NON-LINEAR PHOTOELASTICITY ANALISIS OF FRACTURE MECHANICS PROBLEMS

G. N. Albaut
Novosibirsk State University of Architecture and Civil Engineering (Sibstrin)
630008, Novosibirsk, Russia

ABSTRACT

In this study the problems of fracture mechanics having geometrical and physical nonlinearity were experimentally investigated by non-linear photoelastic methods. The strain changed in the range from -50% till +250% of relative lengthening. Changes in geometry and in thickness of the specimens were taken into consideration. Here, non-compressible birefringent polyurethane rubber was applied. Two schemes of polarizative-optical tests were used: 1) through translucence of rubber specimens; 2) photoelastic coating method for the study of large plastic strains in metals.

The main equations of the method evolved to verify experimental data are presented in this work, and some elastic and plastic problems are studied as examples. The investigation of the rubber plates with cracks and cuts was executed. The cracks transformed into ellipse or circle under deformation. The crack effect upon stress concentration coefficients and strain coefficients at its tip was studied.

The the experimental study of the process of emergence and development of necking under tensioning in plane bars from mild structural steel was studied by the nonlinear photoelastic method in case of large strains (the relative lengthenings accounting for 100%). The polarization-optical study of coating glued on to the specimens and stress-strain distributions in necking was obtained by five of several ways with using of photoelastic experimental data.

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1 INTRODUCTION

Structural design for new rubber-like or composite materials is mathematically hampered especially in view of concentrators or cracks. The same kind of thing applies to metal structural elements working at the prefracture stage under large plastic strains. Some of the assumptions and hypotheses used in structural design must be experimentally tested and verified. The non-linear photoelastic method, Albaut [1, 2], Akhmetzuanov [3], enables to solve similar problems through experiments.

2 SOME ASSUMPTION, DEPENDENCES, METHODIC ASPECTS OF NON-LINEAR PHOTOELASTICITY

2.1. Stress, strain, equations of connection. The deformed form of an object (a photo or an image on a screen) is fixed in investigation of large (final) strains, so data processing is necessary to execute in curvilinear Euler co-ordinates. In elastoplastics (including in coating) strains are measured with the help of degrees of lengthening $\lambda_i=l_i/l_0$ (i=1,2,3), i.e. the ration of length of the deformed element to the undeformed one (λ_1 and λ_2 - in a plane of a sample, λ_3 – along thickness). Transition to any other measures of strains is executed with their help. In Euler co-ordinates, stresses are measured by a tensor of the true stresses referred to the deformed area (σ_1 and σ_2 - principal components in a plane, and σ_3=0 along thickness, a two-dimensional problem), and strains by a tensor of strains of Almanzy-Hamel with principal components $\varepsilon_i=1/2(1-1/\lambda_i^2)$. In Lagrangian co-ordinates accordingly they are measured by a tensor of the conditional stresses referred to the undeformed area, and by a tensor of strains of Green, $\varepsilon_i=1/2(\lambda_i^2-1)$.

Connection equations of stresses and strains in elastoplastics are established with the help of the phenomenological way on the basis of elastic potential Bartenev-Khazanovitch, Albaut [1, 2].

$$\sigma_1 = A(\lambda_1 - \lambda_3);\ \ \sigma_2 = \mathrm{A}(\lambda_2 - \lambda_3). \tag{1}$$

Here A is a constant of Bartenev-Khazanovitch elastic potential. Dependences (1) are well executed for polyurethane SKU-6 and are not suitable for some of foreign rubbers. For more composite cases connection equations with the use of the theory of potentials in variant of Mooney-Rivlin with four constants are selected.

$$\begin{aligned}\sigma_1 &= 4(\mathrm{B}_2 + \mathrm{A}_2(1-2\varepsilon_2))(\varepsilon_1 - \varepsilon_3) + 16(\mathrm{A}_4(1-2\varepsilon_3)^2 - \mathrm{B}_4)(\varepsilon_1 - \varepsilon_3)(1-(\varepsilon_1+\varepsilon_3));\\ \sigma_2 &= 4(\mathrm{B}_2 + \mathrm{A}_2(1-2\varepsilon_1))(\varepsilon_2 - \varepsilon_3) + 16(\mathrm{A}_4(1-2\varepsilon_3)^2 - \mathrm{B}_4)(\varepsilon_2 - \varepsilon_3)(1-(\varepsilon_2+\varepsilon_3)).\end{aligned} \tag{2}$$

Here A_2, B_2, A_4, B_4 are constants of Mooney row.

