaxation of the broken fiber is characterized by three parameters E_b, B, and m, where E_b is the effective stiffness of a broken fiber, and the exponent m characterizes the strength of non-linearity of the element. We study the behavior of the system for the region $m \geq 1$. Assuming global load sharing for the load redistribution, the differential equation governing the time evolution of the load σ on the intact fibers during the fradual breaking process can be cast in the form

$$\sigma\left[\frac{1}{E_f}-\frac{1}{E_b}\left(1-\frac{1}{P(\sigma)}+\frac{p(\sigma)}{P(\sigma)^2}\left(\sigma-\sigma_0\right)\right)\right]=B\left[\frac{\sigma_0-\sigma[1-P(\sigma)]}{P(\sigma)}\right]^m \quad . \qquad (2)$$

Subjecting the undamaged specimen to an external stress σ_0 all the fibers attain this stress value immediately due to the linear elastic response. Hence the time evolution of the system can be obtained by integrating eqn. (2) with the initial condition $\sigma(t=0) = \sigma_0$. Since intact fibers are linearly elastic, the deformation-time history $\varepsilon(t)$ of the model can be deduced as $\varepsilon(t) = \sigma(t)/E_f$, which has an initial jump to $\varepsilon o = \sigma_0/E_f$. It follows that those fibers which have breaking thresholds σ_i smaller than the externally imposed σ_0 immediately break. Similarly to the previous model, two different regimes of $\sigma(t)$ can be distinguished depending on the value of σ_0: if the external load is smaller than a critical value σ_c a stationary solution σ_s of the governing equation exists. If the external load falls above the critical value the deformation rate $\dot{\varepsilon} = \dot{\sigma} / E_f$ remains always positive resulting in a macroscopic rupture in a finite time t_f as it is illustrated in Fig. 3b). It follows that the critical load σ_c of creep rupture coincides with the static fracture strength of the composite (Kun [9]).

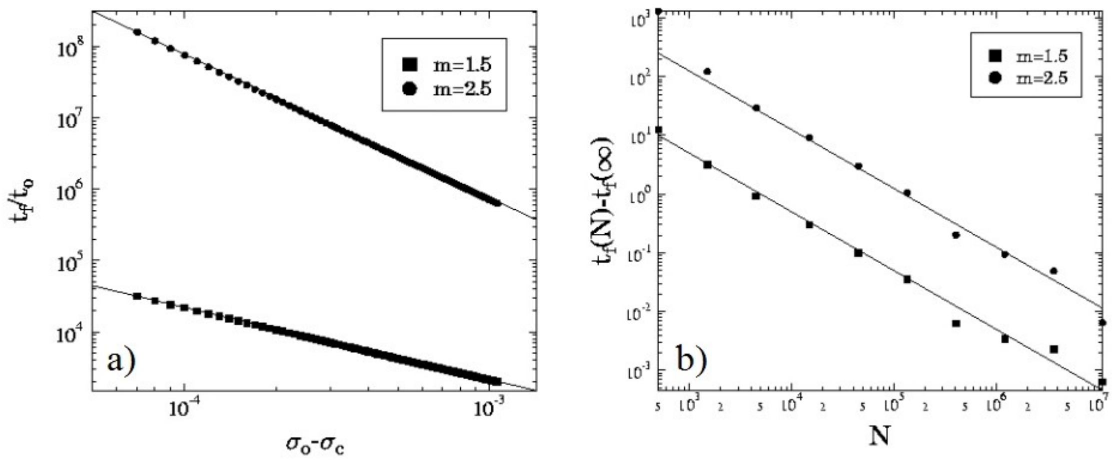

Figure 4: a) Lifetime t_f of the bundle as a function of the distance from the critical point for two values of the stress exponent *m* obtained by computer simulations. b) The scaling behavior of the lifetime t_f at a fixed load value as a function of the number of fibers *N*.

The behavior of the system shows again universal aspects in the vicinity of the critical point. Below the critical point the relaxation of $\sigma(t)$ to the stationary solution σs is governed by a differential equation of the form $d\delta/dt \sim \delta^m$, where δ denotes the difference $\delta(t) = \sigma_s - \sigma(t)$. Hence, the characteristic time scale τ of the relaxation process only emerges if *m=1*, furthermore, in this case also $\tau \propto \left(\sigma_c - \sigma_0\right)^{-1/2}$ holds when approaching the critical point. Similarly to the previous model, it can also be shown that the lifetime t_f of the bundle has a power law divergence when the external load approaches the critical point from above $t_f \propto \left(\sigma_0 - \sigma_c\right)^{-(m-1/2)}$ for $\sigma_0 > \sigma_c$. The exponent is universal in the sense that it is independent on the disorder distribution, however, it depends on the stress exponent *m*, which characterizes the non-linearity of broken fibers, see Fig. 4a). The lifetime t_f of the bundle at a fixed external load above the critical point converges to the lifetime of the infinite bundle $t_f(\infty)$ with increasing number of fibers as *1/N*, which can be seen in Fig. 4b) (Kun [9]).

The process fiber breaking on the micro level can easily be monitored experimentally by means of the acoustic emission techniques. Except for the primary creep regime where a large amount fibers break in a relatively short time, the time of individual fiber failures can be recorded with a high precision. In order to characterize the process of fiber breaking we calculated numerically the distribution *f* of waiting times Δt between consecutive breaks in the two models (Kun [10]). A detailed analyses revealed that $f(\Delta t)$ shows a power law behavior $f(\Delta t) \sim \Delta t^{-b}$ on both sides of the critical point (Kun [10]). The exponents are different below and above the critical load, however, they are independent of the disordered properties of the fibers, see Fig. 5.

3 DISCUSSION

We presented two models of creep rupture of fiber composites with two different microscopic mechanisms that can lead to macroscopic creep behavior. The first model can be relevant for natural fiber composites such as wood, which is composed of viscoelastic fibers, while the second model can provide an adequate description of metal matrix composites reinforced by brittle fibers. Based on the models, we explored universal aspects of the creep response of materials, which are important to clarify the analogy between creep rupture of materials and phase transitions, and can also be relevant for materials design. Our results are in a good qualitative agreement with the experimental findings on the creep rupture of fiber composites (Weber [[11], Gambone [12], Weber [13], Faucon [14]).

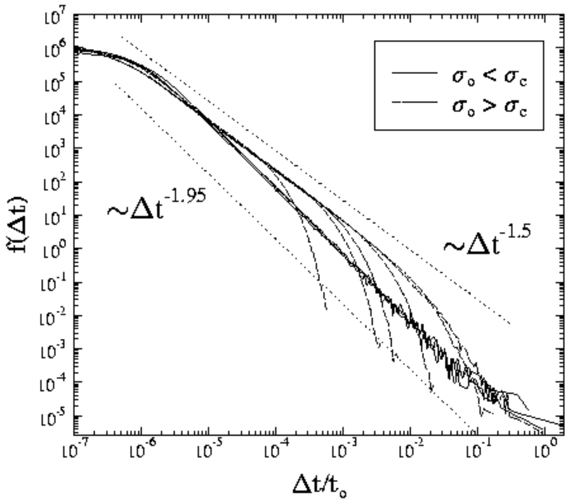

Figure 5: Distribution of waiting times between two consecutive fiber breaks below and above the critical load. Simulation results for 10 million fibers.

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