HOW CRACKS MODULATE NONLINEAR ACOUSTIC SIGNALS

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ABSTRACT

Non-destructive testing for constructed facilities has been attracting researcher's attention in recent decades because that our infrastructure system is deteriorating at an alarming rate. Acoustics is considered to be one of the most promising methods. In particular, linear acoustic techniques such as pulse velocity or amplitude attenuation are commonly used but not reliable especially at the early stage of damage. Non-linear nondestructive testing is different from linear acoustic in that it correlates the presence and characteristics of a defect with acoustical signals whose frequencies differ from the frequencies of the emitted probe signal. The difference in frequencies between the probe signal and the resulting frequencies is due to a nonlinear transformation of the probe signal as it passes through a defect. To consider nonlinear acoustic effect due to interaction between elastic acoustic wave and the damage state (i.e. microcracks) of material, we need to first consider such effect on a single crack. The most basic way to consider a crack-like defect is as a planar interface where there is no adhesion between two semi-infinite elastic materials. Under acoustic interrogation due to longitudinal waves, as the compression phase passes the defect the two sides of the interface are in direct contact and the contact area increases. Similarly, the tensile phase passes through the defect, yet because there is no traction across the interface, the two sides separate and the contact area decreases, thereby modulating the signal amplitude. In reality, these two crack planes are not smooth, neither traction free. The contact area depends on the roughness of the surface and on the magnitude of the cohesive forces that arise from the small crack openings. Such cohesive forces may be attributed to aggregate interlock (in plain concrete), fiber bridging (in fiber reinforced concrete) or both. In this paper, the frequency shifts of the probe elastic wave will be analytically related to the roughness and varying cohesive forces of the crack-like defect.

1 INTRODUCTION

As civil infrastructure ages it is pertinent that engineers evaluate structures on a regular basis. Current non-destructive testing methods permit flaw detection, but are not sensitive to the gradual deterioration of concrete that begins soon after the structure is placed into service (Mindess [1]). Furthermore, it is this micro-crack deterioration that eventually interacts to become readily detectable flaws and shortly thereafter leads to failure (Ashby [2]). Early intervention is safer and generally more economical. It is in this pursuit that new methods of testing concrete are essential. Much work has been done in the field of ultrasonic testing as regards civil infrastructure. This encompasses such testing methods as ultrasonic pulse time-of-flight testing for strength, quality, thickness, or reflective flaws, to acoustic emission monitoring for lifetime health. This paper discusses a new method that uses an acoustic perturbation of any crack-like flaw to modulate an ultrasonic signal. Emphasis is placed on the mechanics of the testing theory.

Actively modulated acousto-ultrasonic (AMA) non-destructive evaluation is the subject of interest. The method, based on the vibro-acoustic modulation method (Donskoy [3]) or the nonlinear wave modulation spectroscopy (Van Den Abeele [4]), uses simple tone burst or very narrow band ultrasonic signal in a pitch catch arrangement to interrogate the material between transmitter and receiver. Where this differs from traditional acousto-ultrasound is in the application of a low frequency acoustic excitation to the specimen, thereby perturbing each flaw through which the probe signal passes. The result of interaction with a crack like flaw is that signal energy is modulated by the generation of harmonics (Buck [5]) of both the probe wave and the acoustic excitation, and by the generation of sidebands of both signals by their co-interaction through the crack interface. Additionally, non-linear effects are seen due to partial signal transmission resulting from the time-varying stresses (mostly from the excitation signal) on the rough crack interface (Donskoy [3], Sutin [6]). Fundamental frequency distortion has been considered to some degree (Daponte [7]).

Experimentation was performed at Wayne State University's Advanced Infrastructure Materials Laboratory using the actively modulated acousto-ultrasonic (AMA) testing method as the basis. Details of the testing program and results have been published (Warnemuende [8]). The focus here is in modeling the primary source of energy dispersion in this test method. We develop a simple one dimensional model that includes both harmonic generation and sideband generation using some idealizations. From there we investigate numerically generated test data and qualitatively compare a computer generated AMA test signal with test data recorded during the experimental regime.

