# AN EXPERIMENTAL / NUMERICAL INVESTIGATION OF THE DYNAMIC FRACTURE CHARACTERISTICS OF PMMA

N. Murphy and A. Ivankovic
Department of Mechanical Engineering, University College Dublin, Ireland

ABSTRACT

A cohesive zone model (CZM) was used in conjunction with the finite volume method to simulate the dynamic fracture of single edge notched tensile (SENT) specimens of PMMA under essentially static loading conditions. A series of experiments was carried out to study the fracture behaviour of specimens prepared from low, standard and high molecular weight sheets with 0.1 mm, 0.5 mm and 1.0 mm notch depths. The speed of crack propagation through the SENT specimens was measured using an electrical resistance method. It was found that the crack speed and failure load increased with reduced notch depths. For a given notch depth, the failure load was independent of the molecular weight. The surface roughness and subsurface damage were seen to increase substantially as the molecular weight was reduced. Large crack velocity fluctuations were associated with increased surface roughness. In addition, successful crack bifurcation was observed in the low molecular weight specimens containing the shorter notch lengths.

In the numerical simulations, global material behaviour was approximated as linear elastic while the cohesive zone model was used to define the local non-linear separation process of the material. Two distinct strategies were investigated. Firstly, the cohesive surfaces were assumed to exist *a priori* between the continuum cells in a fine mesh region to either side of the initial fracture plane. The cohesive characteristic was assumed to contain both an ascending part and a descending part, following Xu and Needleman [1]. These initially elastic cohesive laws alter the effective elasticity of the material and were found to be unsatisfactory in this regard. An alternative approach was taken whereby cohesive cells were only inserted if a specified failure criterion was satisfied, as employed by Camacho and Ortiz [2]. In this work, the cohesive strength and separation energy were held constant and the shape of the cohesive law was varied. This was found to have a profound effect on the dynamic fracture behaviour of the model.

## 1 INTRODUCTION

The continuum theory of linear elastodynamic fracture mechanics fails to explain certain experimental observations that have been made for a wide range of brittle materials, particularly at higher crack speeds. Rather than accelerating to the Rayleigh wave speed, cracks tend to propagate at much lower mean velocities under conditions of increasing energy flux into the tip region. This is accompanied by an expansion of the fracture process region and an increase in fracture surface roughness and subsurface damage. In addition, attempted and successful crack branching is often observed. In an attempt to capture the dynamic evolution of the process region, a cohesive view of material separation was considered, which incorporates a cohesive strength and separation energy into the material description. The implications of the choice of cohesive law are examined here.

## 2 EXPERIMENTAL TEST RESULTS

Two sets of experiments have been undertaken to study the dynamic fracture characteristics of PMMA using small SENT specimens. The first set [3, 4, 5] used a standard grade, whilst the second, more recent set examined the effect of molecular weight ($M_w$) on the dynamic fracture behaviour [6], as discussed below.

In the first set of tests, SENT specimens, 20 mm wide and 8 mm thick, were mounted in a universal testing machine with a grip spacing (gauge length) of 40 mm and extended at a rate of 2 mm/min, until specimen failure occurred. The notch depths ranged from 0.1 to 2.0 mm. Crack speeds were measured using both an electrical resistance method and high speed photography. The results are summarized in Figure 1. Clearly, the terminal crack speed is strongly dependent on the initial notch depth, with peak values of up to 800 m/s observed for the shortest notches. Specimens containing the 2.0 mm notches exhibited much lower crack speeds, with mean values of the order of 350 m/s. As has been reported elsewhere [7], the high-frequency crack speed oscillations have been found to correlate well with the fracture surface roughness, although as discussed in [4, 5], a certain amount of filtering is usually required to remove electrical noise from the crack speed signal, and this can have a significant influence on the results. In addition, the fracture surface characteristics were seen to vary significantly as a function of notch depth. The specimen containing the 2.0 mm notch exhibited a smooth mirror-like surface, while at shorter notch depths the visible damage increased until, at a notch depth of 0.1 mm, the entire surface had a flake-like structure accompanied by extensive subsurface damage.

