STUDY ON THE FRAGMENTATION OF SHELLS

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ABSTACT

Fragmentation, in other words the breaking of particulate material into smaller pieces can be observed in nature and in modern life on a wide range of length scales in all kinds of technical applications. Most studies on dynamic failure focus on the behaviour of bulk systems in one, two and tree dimensions under impact and explosive loading, showing universal power law behaviour of fragment size distribution, however, hardly any studies have been devoted to fragmentation of shells. We present a detailed theoretical and experimental study on the fragmentation of closed thin shells made of disordered brittle material, due to an excess load inside the system. Based on large-scale discrete element simulations with spherical shell systems under different extreme loading situations, we prove a power law for the fragment mass distribution and give evidence that it arises due to an underlying phase transition. Satisfactory agreement is found between the numerical predictions of the exponent of the power law for the fragment mass distribution and the experimental findings.

1 INTRODUCTION

Closed shells made of solid materials are used in every day life, in industrial applications in form of containers, pressure vessels or combustion chambers and also in nature *e.g.*, as nature's oldest container for protecting life - the egg-shell. From a structural point of view, aircraft vehicles, launch vehicles like rockets and building blocks of a space station are also shell-like systems. In many of the applications shell-like constructions operate under an internal pressure and they usually fail due to an excess internal load which can arise as a result of slowly driving the system above its stability limit, or by a pressure pulse caused by an explosive shock inside the shell. The explosive fragmentation of a rocket tank resulting in space debris, endangering other space missions for years onward, is such an application with enormous social impact due to the human costs arising, as a consequence of accidents.

Fragmentation, *i.e.* the breaking of particulate materials into smaller pieces is abundant in nature occurring on a broad range of length scales from meteor impacts through geological phenomena and industrial applications down to the break-up of large molecules and heavy nuclei [1-4]. The most striking observation concerning fragmentation is that the distribution of fragment sizes shows power law behaviour independent on the way of imparting energy, relevant microscopic interactions and length scales involved, with an exponent depending solely on the dimensionality of the system [2-11]. Detailed experimental and theoretical studies revealed that universality prevails for large enough input energies when the system falls apart into small enough pieces [2-6], however, at lower energies a systematic dependence of the exponent on the input energy is evident [10]. Recent investigations on the low energy limit of fragmentation suggest that the power law distribution of fragment sizes arises due to an underlying critical point [6-9]. Former studies on fragmentation have been focused on bulk systems in one, two and three dimensions, however, hardly any studies have been devoted to the fragmentation of shells. The peculiarity of the fragmentation of closed shells originates from the fact that their local structure is inherently two-dimensional, however, the dynamics of the systems, deformation and stress states are three dimensional which allows for a rich variety of failure modes.

2 EXPERIMENTS ON SHELL FRAGMENTATION

Ordinary hen eggs provide an excellent and cheap possibility for the study of fragmentation of thin brittle shells of disordered material. In the preparation, first two holes of regular circular shape were drilled in the bottom and top of the egg through which the content of the egg was blown-out. The inside was carefully washed and rinsed out several times and finally the empty shells were dried in a microwave oven to get rid of all moisture of the egg-shell, resulting in a strong loss of toughness of the cuticula.

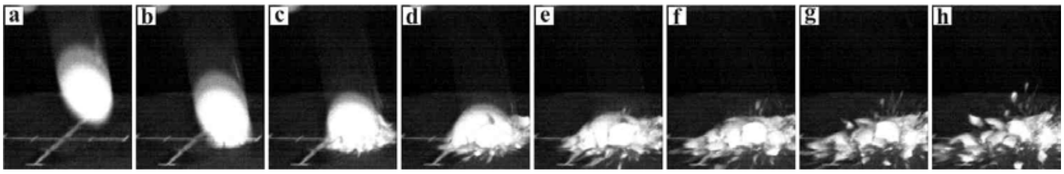

Figure 1: Time series of the gradual collapse of an egg shell due to impact with the hard ground.

