Modelling of anisotropic damage by microcracks: towards a discrete approach within a standard framework.

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ABSTRACT
Modelling anisotropic damage in quasi-brittle materials such as concrete, rocks and ceramics implies representing three-dimensional complex phenomena such as unilateral effect and induced anisotropy. Most of classical macroscopic damage models use a unique second order internal variable to take into account the directionality of the defects. However, the spectral decomposition of this damage variable recently revealed some drawbacks. This paper presents a discrete definition of anisotropic damage, based on nine densities of cracks associated to nine fixed directions (second order tensors). This model avoids some encountered problems while preserving a good representation of unilateral effect. Finally, the prospects of this approach are presented, such as the representation of frictional sliding of microcracks lips and the competition between initial and induced anisotropy.

1 INTRODUCTION

The behaviour of quasi-brittle materials such as concrete and ceramics, related to the appearance and growth of microcracks, remains an important topic of Damage Mechanics. Among the difficulties encountered when modelling these materials, one may mention: (i) induced anisotropy (oriented microcracks), (ii) three-dimensional effects, (iii) crack closure effect (known as "unilateral effect") and its consequences. Many methods have been used to model these phenomena, either micromechanical or 'macroscopic', although frequently based on micromechanical works (see Krajcinovic [1] for an exhaustive synthesis). However certain models were essentially two dimensional (Andrieux and al. [2]) or led to mathematical inconsistencies, in particular when unilateral effect is taken into account as shown by Chaboche [3] (non symmetric stiffness, discontinuity of stress-strain response, non convex reversibility domain …).

During the 90's, some models found a compromise between a thrifty formulation and a physical motivation. One of them was proposed by Dragon and al. [4] and built from an unique damage second order variable **D**. A relevant thermodynamic potential is proposed; its **D**-derivative gives the thermodynamic force associated to damage.

This potential was then enriched in order to describe other phenomena and in particular the unilateral effect by the addition of a closure term (Halm and Dragon [5] and [6]).

However, several problems were detected for this model; for example (i) it is not *standard* in the strict sense defined by Halphen and Nguyen [7]: indeed, the thermodynamic force associated to damage is split into different terms corresponding to positive and negative strains, and only the former part is used in the definition of the elastic domain; (ii) the *convexity* of the reversibility domain depends on restrictions on the values taken by the material parameters and the damage variable; in extreme loading path, the elastic domain may lose its necessary convexity ; (iii) the *free energy may be not unique* in specific configurations (see Cormery and Welemane [8]): indeed, the damage variable **D** is decomposed on its spectral basis $(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3})$: $\mathbf{D} = \sum_{k=1}^{3} \mathrm{D_k} \mathbf{v_k} \otimes \mathbf{v_k}$, where $\mathrm{D_k}$ is the microcracks density normal to $\mathbf{v_k}$ .The damage configuration is then equivalent, in its effect, with a network of 3 orthogonal systems of parallel microcracks. However, this spectral decomposition is not always unique (in the case of isotropy for example), and in these cases the free energy value depends on the chosen decomposition.

These remarks also concern other models in which variables of this type are used (for example Chaboche [9]).

This leads to reconsider anisotropic damage definition in order to unravel these inconveniences while keeping the possibility of describing the different phenomena quoted before. Consequently, this paper presents a discrete damage formulation; some advantages are shown as well as the prospects it offers. This approach is phenomenological while some points are strongly micromechanically motivated; the goal is thus to represent observed macroscopic effects while preserving marked physical features.

2 A DISCRETE APPROACH

The formalism of the thermodynamics of irreversible processes with internal variables is used. A discrete definition for the damage internal variables is proposed below.

2.1 A discrete damage definition

The presented approach consists in substituting p independent couples $(\rho_i, \mathbf{N_i} = \mathbf{n}_i \otimes \mathbf{n})$ for the damage variable **D**, where each orientation $\mathbf{N_i}$ is fixed in physical space and each crack density ρ_i (normal to $\mathbf{n}_i$) in this direction is an internal variable. Two sets are then put forward: the first one of second order tensors $\mathbf{N_i}$ of cracks orientation and the second one of internal variables ρ_i.

The principal difference between this definition and employment of the single tensor **D** lies, in addition to the multiplication of the variables, in the fixed damage directions. Indeed, the single variable **D** has three eigenvectors which evolve during loading; here, these directions are *fixed* and only the densities ρ_i vary: no spectral decomposition is made.