A second set of experiments was carried out to assess the effect of varying the molecular weight on the dynamic fracture characteristics. Three grades of PMMA were considered: low ($1.4\times10^5$ g/mol), standard ($1\text{-}2\times10^6$ g/mol), and high ($5\times10^6$ g/mol) molecular weight. In these tests, specimen dimensions of $20\times20\times3$ mm were used. In addition, notch depths of 0.1 mm, 0.5 mm and 1.0 mm were considered. Although these dimensions are different to both the previous set of tests and the numerical simulations that follow, it is of interest to note the main conclusions of this study for future reference. Firstly, it was found that the fracture stress was essentially independent of $M_w$. Furthermore, the crack velocity characteristics for the high and standard $M_w$ specimens were similar to the values shown in Figure 1. However, the values recorded for the low $M_w$ specimens were considerably higher than those for the other grades. Those associated with the 1.0 mm notches averaged 650 m/s, whilst values close to the Rayleigh wave speed were observed for the 0.1 mm notch depths. In addition, it was found that the surface roughness increased dramatically as the molecular weight decreased. This is arguably related to the number of chain ends and the amount of free volume present at a molecular level in this grade of material. Shorter chain lengths are associated with an increased number of chain ends which may act as nucleation sites for micro-cracks [8]. Finally, crack bifurcation was observed in the low $M_w$ specimens containing the 0.1, the 0.5 mm and occasionally the 1.0 mm notches.

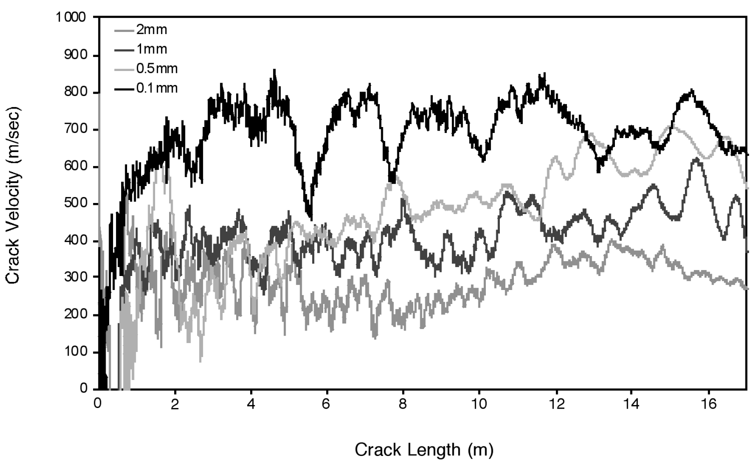


Figure 1. Crack speed data for 40x20x8 mm SENT specimens of standard molecular weight PMMA [4].

## 2 INITIALLY ELASTIC COHESIVE LAWS

In this case, as the cohesive surfaces 'separate', the magnitude of the cohesive traction at first increases, reaches a maximum and then decreases to zero with increasing separation. The cohesive law therefore exhibits an elastic initial response due to the finite initial slope of the traction-separation curve, as shown in Figure 2. This type of traction-separation law has been used extensively since its introduction in this context by Xu and Needleman [1] to model dynamic fracture in brittle solids. When using this type of law, insertion of the cohesive surfaces into a model alters the effective elasticity of the material. Whilst this may be of little consequence if only a single layer of cohesive elements is used, the effect can be significant if the cohesive cells are inserted between many continuum cells in a finite element model, as in [1], or in a finite volume model, as in [4]. When using this type of cohesive law to study the dynamics of the fracture process region, a conflicting situation arises, as noted in [9] and [10]. The length of the cohesive zone ahead of a crack in a material such as PMMA is of the order of microns. If adaptive re-meshing is not used as the crack propagates at high speed through the material, a uniform fine grid must be employed across the entire width of the model and must extend to a sufficient height above and below the initial fracture plane to capture the evolving process region. On the other hand, the cohesive contribution to the stiffness should be small compared to that of the continuum cells. This can be difficult to achieve in practice when thousands of cohesive cells are inserted between the continuum cells in the fine grid region.