In the impact experiments, intact egg shells are catapulted onto the ground at a high speed using a simple catapult made of rubber bands. Eggs are shot directly into a soft plastic bag, contacting the ground, prohibiting secondary fragmentation and assuring that no fragments are lost for further evaluation (Fig. 1b-h). In explosion experiments, initially the egg shell is flooded with hydrogen and hung vertically inside a plastic bag (Fig. 2a). The combustion reaction is initiated by igniting the escaping hydrogen on the eggs top. Due to the escaping hydrogen, oxygen is drawn-up into the egg through the bottom hole. When enough air has entered to form a combustible mixture inside the egg, the flame back-fires through the top hole and the egg explodes (Fig. 2b-c) within 1 msec.

The resulted egg-shell pieces are carefully collected and placed on the tray of a scanner without overlap. In the scanned image fragments occur as, for instance, black spots on a white background (Fig. 2) further analyzed by a cluster searching code. The mass of fragments was determined as the number of pixels of pieces in the scanned image. Shattered fragments of size comparable to normal dust pieces in the air were excluded from the analysis by setting the cut-off size to a few pixels of the scanner.

Several series of impact and explosion experiments with hole diameters between ***1.2*** and ***2.5*** mm were carried out. Fragment mass distributions averaging over 10-20 egg-shells for each curve

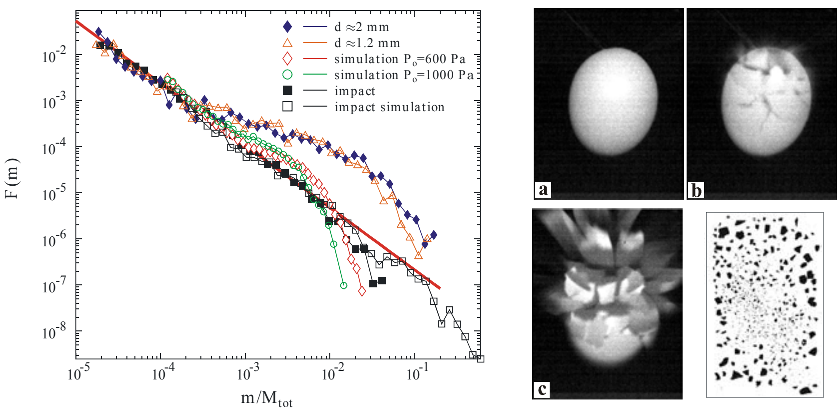

Figure 2: Comparison of fragment mass distributions obtained by explosion experiments with two different sizes of holes and a-c) impact experiments with results of the simulation and time evolution of the explosion of an egg taken by a high speed camera with 1kHz.

are shown in Fig. 2. For the impact experiment, a power law behaviour of the distribution $F(m) \sim m^{-\tau}$ can be observed over three orders of magnitude where the value of the exponent can be determined with a high precision to $\tau = 1.35 \pm 0.02$. Explosion experiments result also in a power law distribution of the same value of τ for small fragments with a relatively broad cut-off for the large ones. Note that the exponent τ is significantly different from the experimental and theoretical results on fragmenting two-dimensional bulk systems where $1.5 \leq \tau \leq 2$ has been found [2-10].

3 SIMULATION OF THE BREAK-UP PROCESS

For simplicity, our theoretical study is restricted to spherical shells, *i.e.* we worked out a three dimensional discrete element model of spherical shells by discretizing the surface of the unit sphere into randomly shaped triangles (Delaunay triangulation) and Voronoi polygons. The nodes of the triangulation represent point-like material elements in the model whose mass is defined by the area of the Voronoi polygon assigned to it. The bonds between nodes are assumed to be springs having linear elastic behaviour up to failure. Disorder is introduced in the model solely by the randomness of the tessellation so that the mass of the nodes, furthermore, the length and cross-section of the springs are determined by the tessellation (quenched structural disorder). After prescribing the initial conditions of a specific fragmentation process studied, the time evolution of the system is followed by solving the equation of motion of nodes. In order to account for crack formation in the model springs are assumed to break during the time evolution of the system when their deformation ε exceeds a fixed breaking threshold ε_c, resulting in a random sequence of breakings due to the disordered spring properties. As a result of successive spring breakings cracks nucleate, grow and merge on the spherical surface giving rise to a complete break-up of the shell into pieces. The process is stopped when the system has attained a relaxed state.