Defining a 'sufficient' number and the specific orientations embodied by $\mathbf{N_i}$ is the next step.

2.2 Choice of $\mathbf{N_i}$

The choice of $\mathbf{N_i}$ should fulfil two objectives; (i) the set of $\mathbf{N_i}$ is postulated to generate the set of tensors of the type $\mathbf{n} \otimes \mathbf{n}$; (ii) an isotropic damage configuration should be represented by p couples $(\rho_0, \mathbf{N_i} = \mathbf{n}_i \otimes \mathbf{n}_i)$, i.e. the same density ρ_0 in each direction $\mathbf{N_i}$ ($\sum_i \rho_0 \mathbf{N_i} = \alpha \rho_0 \mathbf{I}$, where α is a scalar coefficient).

The following set of nine $\mathbf{N_i}$ tensors has been found to meet the above requirements:

$$\begin{aligned}
&\mathbf{N}_1 = \mathbf{e}_1 \otimes \mathbf{e}_1 \,,\; \mathbf{N}_4 = \frac{1}{2}(\mathbf{e}_1 + \mathbf{e}_2) \otimes (\mathbf{e}_1 + \mathbf{e}_2) \,,\; \mathbf{N}_7 = \frac{1}{2}(\mathbf{e}_1 - \mathbf{e}_2) \otimes (\mathbf{e}_1 - \mathbf{e}_2) \,,\\
&\mathbf{N}_2 = \mathbf{e}_2 \otimes \mathbf{e}_2 \,,\; \mathbf{N}_5 = \frac{1}{2}(\mathbf{e}_1 + \mathbf{e}_3) \otimes (\mathbf{e}_1 + \mathbf{e}_3) \,,\; \mathbf{N}_8 = \frac{1}{2}(\mathbf{e}_1 - \mathbf{e}_3) \otimes (\mathbf{e}_1 - \mathbf{e}_3) \,,\\
&\mathbf{N}_3 = \mathbf{e}_3 \otimes \mathbf{e}_3 \,,\; \mathbf{N}_6 = \frac{1}{2}(\mathbf{e}_2 + \mathbf{e}_3) \otimes (\mathbf{e}_2 + \mathbf{e}_3) \,,\; \mathbf{N}_9 = \frac{1}{2}(\mathbf{e}_2 - \mathbf{e}_3) \otimes (\mathbf{e}_2 - \mathbf{e}_3) \,,
\end{aligned} \tag{1}$$

where $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ form an orthonormal basis of the Euclidean space $\Re^3$.

Consequently, this system of nine fixed symmetrical second order tensors, representing nine microcracks directions, associated to nine densities as internal variables is selected to enter the thermodynamic potential and the resulting constitutive relationships.

2.3 Thermodynamic potential and state laws.

The free energy per unit volume w is chosen as thermodynamic potential.

At this stage of model formulation, the material is considered as initially isotropic; microcracks are assumed not to interact and their lips to slide without friction; the effects of residual stresses are not taken into account, contrarily to [4] and [5]. Moreover, the strain-stress response is supposed to be linear at constant damage.

According to the tensorial functions representation theory (Boelher [10]), and considering the hypotheses introduced in the foregoing, the following expression is obtained for w:

$$w(\boldsymbol{\varepsilon},\rho_i) = \underbrace{\frac{\lambda}{2}\mathrm{tr}^2(\boldsymbol{\varepsilon}) + \mu\,\mathrm{tr}(\boldsymbol{\varepsilon}.\boldsymbol{\varepsilon})}_{a} + 2\beta\sum_{i=1}^{9}\rho_i\Big[\underbrace{\mathrm{tr}(\boldsymbol{\varepsilon}.\boldsymbol{\varepsilon}.\mathbf{N_i})}_{b} - \underbrace{\mathrm{tr}^2(\boldsymbol{\varepsilon}.\mathbf{N_i})H(-\mathrm{tr}(\boldsymbol{\varepsilon}.\mathbf{N_i}))}_{c}\Big], \qquad (2)$$

where H is the Heaviside function.