It has been shown in a recent work by the present authors [4] that the effect of this reduction in elastic stiffness can be so great as to completely rule out the use of this type of cohesive characteristic when studying the two dimensional evolution of the fracture process region. Two simulations were undertaken which clearly illustrate the effect of inserting cohesive cells of the type used by Xu and Needleman [1] between 10 micron square cells in a finite volume formulation. When a *single layer* of cohesive cells was used along the initial crack path, the predicted fracture load agreed with the experimental value and the crack accelerated quickly to a speed approaching the Rayleigh wave speed, as predicted by continuum fracture mechanics for a constant specific fracture energy. On the other hand, when many such layers were inserted in the fine grid region, both static and dynamic results were dramatically altered. The predicted fracture stress was reduced by about 40%, and the mean terminal crack propagation speed was reduced to about half the previous value. It was then demonstrated quite clearly for a wide range of cohesive parameters that the speed of crack propagation was of the order of the Rayleigh wave speed calculated on the basis of the modified Young's modulus of the material into which the cohesive cells had been inserted.

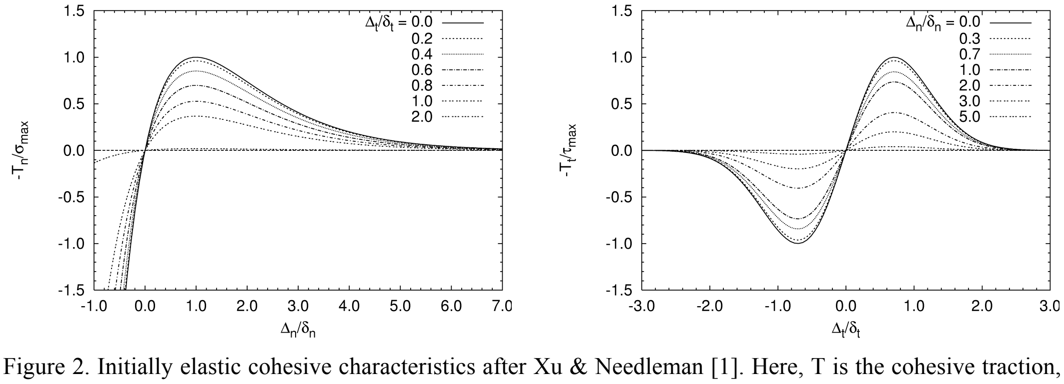


Figure 2. Initially elastic cohesive characteristics after Xu & Needleman [1]. Here, T is the cohesive traction, $\sigma_{max}$ and $\tau_{max}$ are the normal and shear strengths, $\Delta$ is the cohesive separation, and $\delta$ is a characteristic length.

## 3 INITIALLY RIGID COHESIVE LAWS

In this case, four different, rate-independent, cohesive characteristics were considered, as shown for the normal traction component in Figure 3(a)-(d). The cohesive strength and separation energy were held constant at typical values of 80 MPa and 355 J/m$^2$ respectively. The main parameter under investigation in this study was the shape of the cohesive law. For each of the three descending cohesive laws, the initial slope became progressively steeper by a factor of two, whilst the critical separation was held constant at 8.8 μm. The cohesive cells were inserted when the normal traction along any internal cell face in the model exceeded the specified cohesive strength. If, at a given separation level, $\delta^*$, unloading took place, the tractions obeyed a linear unloading relation, as shown in Figure 3(b). Upon subsequent reloading, the unloading path was reversed until the displacement $\delta^*$ was reached, and subsequently the monotonic cohesive relations were followed again. When the critical normal separation was reached, fracture was assumed to have taken place and the cell faces were thereafter treated as traction-free surfaces. The shear traction across the cohesive surfaces essentially followed the same cohesive law as the normal traction, with the value of the normal cohesive strength replaced by the shear traction that prevailed along that face when the cohesive cell was inserted. This resulted in variable shear fracture energy, which usually only accounted for a small proportion of the overall fracture energy value.