In computer simulations two different ways of loading were realized to model the experimental conditions representing limiting cases of energy input rates: *(i) pressure pulse* and *(ii) impact* load starting from an initially stress free state. A *pressure pulse* in a shell is carried out such that a fixed internal pressure P_0 is imposed giving rise to an expansion of the system with a continuous increase of the imparted energy $E_{tot}=P_0\Delta V$, where ΔV denotes the volume change with respect to

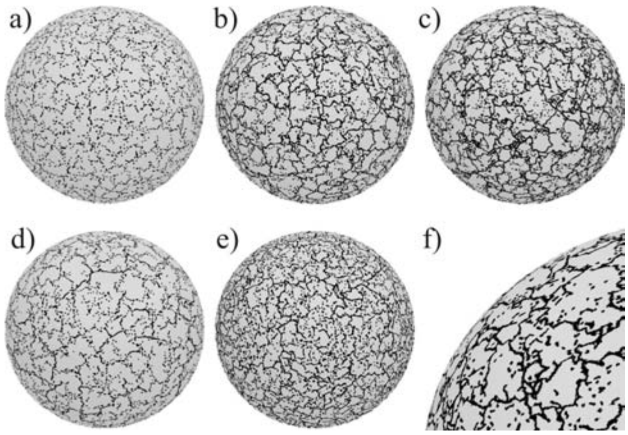

Figure 3: The time evolution of the breakup process under a constant pressure of $P_0/P_c \approx 4.0$ a-c), until the final state c) is reached with a magnified view f). Final states of impact simulations are shown for energies $E_0/E_c \approx 0.8$ and 2.8 d),e). Particle positions are projected on their initial positions. Fragments are identified as shell pieces surrounded by cracks.

the initial volume V_0. Since the force F acting on the shell is proportional to the actual surface area A, the system is driven by an increasing force $F \approx P_0 A$ during the expansion process. The impact loading realizes the limiting case of instantaneous energy input $E_{tot}=E_0$ by giving a fixed initial velocity v_0 to the material elements pointing radially outward. Large scale simulations performed varying the control parameters P_0 and E_0 in a broad range, revealed substantial differences between the two ways of fragmentation. Under both ways of loading a rather uniform stress and deformation state arises so that during the expansion process first overstrained springs break in an uncorrelated manner generating microcracks on the surface. When P_0 exceeds the critical pressure P_c the expanding shell surpasses a critical volume V_c where fragmentation sets on, *i.e.* abruptly a large amount of springs break rapidly forming cracks which grow and join resulting in shell pieces surrounded by a free crack surface (fragment). First large fragments are formed which then break-up into smaller pieces until the surviving springs can sustain the remaining stress, see Fig. 3. The transition from the damaged state where the shell keeps its integrity to the fragmented state where the system disintegrate into pieces occurs abruptly at P_c. Under *impact loading* however, the radial initial velocity implies a prescribed path for the motion of material elements resembling to the strain controlled loading of bulk specimens. Similar to the pressure loading case, simulations revealed that a critical value of the imparted Energy E_c can be identified blow which the shell maintains its integrity suffering only damage, while exceeding E_c gives rise to a complete fragmentation of the shell (see Fig. 3).

4 PREDICTIONS OF FRAGMENT MASSES

Quantitative characterization of the break-up process when increasing the control parameter can be given by monitoring the average mass of the largest fragment normalized by the total mass $<M_{max}/M_{tot}>$. M_{max} is a monotonically decreasing function of both P_0 and E_0, however, the functional forms are different in the two cases, see Fig. 4a-b. At low pressure values in Fig. 4a-b M_{max} is practically equal to the total mass since hardly any fragments are formed. Above P_c however, M_{max} gets significantly smaller than M_{tot}, indicating the disintegration of the shell into

