# INFLUENCE OF THE MATERIAL MESO-STRUCTURE ON 2D AND 3D LATTICE RESPONSE

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#### ABSTRACT

Heterogeneous lattice models are receiving increasing recognition as one of the most suitable models for studying the mechanisms of fracture in disordered materials such as concrete. In concrete failure is induced by formation of micro-cracks, which propagate and coalesce to form macro-cracks. Micro and macro cracking correspond in the load-displacement diagram to the pre- and post-peak regime, respectively. In lattice models concrete is schematized as a network of beams (or springs) that have a linear elastic brittle behavior until failure, and removing at each loading step the element that violates the adopted fracture criterion simulates cracking. Despite the local brittleness, the overall response of the lattice presents softening. Thus, lattice models are useful not only for investigating the factors that influence the strength, but also for understanding the mechanism of softening in the structure. Though, these models are still controversial. The reason is that 2D lattice analyses, although able to reliably predict crack patterns for a wide range of laboratory experiments, give always a too brittle response in terms of load-displacement diagrams in comparison to the experiments. In this paper it is shown that the brittle lattice response is consequence of the rough schematization of the material, which is usually adopted, and the neglect of 3D effect. 3D Lattice analyses performed with varying particle density show that by increasing the particle density an increasing ductility of the lattice response can be obtained. The same type of analysis conducted with a 2D lattice shows a less remarkable influence of the particle density on the overall lattice response. The reason for this may be that percolation of the interface occurs in 2D lattices already for relatively sparse particle distributions. In fact, the same type of response can be obtained with a sparse particle distribution by increasing the thickness of the interface.

# 1 INTRODUCTION

Over the last decades engineers have favoured non-linear homogeneous models for studying fracture of concrete. These models assume that concrete behaves as a homogeneous material, which exhibits softening locally. Introduction of softening entails additional material parameters, besides the elastic parameters and the strength, that are measured in laboratory experiments on specimens of finite size. As concrete is a highly heterogeneous material, and the response that is measured in the laboratory is the response of the specimen as a whole structure, softening is a combined material/structural property. This aspect remains ignored by all those adopting softening models. A more physically sound approach for studying fracture processes in concrete is adopted in heterogeneous models such as lattice models. This paper will deal further with this type of models, specifically with a beam lattice model (Herrmann et al. [1]).

In the beam lattice model concrete is schematized as a network of beams that have a linearelastic brittle behaviour. After a computer-generated particle distribution is overlaid on top of the lattice, different mechanical properties are assigned to the beams falling inside each of the 3 phases, namely aggregate, matrix and interface. Removing at each loading step the beam that violates the assumed failure criterion simulates fracture processes. Despite the local brittleness, the overall response of the lattice presents softening. The model, in its 2D version, has been able to predict the crack pattern in a wide range of experiments where Mode I failure prevails. Though, the response was too brittle in comparison with the experiments. Reasons of the brittle lattice response are the absence of 3D effects and the rough schematization of the material structure.

In the 2D lattice, as soon as one element fails the specimen is cracked through the whole depth. In the 3D lattice more elements have to fail before the crack crosses the entire structure, and the crack surface is oriented in all directions. As the area of the crack surface increases, also the dissipated energy is larger than predicted by the 2D model.

As the particle distributions are obtained usually by randomly positioning the particles inside the area of the specimen, with this procedure it is practically impossible to obtain particle distributions as dense as in real concrete. However, using computer programs where collision rules among the particles are adopted can solve this problem. Furthermore, in order to limit the computational time, relatively coarse lattices are adopted and, as a consequence, small particles are neglected.

The only limitation to introducing 3D effects and material structure effects is the enormous computational time. For taking into account these effects also in a coarse lattice, some lattice models have been proposed where a non-linear material constitutive relation is assigned at element level. Nowadays this is not necessary, as parallel computing easily allows to perform analysis with millions elements. Moreover introducing softening at the lattice beam level confuses matters, since it is the softening that we would like to explain.