Initially, to reduce the computational effort, a 10x40 mm SENT model was employed to simulate the first 10 mm of crack growth in the experimental specimens. A uniform fine grid region was defined, which spanned the width of the model and whose height extended an equal distance above and below the initial fracture plane. The appropriate height of the fine grid region depended on the notch depth and the cohesive characteristic employed, and had to be large enough to contain the evolving fracture process zone. The height of this region ranged from 3 to 6 mm. Within the fine grid region, a square cell of size 10x10 μm was used. Outside this region, the height of the cells increased gradually to 0.8 mm at the top and bottom of the model. The number of continuum cells in the models ranged from 388,000 to 688,000, depending on the size of the fine grid region. Linear elastic behaviour and plane strain conditions were assumed. In each case the top face of the model was subjected to the experimental loading rate of $3.3\times10^{-5}$ m/s (or 2 mm/min). As in the previous simulations, a finite volume formulation with fully implicit time discretisation was employed. Results of particular interest from the transient analysis include the variation of crack front velocity, crack paths, branching and the accumulated damage as a function of crack length. A full discussion of results may be found in [11]. Figure 4 shows the fracture evolution for models containing a 0.1 mm notch. This may be compared with the experimental fracture in Figure 5. The corresponding crack velocity histories are shown in Figure 6.

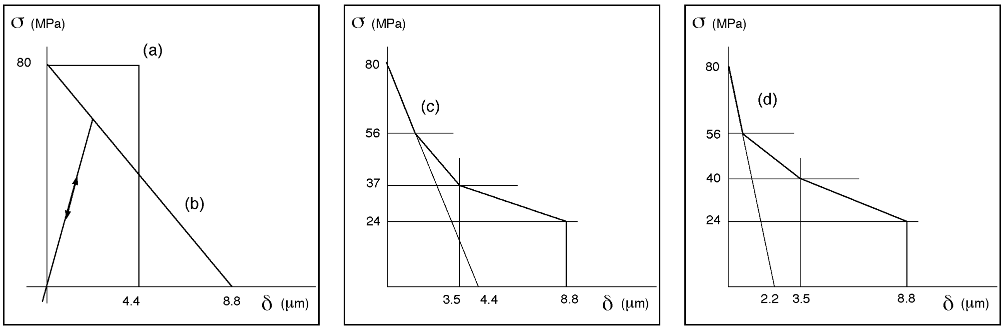


Figure 3. Initially rigid cohesive characteristics [11]. (a) constant stress, (b) linearly descending (including unloading behaviour), (c) trilinear, (d) steeper trilinear.