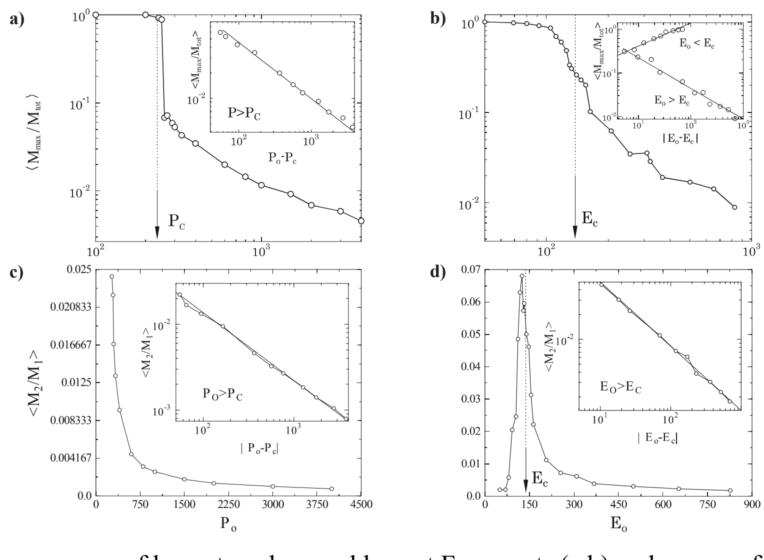

Figure 4: Average mass of largest, and second largest Fragments (a,b) and average fragment mass (c,d) as functions of control parameters, P_0 and E_0

pieces. The value of the critical pressure $\boldsymbol{P_c}$ needed to achieve fragmentation and the functional form of the curve above $\boldsymbol{P_c}$ were determined by plotting $\boldsymbol{<M_{max}/M_{tot}>}$ as a function of the difference $|\boldsymbol{P_0 - P_c}|$ varying $\boldsymbol{P_c}$ until straight line is obtained on a double logarithmic plot. The power law dependence on the distance from the critical point $\boldsymbol{<M_{max}/M_{tot}>} \approx |\boldsymbol{P_0 - P_c}|^{-\alpha}$ for $\boldsymbol{P_0 > P_c}$ is evidenced by the inset of Fig. 4a, where $\boldsymbol{\alpha=0.66\pm0.02}$ was determined. The finite jump of $\boldsymbol{M_{max}}$ at $\boldsymbol{P_c}$ indicates the abrupt nature of the transition between the two regimes. For impact loading $\boldsymbol{<M_{max}/M_{tot}>}$ proved to be a continuous function of $\boldsymbol{E_0}$, however, it shows also the existence of two regimes of the break-up process with a transition at a critical energy $\boldsymbol{E_c}$. In the inset of Fig. 4b $\boldsymbol{<M_{max}/M_{tot}>}$ is shown as a function of the distance from the critical point $|\boldsymbol{E_0 - E_c}|$, where $\boldsymbol{E_c}$ was determined numerically in the same way as $\boldsymbol{P_c}$. Contrary to the pressure loading, $\boldsymbol{<M_{max}/M_{tot}>}$ exhibits a power law behaviour on both sides of the critical point but with different exponents $\boldsymbol{<M_{max}/M_{tot}>} \approx |\boldsymbol{E_0 - E_c}|^{\beta}$ for $\boldsymbol{E_0 < E_c}$ and $\boldsymbol{<M_{max}/M_{tot}>} \approx |\boldsymbol{E_0 - E_c}|^{-\alpha}$ for $\boldsymbol{E_0 > E_c}$ where the exponents were obtained as $\boldsymbol{\beta = 0.5\pm0.02}$ and $\boldsymbol{\alpha=0.66\pm0.02}$. Note that the value of $\boldsymbol{\alpha}$ coincides with the corresponding exponent of the pressure loading.

We also evaluated the average fragment mass $\boldsymbol{<M>}$ as the average value of the ratio of the second $\boldsymbol{M_2}$ and first $\boldsymbol{M_1}$ moments of fragment masses [7]. The behaviour of $\boldsymbol{<M>}$ shows again clearly the existence of two regimes of the break-up process with a transition at the critical point $\boldsymbol{P_c}$ and $\boldsymbol{E_c}$, see Fig. 4c-d. Under pressure loading due to the abrupt disintegration $\boldsymbol{<M>}$ can only be evaluated above the critical point $\boldsymbol{P_c}$, while for the impact case $\boldsymbol{<M>}$ has a maximum at the critical energy $\boldsymbol{E_c}$ typical for continuous phase transitions. In both loading cases $\boldsymbol{<M>}$ has a power law dependence on the distance from the critical point, *i.e.* $\boldsymbol{<M>\sim|E_0-E_c|^{-\gamma}}$, and $\boldsymbol{<M>\sim|P_0-P_c|^{-\gamma}}$ hold with the same value of the exponent $\boldsymbol{\gamma= 0.8\pm0.02}$ (inset of Fig. 4c-d).