2 MODELING

Dilatational (longitudinal) plane waves have the following form if we assume that propagation occurs along the x-axis:

$$\frac{\partial^2 u}{\partial t^2} = c_l^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

The displacement u can then be solved using any function in the differential eqn (1) resulting in a basic solution of the form:

$$u = f(x + c_l t) + f_1(x - c_l t) \tag{2}$$

Eqn (2) is made up of two functions one wave of half amplitude traveling in each direction away from the source. In a real system, we need to have at least one free surface for the transducer so the specimen is not really infinite, but if sufficiently large could be considered similar enough to apply these results. With the one free surface we may apply a forcing function and find that the forward moving half of the displacement function $f_1(x - c_l t)$ is the only part of significance, because the backward moving part $f_1(x + c_l t)$ is generally absorbed by design in a transducer assembly (Krautkrämer [9]). The particle velocity due to the stress wave is:

$$\dot{u} = \frac{\partial u}{\partial t} = -c_l f_1'(x - c_l t) \tag{3}$$

where $f_1'(x - c_l t)$ is the prime means derivative of $f_1(x - c_l t)$ with respect to $(x - c_l t)$. Then, by equating the potential energy and the strain energy we get:

$$\sigma_x = -\rho c_l \dot{u} \tag{4}$$

The displacement, velocity (eqn 3), and pressure front (stress, eqn 4) of the wave are sinusoidal and carry the same frequency as the displacement. Consider the infinite linear-elastic body above in the description of the propagation of plane dilatational waves. Defining a planar crack at the origin and orthogonal to the axis of the stress wave propagation we can investigate the most simplified form acoustic interaction with a crack. Though the body behaves in a linear elastic manner in response to acoustic stress wave propagation, yet under some conditions the interface will produce a response that is not linear. In other words, though you would expect a zero-phase lag sinusoidal strain response (linear response) from a sinusoidal stress input, depending on the pressure holding the interface together, the strain response may be non-sinusoidal across the interface. Following Richardson's approach (Richardson [10]) to the problem and including a more complex closing stress state, we begin with the small displacement, elastic wave equations discussed above resulting in a stress-strain relationship as in eqn (4). The gap closing stress s_g acts in the medium to close the interface.

$$\sigma_x = \lambda_E \frac{\partial u}{\partial x} + \sigma_g \tag{5}$$

Here, l_E is an elastic constant and s_g may be a function of time and/or crack separation.

The closing stress function can be written as a function of the static loading on the structure and the spring like resistance to the crack opening that is offered by the surrounding sound material. For this function we turn to the simple relationships developed in linear fracture mechanics for an elliptical crack in an infinite material (Broek [11]). We can use this idealization when the crack is small compared to the cross-sections of the member. Here we find a relationship for the crack opening displacement under any uniform internal pressure applied to the inner surfaces of an elliptical crack (Broek [11]).

$$\delta(x) = \frac{4p}{E}\sqrt{a^2 - x^2} \tag{6}$$

Here $d(x)$ is the total opening separation of the crack interface at the location x units of distance away from the center of an elliptical crack of $2a$ units of distance in width. The value p is the positive internal pressure exerted on the crack. Using this relationship we can find a simple expression for the resistance to a mean crack opening displacement as:

$$\sigma_R(\delta) = \frac{\delta E}{a\pi} \tag{7}$$

In this case we can simply write the total closing stress at the crack interface as the sum of the stress exerted by external loading and the crack separation resistance. Note that the external loading is assumed to be effectively static.

$$\sigma_g(\delta) = \sigma_R(\delta) + \sigma_0 = \frac{\delta E}{a\pi} + \sigma_0 \tag{8}$$

However, as the crack size approaches zero the resistance to the crack opening goes to infinity at the same rate resulting in the intermittent stress wave transmission becoming constant with respect to crack size. This due to the one-dimensional assumptions, for if we simply included a cross-sectional area term for the wave front, the area of the crack interaction with the whole wave front would then go to zero with the crack size and the proportion of the energy modulation due to the infinitesimally small crack would become zero as well.

3 DISCUSSION OF MODEL AND EXPERIMENTAL RESULTS

Using the numerical approach, with both 10kHz and 100kHz sine waves as the interrogating signal in mortar that has similar properties to that of the mortar used in experimentation, we get the frequency spectrums in Fig. 1 for increasing external pressure normalized with respect to the peak stress developed by the combined interrogating signals where the size $a \to \infty$. Experimentation suggested that the acoustic perturbation must have significantly larger amplitude than the ultrasonic probe, so we used a ratio of 4 to 1 in Fig. 1.

Notice that for the case where the external closing pressure is equal to the maximum stress developed due to the interrogating signal, the crack never opens and the spectrum consists two spikes. This is, of course, typical of linear systems, resulting in the pure superposition of waves, the combination of these waves in the nonlinear system where the crack opens is not a superposition, but is rather characterized by modulation as can be seen in all the other cases. This modulation consists of two parts, multiple harmonic generation, and multiple sideband generation. Of course, the other special case in Fig. 1 is when the external closing pressure is zero, in which case no harmonics, and no sidebands are generated, in fact no signal is transmitted across the crack because it never closes.