The term (a) represents the free energy without any damage effect; the latter intervenes through the term (b). Note that the term (c) induces recovery effects due to crack closure and can be compared to similar micromechanically obtained expressions (Kachanov [11]): cracks close when the normal strain $\mathrm{tr}(\boldsymbol{\varepsilon}.\mathbf{N_i})$ becomes negative and then the term (c) restores the bulk modulus (this latter recovery condition has been experimentally observed by Sibaï and al. [12]) and the Young's modulus the direction $\mathbf{n}_i$.

The free energy w (2) fulfils the continuity requirements imposed by the theory of multilinear functions (Curnier and al. [13]):even if the elastic properties are discontinuous when passing from open to closed microcracks (and inversely), both free energy and stress strain response remain continuous.

The nine thermodynamic forces corresponding to each density variable are derived from w:

$$F^{\rho_i} = -\frac{\partial w(\boldsymbol{\varepsilon},\rho_i)}{\partial \rho_i} = -2\beta\Big[\mathrm{tr}(\boldsymbol{\varepsilon}.\boldsymbol{\varepsilon}.\mathbf{N_i}) - \mathrm{tr}^2(\boldsymbol{\varepsilon}.\mathbf{N_i})H(-\mathrm{tr}(\boldsymbol{\varepsilon}.\mathbf{N_i}))\Big] \qquad (3)$$

The expression of stress is:

$$\boldsymbol{\sigma} = \frac{\partial w(\boldsymbol{\varepsilon},\rho_i)}{\partial \boldsymbol{\varepsilon}} = \lambda\,\mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon} + 2\beta\sum_{i=1}^{9}\rho_i\big[\boldsymbol{\varepsilon}.\mathbf{N_i} + \mathbf{N_i}.\boldsymbol{\varepsilon} - 2\mathrm{tr}(\boldsymbol{\varepsilon}.\mathbf{N_i})H(-\mathrm{tr}(\boldsymbol{\varepsilon}.\mathbf{N_i}))\mathbf{N_i}\big] \qquad (4)$$

λ,μ,β are material constants to be identified.

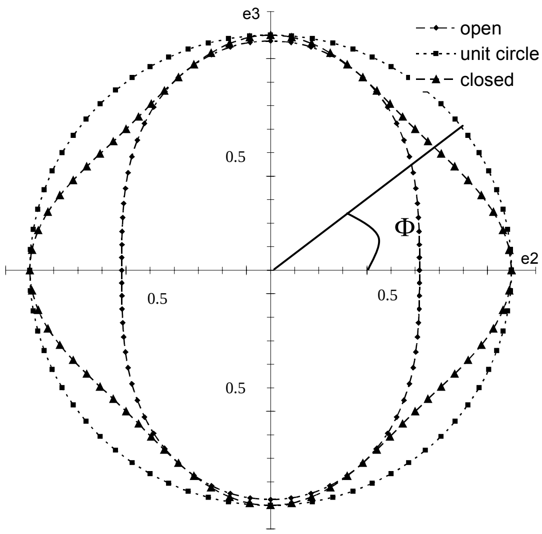
Consider the simple case of a material weakened by a single system of parallel microcracks with unit normal $\mathbf{e_2}$. The influence of such a system on Young's modulus (in the $(\mathbf{e_2},\mathbf{e_3})$ plane) is shown in Figure 1 for open and closed cracks configurations. The initial Young's modulus (without damage effect) is represented by the unit circle: its value is initially identical in all directions (isotropy).

This figure shows that the Young's modulus in the direction normal to parallel microcracks is affected by microdefects change of state. In particular, it recovers its initial value at the microcracks closure.

It is also shown that the restitution is partial in the other directions; in these directions the sliding of cracks lips still affects the material properties.

Finally a system of parallel microcracks has quasi no influence on the Young's modulus in the plane of the defect (here the Young's modulus in direction 3 is quasi not affected).

These remarks are in accordance with the micromechanical considerations (Krajcinovic [14], Kachanov [15]).
In conclusion this potential seems to represent correctly the unilateral effect.



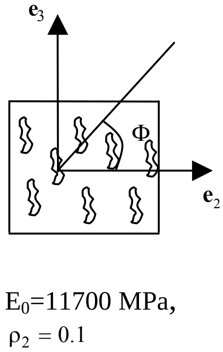


Figure1: Generalized Young's modulus normalized by its initial value E/E_0.