In this paper analyses conducted with a 2D and 3D beam lattice model for varying particle densities are described. The 3D lattice analyses show that by increasing the particle density an increasing ductility of the lattice response can be obtained. The same type of analysis conducted with a 2D lattice shows a less remarkable influence of the particle density on the overall lattice response. The reason for this may be that percolation of the interface occurs in 2D lattices already for relatively sparse particle distributions. In fact, the same type of response can be obtained with a sparse particle distribution by increasing the thickness of the interface.

### 2 CASE STUDY

The case study considered in this paper is a uniaxial test. In the 2D case the specimen is a square of 60 mm constructed of short beams of 0.25 mm. In order to limit the 3D analysis to approximately the same number of degrees of freedom as in the 2D analysis, in the 3D case the specimen is a 24 mm cube, and the maximum length of the elements is 1.3 mm. The nodes of the upper and bottom face are supported in all directions, and a vertical displacement is applied to the nodes of the upper face. As the beam length and the minimum diameter of the particle that can be represented in the lattice follow the relation  $l_{beam} \leq 1/3 \cdot d_{a,min}$ , particle distributions with diameter varying in the range 1-12 mm and 4-12 mm were considered, respectively in the 2D and 3D case. Three different particle densities were considered:  $P_{k,eff}^* = 35\%$ , 51% and 67% in the 2D case,  $P_{k,eff}^* = 34\%$ , 48% and 62% in the 3D case. The mechanical properties assigned to the elements were  $f_{t,a}/f_{t,m}=10/5$ ,  $f_{t,b}/f_{t,m}=1.25/5$ ,  $E_a/E_m=70/25$  and  $E_b/E_m=25/25$ , for the tensile strengths and the elastic moduli of aggregate, matrix and bond, respectively. For studying the effect of the interfacial transition zone (ITZ) on the overall response of the lattice, the thickness and the strength of the ITZ were varied. In the 2D case with the sparsest particle density the thickness was varied from 0.25 mm to 1.00 mm. The 3D analyses were repeated considering a strength of the interface  $f_{t,b} = f_{t,a} = 5.00$ . The analyses were repeated on 3 different samples for each parameter combination.

## **3 RESULTS FROM 2D ANALYSES**

## 3.1 Effect of particle density

In Figure 1 the load-displacement diagram and the corresponding dimensionless diagram (obtained by dividing the load and the displacement for the values at peak), with the scatter among the 3 analyses, are shown for each value of the particle density. In Figure 2 the crack patterns at the peak and at 25  $\mu$ m displacement are shown for the 2 extreme particle densities. Finally, the peak load is shown in the inset.

Independently of the particle density, at the peak micro-cracks are localized in the interface between aggregate and matrix. These cracks are differently distributed over the specimen: they are distributed over the whole area of the specimen in case of sparse particle density, and concentrated in a band in the case of dense particle density. In the post-peak regime micro-cracks propagate through the matrix, forming macro-cracks and, finally, failure of the specimen. In the load-displacement diagram this corresponds to the steep post-peak branch. Branching of the cracks and bridging occurs in the tail of the softening branch (Van Mier [2]). Higher particle densities tend to decrease the peak load, while in the post-peak regime the drop of load after the peak is limited. Nevertheless, the post-peak regime remains still quite brittle.



Figure 1: 2D Results for different particle densities



Figure 2: Crack patterns at peak, (a) and (c), and 25 µm displacement, (b) and (d)

### 3.2 Effect of interface thickness

The results of the analyses conducted varying the thickness of the interface are shown in Figure 3 and Figure 4. Comparing these results with the results discussed in the previous paragraph, suggests that the effects produced in a specimen with sparse particle distribution when the thickness of the interface is doubled from 0.25 mm to 0.50 mm is analogous to the effects produced when a denser particle distribution is adopted. Also the crack patterns are similar, namely cracks are concentrated in bands, rather than distributed over the whole specimen. Further increase of the ITZ thickness does not affect the results of the analyses.

