

# A CRITERION FOR INTERFACIAL DEBONDING IN THE VICINITY OF A MATRIX CRACK

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## ABSTRACT

The nucleation of an interfacial crack is analysed in the vicinity of a matrix crack. The selected geometry is an axisymmetric fibre/matrix cell submitted to a tensile loading. For a given value of the matrix ligament, an energetic approach provides a nucleation condition comparing the ratio of the interfacial toughness over the matrix toughness to a critical value depending on the elastic mismatch between the fibre and the matrix. A strength condition is used to determine the decohesion length. Predictions of the decohesion condition are then presented in the case of a stationary matrix crack.

## 1 INTRODUCTION

Deflection of matrix cracks to the fibre/matrix interface is essential for achieving a tough behaviour of ceramic matrix composites (Naslain 1993 [1]). As a matter of fact, debonding allows crack bridging as the fibre is left intact behind the crack tip. Enhanced understanding of crack deflection is necessary to design the coating which is applied to the fibre to promote decohesion. (Kerans et al. 2002 [2]). However, detailed fracture observations are difficult and the sequence of events for crack deflection in brittle matrix composites remains speculative.

Previous studies assume that debonding is delayed until the crack impinges on the interface. The crack can either propagate into the fibre or be deflected along the interface. The requirements to achieve the latter failure mode are obtained with the help of conditions based on strain energy stored in the composite constituents (He and Hutchinson 1989 [3]). However, particular crack extensions must be assumed to assess the competition between deflection and penetration at the interface and the corresponding energy conditions depend on this arbitrary choice (Martin et al. 2001 [4]). Furthermore, tensile debonding ahead of a crack tip was evidenced by several experiments like the recent work of Xu et al. 2003 [5] which clearly demonstrates that an interfacial crack may be nucleated through the remote interaction of an incoming crack with a weak interface.

The aim of this paper is to provide conditions for the initiation of interfacial failure in the vicinity of a matrix crack in brittle matrix composites. The interface is assumed to be free of defect and a finite fracture mechanics approach is used to describe the nucleation process within a representative cell. The debonding length is determined with the help of an additional interfacial strength condition.

## 2 THE GEOMETRY OF THE COMPOSITE CYLINDER

The geometry of the cracked composite cylinder considered is given in figure 1a. It consists of a single fibre (Young's modulus  $E_f$  and Poisson's ratio  $\nu_f$ ) of radius  $R_f$  and infinite length surrounded by a concentric cylinder of matrix (Young's modulus  $E_m$  and Poisson's ratio  $\nu_m$ ) with outer radius

$\frac{R_f}{\sqrt{V_f}}$  where  $V_f$  is the fibre volume fraction. An annular matrix crack is introduced in the plane

$z=0$  and the distance between the crack tip and the fibre/matrix interface is denoted  $l$ . A prescribed displacement is applied on both ends of the cylinder and the external cylindrical surface of the matrix is stress free.

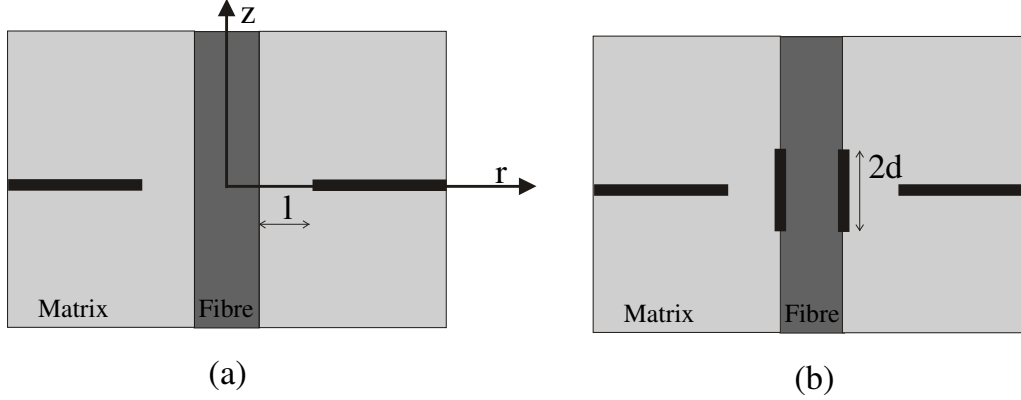


Figure 1: Geometry of the cracked composite cylinder: a) Introduction of an annular crack, b) Nucleation of an interfacial crack in the vicinity of the matrix crack.

A linear elastic and brittle behaviour is assumed for each constituent of the bimaterial. The radial stress  $\sigma_{rr}(l, z)$  along the fibre/matrix interface is given by :

$$\sigma_{rr}(l, z) = k_{rr}(l, z)\sigma \quad (1)$$

where  $\sigma$  is the applied stress. As a result of the stress concentration induced by the matrix crack, an interfacial crack of length  $2d$  may nucleate for a given value of the applied stress as depicted in figure 1b. The energy release rate  $G_i(l, d)$  for the propagation of this interfacial crack in the vicinity of the matrix crack is denoted :

$$G_i(l, d) = A_i(l, d)\sigma^2. \quad (2)$$

### 3 THE DEFLECTION CRITERION

If an interfacial crack of length  $2d^*$  is nucleated, an energy balance states that :

$$\Delta W(l, d^*) \geq 2\pi R_f d^* G_i^c \quad (3)$$

where  $\Delta W(l, d^*)$  is the change in potential energy between the two states schematised in Figure 1 and  $G_i^c$  is the toughness of the fibre/matrix interface. Introducing (assuming  $d \neq 0$ )

$$\bar{A}_i(l, d) = \frac{1}{d} \int_0^d A_i(l, z) dz, \quad (4)$$

allows to obtain :

