ON THE SELF-FORCE OF A NON-UNIFORMLY MOVING DISLOCATION

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ABSTRACT

The self-force on a generally non-uniformly moving dislocation is defined from a conservation law of elastodynamics (based on Noether's theorem) in terms of path independent integrals. By choosing the infinite strip, the integral is evaluated. In order to regularize the effect of the logarithmic singularity associated with the acceleration, a ramp-core model is used, from which the core structure can be determined by matching self-forces calculated from continuum and lattice scales.

1. INTRODUCTION

Why has research eluded the definitive answer to the dynamic self-force of a dislocation is due to a lack of proper definition, the complexity of the analysis and the fact that dislocations are modeled by means of continuum mechanics that do not adequately grasp the lattice structure of the moving defect. Volterra dislocations, i.e. step-function displacement discontinuities at the core, moving uniformly (steady-state) or non-uniformly, i.e., with acceleration, have been studied subsequently to Frank [7] and Eshelby [5], among others by Weertman [17], Mura [15], Markenscoff [11] and co-workers (list of references may be found in the review article of Weertman & Weertman [17], and Eshelby [4]. The radiated elastic fields from non-uniformly moving straight dislocations and dislocation loops have been obtained given a general accelerating motion subsonic or supersonic, in isotropic solids and anisotropic crystals (Markenscoff and Ni, [10] [12]. However, the self-force on a moving dislocation or, equivalently, the "effective mass" of a dislocation, is still not fully understood. Indeed, in the absence of dissipative effects, in steady-state motion that has started at $t \longrightarrow -\infty$, the dislocation is emitting (per unit time) equal radiated energy as the one that it receives from previously emitted wavelets emitted during the path of its motion. However, if the dislocation is starting at rest, the emitted energy is greater than what the dislocation receives in its core from wavelets that itself has emitted in the past. ("The dislocation is haunted by its past," Eshelby). This difference in the energy rates has to be externally supplied, and accounts for the "effective mass" (Clifton and Markenscoff [3], and Markenscoff [14]. For motion of a dislocation jumping from rest to a constant velocity, Clifton and Markenscoff [3] computed the energy-release rate $\dot{E}$ and the self-force as $F \equiv \frac{\dot{E}}{v_d}$, and found, for a screw dislocation

$$\dot{E} = -\frac{\mu b_y^2}{2\pi t}\left(\frac{1-\left(1-v_d^2/c_2^2\right)^{\frac{1}{2}}}{\left(1-v_d^2/c_2^2\right)^{\frac{1}{2}}}\right) + \sigma_{yz}^a b_y v_d ,$$

and for an edge

$$\dot{E} = -\frac{\mu b_x^2}{2\pi t}\left\{\frac{12-8\alpha^2}{\beta^2\left(1-\alpha^2\right)^{\frac{1}{2}}} - \frac{\left(2-\beta^2\right)\left(6-7\beta^2\right)}{\beta^2\left(1-\beta^2\right)^{\frac{1}{2}}} - 2\left(1-\frac{\alpha^2}{\beta^2}\right)\right\} + \sigma_{xz}^a b_x v_d ,$$

$$\alpha = \frac{v_d}{c_1}, \beta = \frac{v_d}{c_2}.$$

which show that to sustain a motion jumping from rest to a constant velocity, an energy rate needs to be supplied that varies in time as $\frac{1}{t}$.

2. SELF-FORCES ON MOVING DISLOCATIONS IN GENERAL NON-UNIFORM MOTION

For general non-uniform motion, Eshelby in a seminal paper [5], provided an intuitive argument for the force necessary to be applied on the slip-plane of a screw dislocation to ensure a motion $x = \xi(t)$; in his notation,

$$F = b\, p_{zy}^A = \left(1 - \frac{\dot{\xi}^2}{c^2}\right)^{-3/2} \frac{\rho b^2}{4\pi}\{ln\ f(t)\}\frac{\partial^2 \xi}{\partial t^2} + g(t). \tag{1}$$

The coefficient of the acceleration $\frac{\partial^2 \xi}{\partial t^2}$ may be considered as the effective mass, and the term $g(t)$ as a "radiation reaction" term that depends on the history of the motion since "the dislocation is continually catching up the radiation it has already emitted." The functions $f(t)$ and $g(t)$ were not given by Eshelby. We propose here to determine them within the framework of a rigorous definition of the "self-force" on a moving singularity.