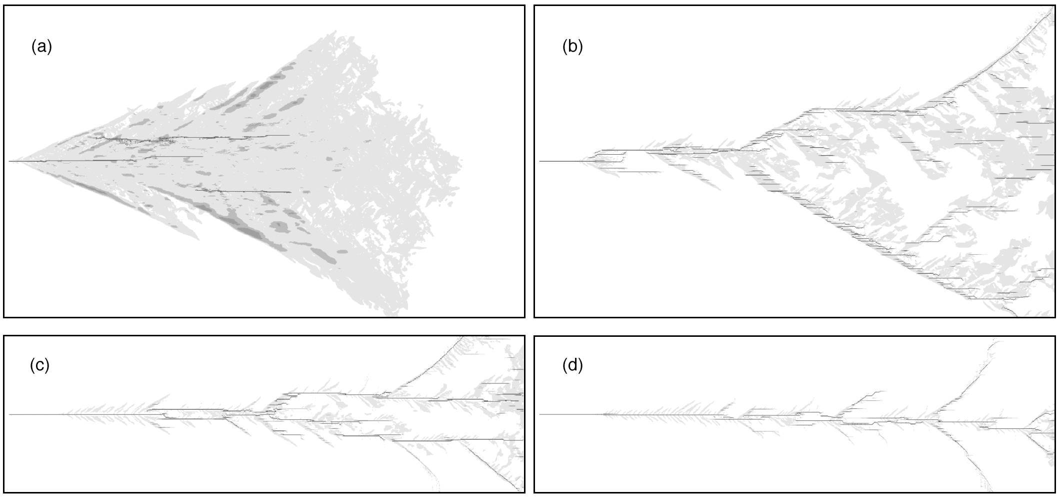


Figure 4. Crack path and damage evolution for 0.1 mm notches [11]. (a) constant stress – simulation stopped at 5.6 mm, (b) linearly descending, (c) trilinear, (d) steeper trilinear. Window sizes 10 mm × 6 mm for (a) and (b), and 10 mm × 3 mm for (c) and (d). Legend: Light grey – cohesive separation 0 to 20% of fully-separated value, Mid-grey: 20 to 40%, Dark grey: 40 to 99%, Black: fully broken.

## 4 DISCUSSION AND CONCLUSIONS

It is clear from Figures 4(a) and 6(a) that the Dugdale-type cohesive law predicts extensive damage and low crack velocities which are not associated with the dynamic fracture of brittle materials such as PMMA and is therefore considered unsuitable for use in this type of formulation. The three descending cohesive laws produced much more realistic behaviour. However whilst the cohesive strength and fracture energy were held constant, it is clear that the *shape* of the cohesive law has a profound effect on the fracture behaviour. In particular, it has been found that the initial slope of the decreasing part of the traction-separation curve is an important parameter. As the slope became steeper, the terminal crack speed increased and the extent of the damage decreased. A recent study by Falk et al. [10] compared crack branching predictions in finite element models containing either cohesive cells with an initially elastic or an initially rigid traction-separation law. They observed that macroscopic branching occurred in the former case but not in the latter. There it was proposed that the lack of branching in the latter case was possibly a consequence of the numerical implementation. In that case an explicit finite element method was used. This problem was not observed with the current fully implicit finite volume formulation.

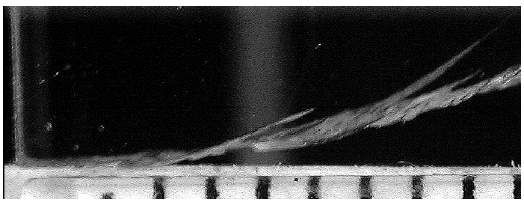

Figure 5. Crack bifurcation in a 20 mm wide, low molecular weight specimen containing a 0.1 mm notch [6]. For ease of comparison with the numerical results, only the first 10 mm of crack growth is shown, after which the crack has deviated approximately 2 mm from the mid-plane. Window size 10 mm × 3 mm.