$$(\sigma^c)^2 \bar{A}_i(l, d^*) \geq G_i^c \quad (5)$$

which reveals an incremental condition in which the infinitesimal energy rates of the classical Griffith's condition are replaced by finite energy increments. Furthermore, the applied stress at

decohesion  $\sigma^c$  and the decohesion length  $d^*$  are unknown and an additional condition must be established. This supplementary relation can be derived from a strength condition which states that the radial stress along the anticipated path of crack nucleation is greater than the interfacial strength  $\sigma_i^c$  (Leguillon 2002 [6]) :

$$\sigma_{rr}(l, d^*) = k_{rr}(l, d^*) \sigma^c \geq \sigma_i^c . \quad (6)$$

Combining (5) and (6) leads to :

$$\frac{\bar{A}_i(l, d^*)}{k_{rr}^2(l, d^*)} = \frac{G_i^c}{(\sigma_i^c)^2} \quad (7)$$

which defines the decohesion length  $d^*$ . The second term in (7) is related to a characteristic size

$$d_i^c = \frac{E_i G_i^c}{(\sigma_i^c)^2} \text{ with } \frac{2}{E_i} = \frac{1 - \nu_f^2}{E_f} + \frac{1 - \nu_m^2}{E_m} .$$

The competition between the matrix crack propagation

and the interfacial decohesion is assessed by considering a stationary matrix crack. The applied stress is monotonically increased and the interfacial nucleation will take place preferentially to the propagation of the matrix crack if the following conditions are satisfied :

$$(\sigma^c)^2 \bar{A}_i(l, d^*) = G_i^c \text{ and } G_m(l) = A_m(l) (\sigma^c)^2 < G_m^c \quad (8)$$

where  $G_m(l)$  is the energy release rate for the propagation of the matrix crack and  $G_m^c$  is the matrix toughness. Condition (8) leads to :

$$\frac{G_i^c}{G_m^c} < \frac{A_i(l, d^*)}{A_m(l)} \quad (9)$$

which must be fulfilled to promote decohesion.

#### 4 RESULTS

A dedicated numerical procedure (Martin et al. 1998 [7]) was developed to estimate the normalised values  $\sigma_{rr}(l, z)$  and  $A_i(l, d)$ . Equation (7) was used to evaluate  $d^*(l, d_i^c, E_f, E_m, \nu_f, \nu_m)$  as plotted in figure 2a. This plot shows that the debonding length is markedly affected by the elastic mismatch between the fibre and the matrix. A stiffer matrix leads to a smaller debonding zone. Conversely, the presence of a stiffer fibre increases the debonding zone. Figure 2b plots the critical ratio  $\frac{A_i(l, d^*)}{A_m(l)}$  versus the ratio  $\frac{E_f}{E_m}$ . The curves in figure 2b thus

delineate domains for interfacial nucleation for two different values of the ligament. For a stiffer matrix, debonding is only possible for a low value of the interfacial toughness whatever the value of the ligament. Conversely, the critical ratio is strongly dependent on the ligament for a stiffer fibre. In terms of toughness, the decohesion is facilitated by a stiffer fibre and a smaller ligament.

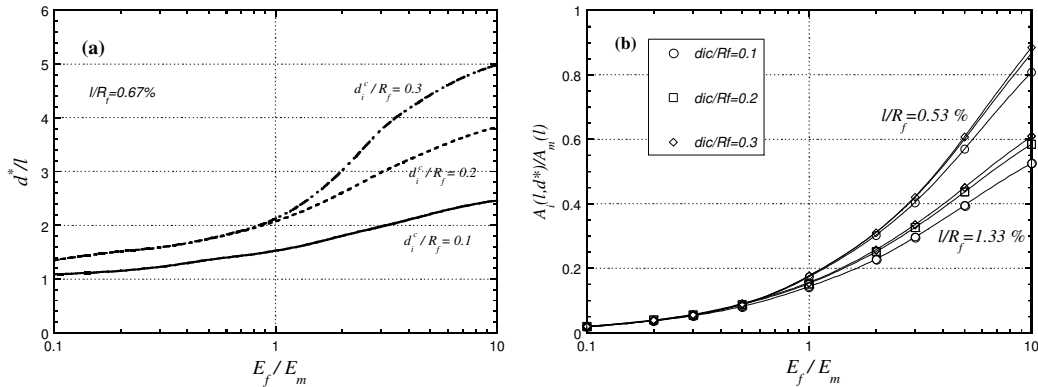


Figure 2: a) Decohesion length as a function of the modulus ratio. b) Critical ratio for nucleation of the debond as a function of the modulus ratio. The fibre volume fraction is  $V_f = 0.4$  and  $\nu_f = \nu_m = 0.2$ .

## 5 CONCLUSION

The onset of interfacial debonding in the vicinity of a matrix crack is analysed in the case of an axisymmetric fibre/matrix cell submitted to a tensile loading. The interface is assumed to be free of defect and an energetic analysis is used to describe the nucleation process. An additional strength condition is used to determine the decohesion length.

A decohesion criterion is then derived for a stationary crack submitted to an increasing loading. In the case of a stiffer matrix, the decohesion is only predicted for low toughness interfaces with

$$\frac{G_i^c}{G_m^c} < 0.02 - 0.2 \text{ depending on the elastic contrast between the fibre and the matrix. The critical}$$

ratio is weakly dependent on the ligament value. In the case of a stiffer fibre, the decohesion is facilitated by decreasing the ligament.

Use of the condition for interfacial debonding requires the identification of two material parameters : the interfacial toughness and the interfacial strength. It is expected that these parameters can be evaluated with the help of initiation loads obtained from two different micromechanical tests (push-out and pull out experiments for example) performed on a fibre/matrix system.

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