By increasing the ITZ thickness, clusters of ITZ are created and form continuous paths, where cracks will propagate. This effect is known as percolation of the ITZ. A similar effect can be obtained when the relative distance of the particles decreases, as in denser particle distributions. Thus, no variation in the overall response of the lattice has to be expected after percolation has occurred, i.e. after a critical value of the particle density or of the interface thickness has been reached. The results, which show that the ITZ determines the strength of the material and also influences the post-peak behavior, suggest that a more realistic schematization of the interface should be adopted in the lattice analyses. As a matter of fact, the thickness of the actual ITZ is up to 1/50 of that considered in the analyses.



Figure 3: Results for different interface thicknesses



ITZ=1.00 mm

Figure 4: Crack patterns at 25 µm displacement

#### 4 RESULTS FROM 3D ANALYSES

The results of the 3D analyses are shown in Figure 5. The load-displacement diagrams differ from those obtained in the 2D analyses in the pre-peak regime as well as in the post-peak regime. In the 2D analyses the first crack occurs at a high value of the load and the pre-peak regime is nearly linear until the peak load. In the 3D analyses the first crack occurs at a relatively low load. As a result, the global stiffness of the specimen decreases and the diagram is kinked. A second nearly linear branch follows the kink. In this phase cracks continue to develop in the interface and, as the number of elements that fail is larger for denser particle distributions, the specimen softens and the slope of the corresponding diagram decreases. Again, the peak load decreases with the density of the particle distribution. In the post-peak regime the load does not drop suddenly as in the 2D analyses but decreases gradually. The overall response of the lattice is more ductile than in the 2D case and the ductility continuously increases with increasing the particle density up to 62 %. An analysis of the crack patterns in this phase would show that cracks propagate mainly in the matrix in sparse particle distributions, while de-bonding prevails in the case of dense distributions.



Figure 5: 3D Results for different particle densities (Lilliu & Van Mier [3])



The same 3D analyses, repeated for the 2-phase material, where  $f_{t,b} = f_{t,a} = 5.00$ , show that the diagrams remain linear up to the peak load. As the ITZ is stronger than in the case of 3-phase material, the maximum load is higher. The difference with the cases already considered is that the peak load slightly increases with the particle density. Again, the load drops suddenly after the peak. Thus, the overall response of the 2D lattice, when the particle density varies. Although these analyses give a more ductile response than the 2D analyses, they are still brittle in comparison to the results from the 3-phase material.

#### **5** CONCLUSIONS

2D and 3D lattice analyses have been conducted with a beam lattice where a linear elastic purely brittle behaviour is adopted at the element level. After a computer-generated particle distribution is generated and overlaid on top of the lattice, different mechanical properties are assigned to the elements falling in each phase, e.g. aggregate, matrix and bond. Removing at each loading step the element that violates the adopted fracture criterion simulates cracking. Analyses were conducted varying the particle density both in the 2D and 3D case. In order to investigate the effect of the interfacial transition zone (ITZ) on the lattice response, the thickness of the ITZ was varied in the 2D analyses, and the strength in the 3D analyses, respectively.

The results show that the adopted particle density affects the peak load and the post-peak regime. By increasing the density of the particle distribution the peak load decreases and the overall response of the lattice is more ductile in the 3D analysis. 2D analyses give still a relatively brittle response. When a sparse particle distribution is considered, and the ITZ becomes thicker, the response of the lattice is similar to the response obtained with denser particle distributions. 3D analyses conducted with a 2-phase material (where the ITZ has the same strength as the matrix) show still a relatively brittle response, although more ductile than the 2D response.

As the ITZ appears to be the factor that determines the overall response of the lattice, a more realistic description of the ITZ, together with 3D effects, are expected to improve the performance of the model. Then, the lattice model could be used for investigating the factors that determine the strength measured in the laboratory, as well as the softening behaviour. In this respect, the lattice model would be a very powerful tool for understanding the physics behind the "material parameters" that are used in homogeneous models.

# REFERENCES

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