The most important characteristic quantity of our system is the mass distribution of fragments $\boldsymbol{F(m)}$. For *impact loading* representative examples are shown at an energy value below, above, and close to $\boldsymbol{E_c}$, Fig. 5a. For *pressure loading* $\boldsymbol{F(m)}$ can only be evaluated above $\boldsymbol{P_c}$, see Fig. 5c.

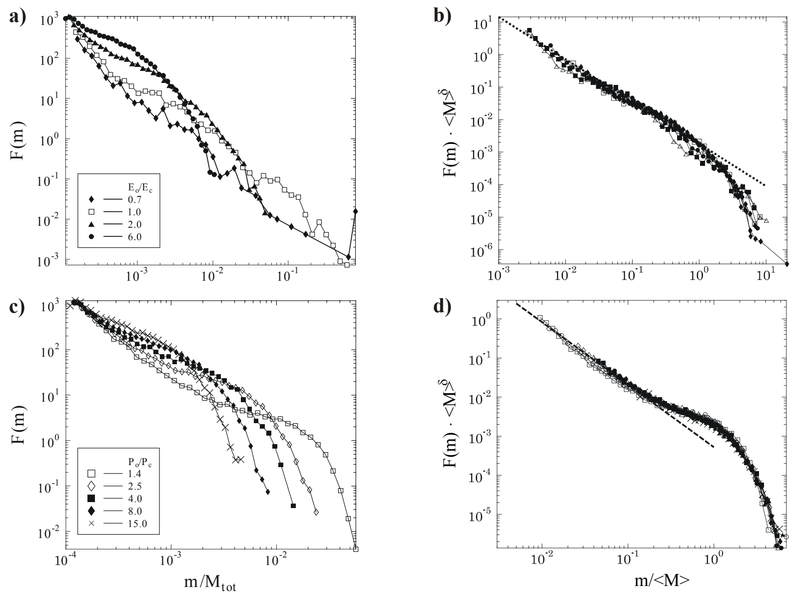

Figure 5: Mass distributions of fragments (a,b) and relative data collaps (c,d) for various values of the control parameter $\boldsymbol{E_0}$ and $\boldsymbol{P_0}$.

At energies $E_0<E_c$ it can be observed that $F(m)$ has a pronounced peak at large fragments indicating the presence of large damaged pieces. Approaching the critical point E_c the peak gradually disappears and the distribution becomes a power law at E_c. Above the critical point the power law remains for small fragments followed by an exponential cut-off for the large ones. Note the agreement of functional forms of $F(m)$ in Figs. 5a,c with the experimental findings on the impact and explosion of eggs (Fig. 2). Figs. 5b,d demonstrate that rescaling the distributions above the critical point by plotting $F(m)<M>^{\delta}$ as a function of $m/<M>$ an excellent data collapse is obtained with $\delta=1.6\pm0.03$. The data collapse implies the validity of the scaling form $F(m)\sim m^{-\tau}\cdot f(m/<M>)$, typical for critical phenomena. The cut-off function f has a simple exponential form $\exp(-m/<M>)$ for impact loading, and a more complex one containing also an exponential component for the pressure case. The rescaled plots make possible an accurate determination of the exponent τ, where $\tau=1.35\pm0.03$ and $\tau=1.55\pm0.03$ were obtained for impact and pressure loading, respectively. Beside the qualitative agreement of the distributions with the experimental results on egg fragmentation, a good quantitative agreement of the exponent τ is evidenced for the impact loading of shells.

5 SUMMARY

We presented a theoretical and experimental study of the fragmentation of closed brittle shells arising due to an excess load inside the shell. We performed experiments on explosion and impact fragmentation of hen egg-shells resulting in power law fragment size distributions. Based on molecular dynamics simulations of a discrete element model of shells we give evidence that power law fragment mass distributions arise due to an underlying phase transition which proved to be abrupt for explosion and continuous for impact. It is shown that the fragmentation of closed shells belongs to a universality class different from that of the two and three dimensional bulk systems.

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