The non-changeable volume condition of rubber is used as:

$$\lambda_1\lambda_2\lambda_3 = 1. \tag{3}$$

2.2. Optical dependences of the nonlinear photoelasticity. It was established empirically that the general law of photoelasticity for piesooptical rubbers has form (4), where change of thickness was taken into consideration with the help λ_3, and optical-strain dependences (5) or (6) were determined by using elastic potential, i.e. they were obtained by substitution of (1) into (4) or (2) in (4).

$$\delta = C_\sigma \lambda_3 h_0 (\sigma_1 - \sigma_2); \tag{4}$$

$$\delta = C_\varepsilon \lambda_3 h_0 (\lambda_1 - \lambda_2); \tag{5}$$

$$\delta = C_\varepsilon h_0 \lambda_3\ (2(A_2 + B_2\lambda_3^2)(\lambda_1^2 - \lambda_2^2) + 4(A_4 + B_4\lambda_3^4)(\lambda_1^4 - \lambda_2^4)). \tag{6}$$

δ is an optical difference of path, C_σ and C_ε are stress and strain optical constants respectively, h_0 - initial thickness of a sample.

A picture of interference fringe patterns and a field of isoclinic lines (angles of inclination of principal stresses) are obtained as a result of polarization-optical experiment which further are used for the division of stresses and strains.

2.3. Methods of stress and strain division in nonlinear photoelasticiyty.

2.3.1. Method of measuring of cross strain. The measuring of coating thickness or plane plate thickness made of metal can be performed in different ways from the technical point of view. By adding dependence (3) and (5) or (6) to the value λ_3, one can solve the equations' system as far as λ_1, λ_2, λ_3 are concerned. After that the stresses in rubber or natural structure are determined by the known strains.

2.3.2. Method of coating's cutting. One can measure value δ in the one-piece coating, then cut in, which is easily performed technically for rubber, and measure δ_x (axis x coincides with notch's direction). This will enable to find the lengthening degree λ_x along the notch by means of tested graph. Now, by using (3), (5) or (6) and λ_x one can determine all the strains' components and then those of stresses in a point. λ_x connects with λ_1 and λ_2 in Eiler co-ordinate with the help of dependence:

$$\frac{1}{\lambda_x^2} = \frac{1}{2}\left(\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}\right) + \frac{1}{2}\left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}\right)\cos 2\alpha. \tag{7}$$

2.3.3. Method of numerical integration of equilibrium equations. The differential equilibrium equations for the plane problem in the coordinates' system XY of the deformed body (Eiler) are as follows:

$$\frac{\partial(\lambda_3\sigma_x)}{\partial x} + \frac{\partial(\lambda_3\tau_{xy})}{\partial y} = 0;\quad \frac{\partial(\lambda_3\tau_{yx})}{\partial x} + \frac{\partial(\lambda_3\sigma_y)}{\partial y} = 0. \tag{8}$$

The making use of the data of polarization-optical study the conditional tangent $\lambda_3\tau_x$ and the differences of normal conditional stresses $\lambda_3(\sigma_x\text{-}\sigma_y)$ are being determined. Further, by means of the numerical integration of equilibrium equations (8) the separate values of conditional stresses in the coordinates' system xy are calculated, and after that so do the principal conditional stresses $\lambda_3\sigma_1$ and $\lambda_3\sigma_2$. By adding two differences (1) or (3) to them, we obtain a complete system of equations for determining all the stresses' and strains' components σ_1, σ_2, λ_1, λ_2, λ_3. The solution is reduced to determining of radicals λ_3 of equation

$$\lambda_3^4 + \lambda_3^2(a+b) - \lambda_3 + ab = 0\,, \tag{9}$$

where $a=\lambda_3\sigma_1/A$; $в=\lambda_3\sigma_2/A$. After λ_3 has been calculated the remaining unknowns are easily expressed in an apparent aspect. One can find the examples of the division of stresses in plates with a crack by a given method in another abstract of the present proceedings [4]. The program for the personal computer realizing the whole process of calculating has been produced for deciphering the data by means of this scheme.

3 STRETCHING OF A RUBBER STRIP WITH SHARP SIDE NOTCHES

Figure 1 gives the results of the study of change of concentration coefficients of true K_σ and conditional K_σ^L stresses and strains K_ε (relative lengthening) in a rubber strip with side notches in depending on nominal strains. One can observe bluntness and disclosing of crack-notches in interference fringe patterns pictures. When analyzing the graphs it is possible to note that λ_3 as the strains increase values of K_σ, K_σ^L, K_ε smoothly decrease, approaching some asymptotic values.

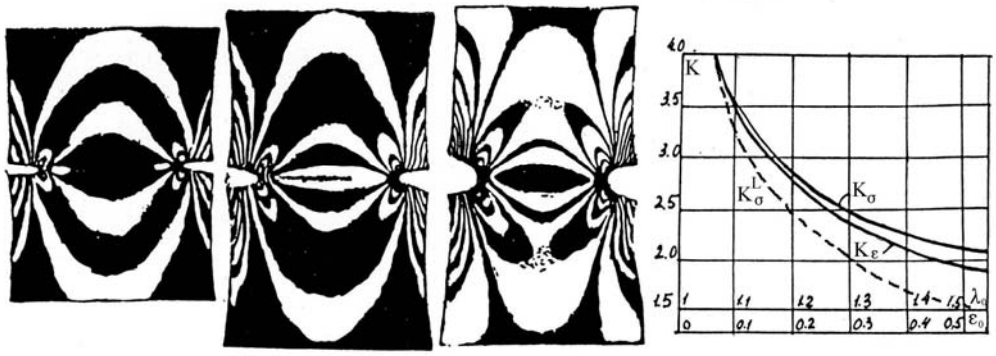


Figure 1: The change of K_σ, K_σ^L, K_ε under the tension of the strip with side cuts.