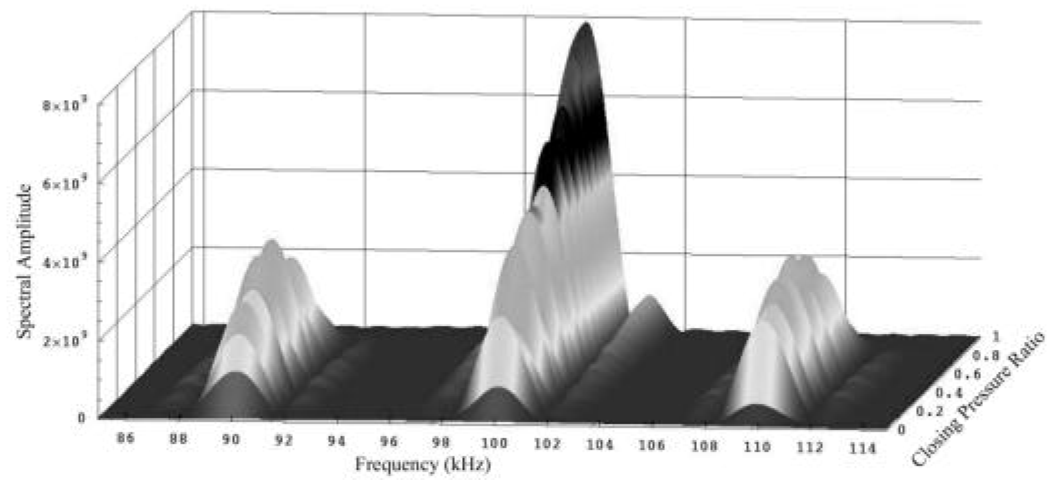

Figure 1: Frequency spectrums of model generated bi-tonal signal with increasing external pressure and no crack separation dependency

Let the effective crack opening stiffness y_{crack} be defined as follows (refer to eqn 8):

$$\Psi_{crack} = \frac{E}{a\pi} \tag{9}$$

We get the following frequency spectrums (Fig. 2) for increasing effective crack opening stiffness y_{crack} (decreasing crack size) when we set the external closing pressure (s_0) to zero.

Of most significance here is the case where the crack size is similar to the expected size in lightly damaged concrete mortar. Initial flaws on the order of 1mm to 2mm in size (Shah [12]) (length not separation) should be considered significant defects in concrete. The following spectrum (Fig. 3) includes two signals, one generated using the model with $a = 2\,mm$ and $s_0 = 0$, and the other recorded during the experimental regime for a lightly damaged mortar specimen with non-interacting micro-cracking (Warnemuende [8]). This time the frequency of the probe signal was set to 86kHz (the same as the measured frequency from the experimentation). The acoustic perturbation in the experimental regime was a tuned impact which covered several kHz in the low frequency range, this, however is rather complex to reproduce, so we use the median frequency of the tuned impact as a 5kHz tone. Notice that in both cases recognizable side-bands are generated.

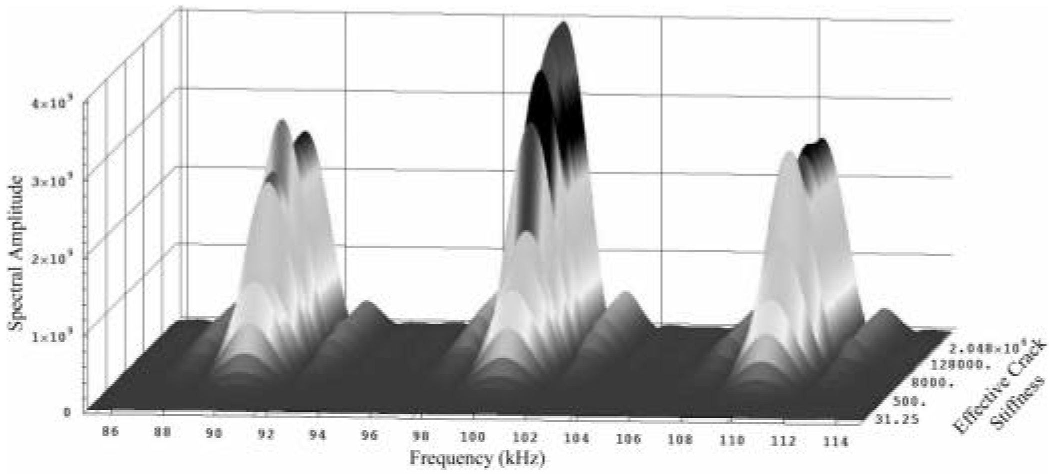

Figure 2: Frequency spectrums of model generated bi-tonal signal with increasing effective crack opening stiffness and no external closing pressure