2.4 Evolution laws
The reversibility domain associated with each density is written in the space of thermodynamic forces:

$$f_i(F^{\rho_i}, \rho_i) = F^{\rho_i} - (C_0 + C_1\rho_i) \leq 0, i \in [1,9]. \quad (5)$$

C_o corresponds to the initial elastic limit and C_1 is a 'ductility' parameter.

The evolution of the different variables follows a normality rule:

$$\dot{\rho}_i = \dot{\Lambda}_i \frac{\partial f_i}{\partial F^{\rho_i}} = \dot{\Lambda}_i, i \in [1,9] \quad (6)$$

3 ADVANTAGES OF THE APPROACH

This approach solves allows to remove some drawbacks of anisotropic damage modelling employing a single damage tensor, without losing its advantages.
First (i) the standard character of the model is ensured; indeed Eqns. (3) and (5) show that no decomposition of the thermodynamic forces is made nor that of the strain tensor, contrarily to the previous model by Dragon and Halm [6].
Moreover (ii) the elastic domain defined by Eqn. (5) is convex in the space of the thermodynamic forces, whatever the values taken by the various parameters and variables. Consequently the dissipation (7) remains always positive:

$$\mathcal{D} = \sum_{i=1}^{9} F^{\rho_i} \dot{\rho}_i \geq 0 \quad (7)$$

Indeed, thanks to the normality rules chosen for the densities evolutions, all terms $F^{\rho_i}\dot{\rho}_i$ are unconditionally positive.
Moreover, (iii) it ensures the continuity of the free energy and of the strain-stress response; indeed, Eqn. (2) shows that w is a continuous function of the set of its arguments. In addition, thanks to the evolution laws, and to the selected orthonormal basis of the Euclidean space $\Re^3$, discretized fixed orientation-tensors $\mathbf{N_i}$ as well as the variables ρ_i have a unique perfectly defined value. Previously, in the model proposed by Halm and Dragon [6], w was also a continuous function of the set of its arguments, but the damage variable **D** could have different values depending on its spectral decomposition, that led to the w non uniqueness.
Finally, the closure term (c) with the discrete damage definition appears directly using the tensorial functions representation theory (Boelher [10]), while this term had to be explicitly added to those obtained by this theory when the unique damage variable **D** was employed. Indeed, Halm and Dragon [6] imposed a complementary fourth-order tensor entity assembled with the eigenvalues and eigenvectors of **D** to account for the unilateral effect; a term containing this imposed entity is added to the previous free energy, and leads to modifications of the stiffness matrix. The same effects on stiffness are directly obtained in the framework of the discrete definition (via invariant $\mathrm{tr}^2(\boldsymbol{\varepsilon}.\mathbf{N_i})$), without postulating the existence of a supplementary entity.

4 CONCLUSION AND PROSPECTIVE WORK

A standard model for representing anisotropic damage in quasi-brittle materials is presented. The discrete damage definition, which employs nine microcrack densities associated with nine fixed directions, already permits to represent unilateral effect while avoiding some inconveniences encountered by models using the unique second order tensor variable **D** and its spectral decomposition.
This approach is able to represent the induced anisotropy; indeed, the use of decoupled densities leads to different values for these internal variables. This remark should be confirmed by testing the model for a number of complex loading paths.
The three-dimensional character of the phenomena is correctly represented via the adequate formulation of th thermodynamic potential.
Finally, the first available results show that the unilateral effect is properly represented by the approach.

The enrichment of the model to account for complementary phenomena is the most important prospect. Dissipative friction on microcracks lips leads to the appearance of a blocked energy (Andrieux et al [2]) and residual effects; it will be described by a sliding internal variable, in the spirit of the contribution of Halm and Dragon [16] and [6].
The interaction between initial and induced anisotropy, essential for example for the Ceramic Matrix Composites, may be modelled by adding fabric tensors in the framework of the tensorial functions representation theory (Boelher [10]), as done by Halm and al. [17] and [6]; the fabric tensors $\mathbf{A_i}\,(\mathbf{a_i}\otimes\mathbf{a_i})$ indicate the direction of reinforcement of the composite, and set $\mathbf{N_i}$ indicate damage orientation. Some optimization regarding the set $\mathbf{N_i}$ vs. the set $\mathbf{A_i}$ could allow for further simplification of the advanced theory.
Coupling initial anisotropy to closure effects would certainly be the most significant prospect of this model.

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