For a dislocation jumping from rest to a constant velocity, Clifton and Markenscoff [3] computed the (net) energy flux through the core of the dislocation by the energy release rate expression used for moving cracks (Freund [8]),

$$\dot{E} = \lim_{S_d \to 0} \int \left[\sigma_{ij}\ \dot{u}_i\ n_j + v_n\left(\frac{1}{2}\sigma_{ij} u_{i,j} + \frac{1}{2}\rho\ \dot{u}_i\ \dot{u}_i\right)\right] dS \tag{2}$$

where S_d is a contour surrounding the dislocation, moving with the dislocation and shrinking to zero. In (2), v_n is the component of the dislocation velocity on the normal to the contour. It was shown by Clifton and Markenscoff that for dislocations jumping from rest to constant velocity, the integral (2) is independent of the shape of S_d by considering the area integral enclosed by two different contours,

$$\int_{R\bullet} \left\{\rho \dot{u}_i \left(\ddot{u}_i + v_n\ \dot{u}_{i,n}\right) + \sigma_{ij}\left(\dot{u}_i + v_n\ \dot{u}_{i,n}\right)_{,j}\right\} dA \tag{3}$$

and showing that it vanishes as the two contours approach each other. In the case of steady-state motion, the integrand in (3) vanishes identically, and not only in the limit, because of the grouping of the terms $(\ddot{u}_i + v_n \dot{u}_{i,n})$ and $(\dot{u}_i + v_n \dot{u}_{i,n})$ which vanish identically. For motion with constant velocity starting from rest, this is true only in the limit $R_* \to 0$, while for motion with velocity that is general function of time, the integral (3) diverges logarithmically. Thus, the integral (2) is not path-independent any longer and defining through it the energy flux through the core as the configurational force for a generally moving dislocation is not possible (for cracks, it is because the singularity is of lesser order).

To define a configurational force for a generally moving dislocation, we apply the conservation law of elasto-dynamics (Fletcher [6], derived by the application of Noether's theorem

$$\frac{\partial}{\partial t}\left(\rho\, \dot{u}_j\, u_{j,i}\right) + \frac{\partial}{\partial x_k}\left(-u_{j,i}\sigma_{j,k} + L\delta_{ik}\right) = 0 \qquad (4)$$

with $\quad L = \frac{1}{2} C_{ijkl} u_{i,j} u_{k,l} - \frac{1}{2}\rho\, \dot{u}_i \dot{u}_i$,

valid in a domain D. We integrate (4) in a domain D_o with boundary ∂D_o which is any bounded regular sub-region of D, and we chose $D_o \equiv D_2 - D_1$, where the domain D_1 with boundary ∂D_1 (moving with the instantaneous velocity of the dislocation) encloses the moving dislocation (the boundary of D_2 also moves with the instantaneous velocity of the dislocation) and obtain

$$\frac{d}{dt}\int_{D_1} \rho\, \dot{u}_j\, u_{j,i}\, dx + \int_{\partial D_1} \left(L n_i - u_{j,i}\,\sigma_{jk}\, n_k\right) dS =$$

$$\frac{d}{dt}\int_{D_2} \rho\, \dot{u}_j\, u_{j,i}\, dx + \int_{\partial D_2} \left(L n_i - u_{j,i}\,\sigma_{jk}\, n_k\right) dS \equiv F_i \qquad (5)$$

The sum of the above two integrals, i.e., the volume integral and the surface integrals are path-independent and we define this as the dynamic configurational force *F* on a moving dislocation, or as the dynamic *J* integral. For moving cracks, it coincides (Bui [1]) with the energy-release rate definition given by equation (2).

3. COMPUTATION OF THE SELF-FORCE (CONFIGURATIONAL FORCE) FOR A GENERALLY NON-UNIFORMLY MOVING DISLOCATION

The expressions for the transient fields of stress, strain, velocity that need to be inserted into the expression (5) are given as integrals over the history of the motion.

As an example, we show how the computation will proceed for a screw dislocation; for edge and loop, it will be the same conceptually, but with more terms. For rectilinear motion $x = l(t)$, for the screw dislocation on the slip plane we have for the strain component (and similarly for the velocity field):

$$\frac{\partial u_y}{\partial z}(x,z,t) = -\frac{\Delta u}{2\pi}\int_0^\infty \frac{(t-\eta(\xi))(x-(\xi))^2 H(t-\eta(\xi)-r\,b)}{r^4\left[(t-\eta(\xi))^2 - r^2 b^2\right]^{1/2}} d\xi$$

$$+\frac{\partial u}{\partial z} z^2 \frac{\partial}{\partial t} \int_0^{\infty} \frac{(t-\eta(\xi))^2 H(t-\eta(\xi)\xi - r b)}{r^4 \left[(t-\eta(\xi))^2 - r^2 b^2\right]^{1/2}} d\xi - \frac{\Delta u}{2\pi} \frac{x}{x^2+z^2} \tag{6}$$

where $r^2 = (x-\xi)^2 + z^2$, and $t = \eta(x)$ describes the motion equivalent to $x = l(t)$, Δu is the Burgers vector and b the shear-wave slowness.