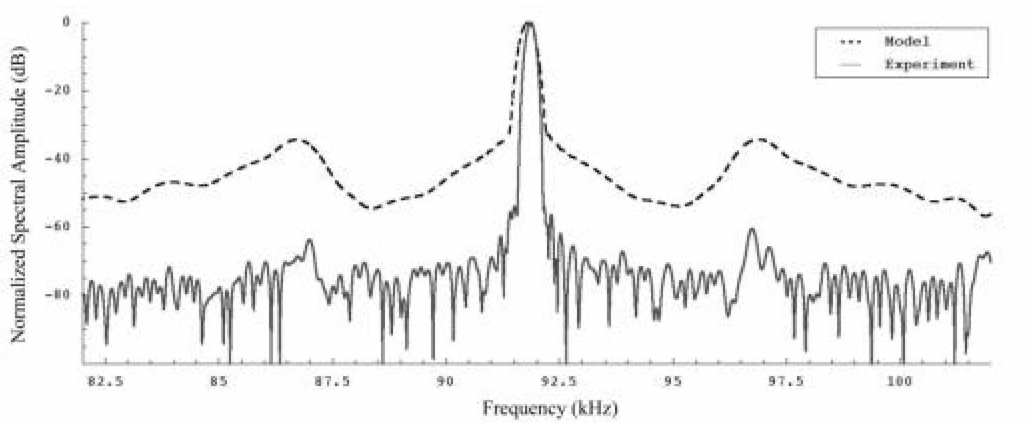

Figure 3: Comparison of the frequency spectrum of a bi-tonal signal generated by the discussed model and by AMA testing

4 CONCLUSION

In the preceding discussions we investigated an experimentally observed phenomenon in the application of actively modulated acousto-ultrasonic non-destructive testing of concrete. Signal energy dispersion was linked to multiple harmonic generation and multiple sideband generation by the intermittent transmission of a probe stress signal across a smooth crack that has been excited by an acoustic perturbation. A model was created for the one-dimensional simplification of the problem and used in direct comparison to experimentally recorded signals and found to behave in a similar, if not identical, manner.

5 REFERENCES

1.Mindess, S. and Young, J.; *Concrete*, 353-379, Prentice-Hall Inc., Englewood Cliffs, 1981

2.Ashby, M.F. and Hallam, S.D.; "The Failure of Brittle Solids Containing Small Cracks Under Compressive Stress States", Acta Metall, **34**, 497-510, 1986

3.Donskoy, D.; Sutin, A.; Ekimov, A.; “Nonlinear Acoustic Interaction on Contact Interfaces and Its Use for Nondestructive Testing”, *NDT&E International*, **34**, 231-238, 2001

4.Van Den Abeele, K.E-A.; Sutin, A.M.; Carmeliet, J.; Johnson, P.A. ; “Micro-damage diagnostics using nonlinear elastic wave spectroscopy”, *NDT&E International*, **34**, 239-248, 2001

5.Buck, O.; Morris, W.L.; Richardson, J.M.; “Acoustic Harmonic Generation at Unbonded Interfaces and Fatigue Cracks”, *Applied Physics Letters*, **33**, 371-373, 1978

6.Sutin, A.M.; Nazarov, V.E; “Nonlinear Acoustic Methods of Crack Diagnostics”, *Radiographics and Quantum Electronics*, **38**, 109-120, 1995

7.Daponte, P.; Maceri, F.; Olivito, R.S.; “Ultrasonic Signal-Processing Techniques for the Measurement of Damage Growth in Structural Materials” *IEEE Transactions on Instruments and Measurement*, **44**, 1003-1008, 1995

8.Warnemuende, K. and Wu, H.-C., “Actively modulated acoustic nondestructive evaluation of concrete”, *Cement and Concrete Research*, Accepted for publishing, 2003

9.Krautkrämer, J.; Krautkrämer, H.; Grabendörfer, W.; Niklas, L.; *Ultrasonic Testing of Materials*, Chapters 1-2, Springer-Verlag New York Inc., Berlin, 1969

10.Richardson, J.M., “Harmonic Generation at an Unbonded Interface – I. Planar Interface Between Semi-Infinite Elastic Media”, *International Journal of Engineering Science*, **17**, 73-85, 1979

11.Broek, D.; *Elementary Engineering Fracture Mechanics*, 75-94, Kluwer Academic Publisers., Dordrecht, 1986

12.Shah, S. P.; Swartz, S. E.; Ouyang, C.; *Fracture Mechanics of Concrete: Applications of Fracture Mechanics to Concrete, Rock, and other Quasi-Brittle Material*s, 88-107, John Wiley & Sons, Inc., NY, 1995