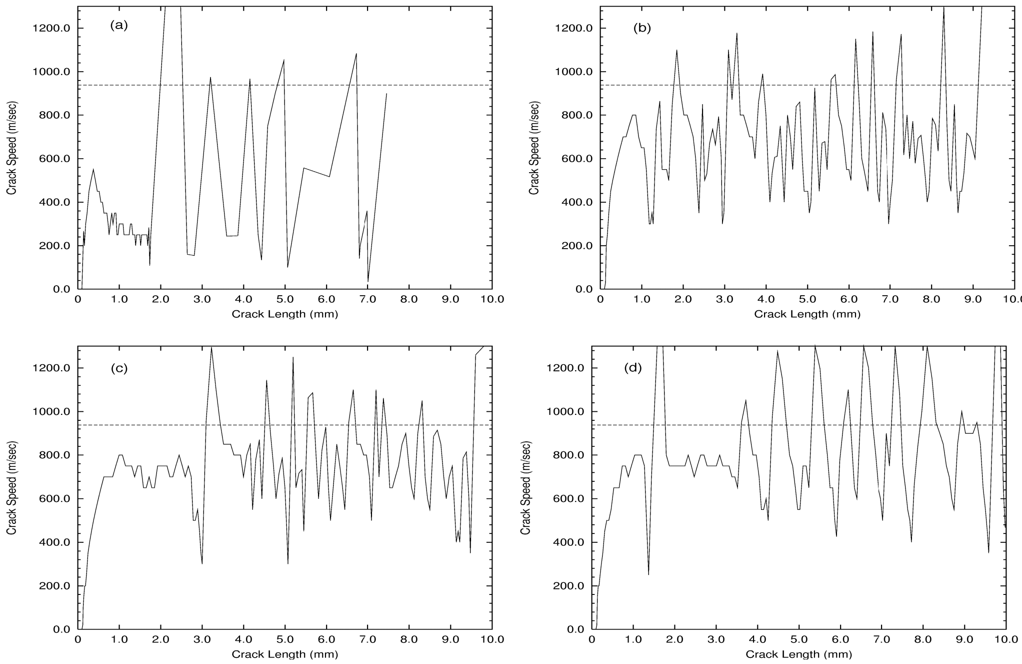


Figure 6. Crack speed histories for 0.1 mm notches [11]. (a) constant stress, (b) linearly descending, (c) trilinear, (d) steeper trilinear. The Rayleigh wave speed of 930 m/s is also shown.

## REFERENCES

[1] Xu, X.P., Needleman, A. Numerical simulations of fast crack growth in brittle solids. J. Mech. Phys. Solids. 42:1397-1434, 1994.

[2] Camacho, G.T., Ortiz, M. Computational modeling of impact damage in brittle materials. Int. J. Solids Struct. 33:2899-2938, 1996.

[3] Manmek, S. Experimental investigation of crack tip velocity variations. MSc Thesis. Imperial College, London, 1999.

[4] Ivankovic, A., Murphy, N., Hillmansen, S. Evolution of dynamic fractures in PMMA: experimental and numerical investigations. In: Aliabadi, M.H., Ivankovic, A., eds. Crack dynamics. Advances in Fracture Mechanics, vol. 9. Southampton, UK: WIT Press/Computational Mechanics Publications, 2004.

[5] Ivankovic, A., Hillmansen, S. Evolution of dynamic fractures in poly(methyl methacrylate). Plast. Rubber Compos. 30:88-93, 2001.

[6] Al-Hamdani, B. Investigating the damage and fracture characteristics of PMMA specimens of varying molecular weights. MSc Thesis. Imperial College, London, 2003.

[7] Fineberg, J., Gross, S.P., Marder, M., Swinney, H.L. Instability in propagation of fast cracks. Phys. Rev. B. 45:5146-5153, 1992.

[8] Newman, S. B., Wolock, I. Fracture phenomena and molecular weight in polymethyl methacrylate. J. App. Phys., 29:49-52, 1958.

[9] Klein, P.A., Foulk, J.W., Chen, E.P., Wimmer, S.A., Gao, H.J. Physics-based modeling of brittle fracture: cohesive formulations and the application of meshfree methods. Theoret. Appl. Fract. Mech. 37:99-166, 2001.

[10] Falk, M.L., Needleman, A., Rice, J.R. A critical evaluation of cohesive zone models of dynamic fracture. J. Phys. IV. 11:Pr5-43-50, 2001.

[11] Murphy, N., Ivankovic, A. The prediction of dynamic fracture evolution in PMMA using a cohesive zone model. Eng. Fract. Mech., in press.