While introducing expressions such as the integrals in (6) into the integrands of (5) seem horrendous, by appropriate choice of contour D_1 the calculation is reduced to evaluating the near field, i.e., the field near the current position of the dislocation $x = l(t)$, by use of singular asymptotics of integrals developed by Callias and Markenscoff [2]. The singular asymptotics give also a $ln\varepsilon$ term in the near field expansion, which is due to the acceleration, and is not present in steady-state motion. These near field expansions to $O(1)$ will be performed for screw and edge straight dislocations as well as for dislocation loops. Once the near field is obtained, the integrals in the definition of the self-force (5) will be evaluated.

Due to the singularities, the volume integral will diverge, which immediately implies that the Volterra dislocation is too strong of a core-model, and thus either a regularization or a ramp-like model of the core will be required. For the surface integral in (5), we will need to choose the most convenient contour (since we have path independence) and as such, we chose the infinite strip on a slip-plane with thickness $2e$ in z.

For a Volterra screw dislocation, the contribution of (6) to the surface integral in (5) for the strip contour is evaluated in the sense of distributions

$$\int_{S\pm} \left(-\sigma_{32} u_{3,1}\right) dx = \int_{\infty}^{\infty} \left[\sigma_{32} u_{3,1}(x,\varepsilon) - \sigma_{32} u_{3,1}(x,-\varepsilon)\right] dx =$$

$$= \int_{\infty}^{\infty} \sigma_{32}(x,\varepsilon)\left[\delta(x - l(t)) + \varepsilon\delta'(x - l(t)) + \varepsilon^2\delta''(x - l(t)) + \ldots\right] dx$$

$$= \left(A\frac{1}{\varepsilon} + A_2\, ln\varepsilon + 0(1)\right) + 0(\varepsilon), \tag{7}$$

so that the contribution to the configurational force is

$$F = \frac{\Delta u}{4\pi} b^2 \frac{1}{\left(1 - \dot{l}(t)^2 / c_2^2\right)^{3/2}} \ddot{l}(t)\, l n\, \varepsilon + 0(1) \tag{8}$$

since the $\frac{1}{\varepsilon}$ terms do not contribute due to symmetry reasons.

The $O(1)$ terms for a screw dislocation are calculated by Markenscoff [14], but are too lengthy to be shown here. The logarithmic singularity in (8) (associated with the acceleration) implies that the Volterra dislocation is too strong of a core model to produce a finite self-force for an accelerating dislocation. Thus a more physically realistic core-model is needed, and as such, we will use a ramp-core one.

4. CONFIGURATIONAL FORCE FOR A RAMP-CORE DISLOCATION MODEL AND MATCHING WITH A DISCRETE LATTICE ONE

The Volterra dislocation is modeled as a step function discontinuity in the displacement moving within the elastic solid according to $H(x-l(t))$. The step function discontinuity is too strong of a singularity and may be replaced by a "spread" core model of a sequence that will have the step function as its limit. We may thus have a delta sequence

$$H_\delta(x) = (1/\pi)\arctan(x/\delta) \tag{9}$$

where δ may either be a constant (rigid core) or function of time $\delta(t)$. If $\delta \neq 0$ the singularities are eliminated and the volume integral in (5) vanishes as $D_1 \to 0$.

To compute the configurational force for the rigid ramp model, we need to perform a convolution on the Volterra dislocation solution (8) (Markenscoff & Ni [13]), which yields for a rigid core

$$F = \frac{\Delta u}{4\pi c_2^2} \frac{\ddot{l}(t)}{\left(1-\frac{\dot{l}^2}{c_2^2}\right)^{3/2}} ln\delta, \tag{10}$$

while for time-independent core $\delta(t)$, by using the expression obtained by Markenscoff & Ni [13], after convolution we have

$$\frac{\mu\Delta u}{4\pi c_2^2}\int_0^t \frac{\ddot{l}(t-w)}{\left(1-\frac{\dot{l}^2(t-w)}{c_2^2}\right)^{3/2}} l'(w)\, ln\,\delta(w)d\,w \tag{11}$$

The $O(1)$ terms will also be calculated by convolution, and they contribute to the acceleration $\ddot{l}(t)$ as well to integrals over the history of the motion.

In order to determine the ramp-core function $\delta(t)$, we have to match the configurational force calculated from the continuum model to the one from the discrete lattice one, such as the one developed by Kresse & Truskinovsky [9]. They used a lattice model of Rosenau [16] of mass-spring chains with arbitrary inter-particle and substrate potentials to describe a systematic approach to derive the equation of motion near the continuum limit for the Frankel-Kantorova dislocation model. From first principles they computed a functional relation between the microscopic configurational force and the velocity of the defect. The discrete model is purely conservative and contains information only about elasticities of the constitutive elements. The apparent dissipation is due to the pressure of micro-instabilities and the non-linearity-induced tunneling of the energy from long to short wavelengths. This type of "radiative damping" is generic. Current research, in collaboration with Lev Truskinovsky is going on on matched asymptotic expansions in order to match the configurational force obtained by Kresse and Truskinovsky [9] to the one we obtained by our continuum model.

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