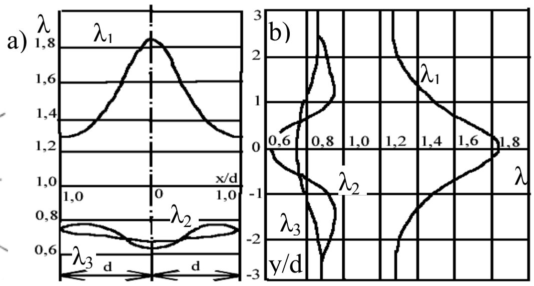
$\alpha=45^0$ $\alpha=55^0$ $\alpha=55^0$ $\alpha=57^0$ $\alpha=58^0$ $\alpha=62^0$

Figure 2: The pictures of interference fringe patterns in the coating; there is a destroyed sample on the right.

4 STRAINS IN A NECKING OF A THIN STEEL BAR

4.1. Experimental results. The study of strains in necking is executed by a method of photoelastic coating made of sensitive optical polyurethane SKU-6 with thickness 1÷2mm.

Pictures of interference fringe patterns under step loading are shown in Figure 2 and fields of isoclinic lines of strains (above) and their increments (below) are given in Figure 3. The sliding



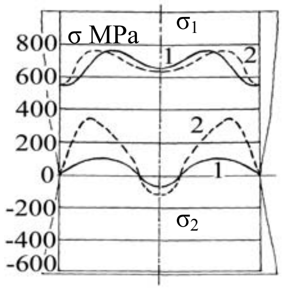


Figure 3: Isoclinics. Figure 4: The epures on the necking symmetry axis. Figure 5: σ_1 and σ_2.

lines are fixed in the first photo, Figure 2. The next photos are made after regluing of a new coating on the specimen, which was stretched through the zone of yielding and hardening, and consequently, only the strains localized in necking are reflected on them. Angles of necking inclination from the vertical increase from 55^0 in the origin of its formation up to 62^0 at fracture.

4.2. Division of strains in longitudinal and cross section of necking at a stage of prefracture was executed by two methods described in item 1.4 of the present paper, which gave practically identical results.

The relevant epures $\lambda_{1,}$ $\lambda_{2,}$ λ_3 are drawn in Figure 4. Maximum lengthening in the center of the necking reach 85%, the greatest narrowing is 37% in the same place. It is necessary to note that cross section strains $\lambda_{2,}$ λ_3 are not equal to each other. Hence, the hypothesis about their equality decided for example in the work Bridgman [5] is not proved by experiment.

5 DETERMINATION OF STRSSES IN NECKING

There is no precise test solution of a problem concerning a stress distribution in plane necking. In the present paper they are determined by five different methods, Alexsandrov [3], with the use of objective data of polarization-optical experiment.

5.1. Determination of stresses using known strains can be executed employing epures in Figure 4 and the diagram of tensioning with the help of equations of the deformation plasticity theory (10), or yield theory (11). Velocities of logarithmic strains and their intensity $\dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3$ and $\dot{\varepsilon}_i$ are substituted by the relevant increments.

$$\sigma_1 = \frac{2}{3}\frac{\left(\varepsilon_1 - \varepsilon_3\right)}{\varepsilon_i}\sigma_i; \qquad \sigma_2 = \frac{2}{3}\frac{\left(\varepsilon_2 - \varepsilon_3\right)}{\varepsilon_i}\sigma_i; \tag{10}$$

$$\sigma_1 = \frac{2}{3}\frac{\left(\dot{\varepsilon}_2 - \dot{\varepsilon}_3\right)}{\dot{\varepsilon}_i}\sigma_i; \qquad \sigma_2 = \frac{2}{3}\frac{\left(\dot{\varepsilon}_2 - \dot{\varepsilon}_3\right)}{\dot{\varepsilon}_i}\sigma_i. \tag{11}$$

Here σ_i and ε_i - intensities of the true stresses and logarithmic strains. The epures of stresses in cross section are given in Figure 5 (continuous lines correspond to an equation (10), and dotted - to an equation (11)).

5.2. A numerical integration of differential equilibrium equations. This way of determining of stresses is the most independent one from equations and hypotheses of plasticity theories. Only one hypothesis here is used here: either about co-axial stresses and strains (the deformation theory of plasticity), or about co-axial stresses and velocities of strains (the theory of a plastic flow), an isoclinic lines are in Figure 3. Stresses are defined with the help of a numerical integration simultaneously by two equilibrium equations (8) along a field of a plane steel specimen considering of change λ_3.

Boundary conditions are defined by experimental data in coating and the diagram of tension. The obtained fields of stresses σ_1 and σ_2 are given in Figure 6a and 6b, epures in cross section in Figure

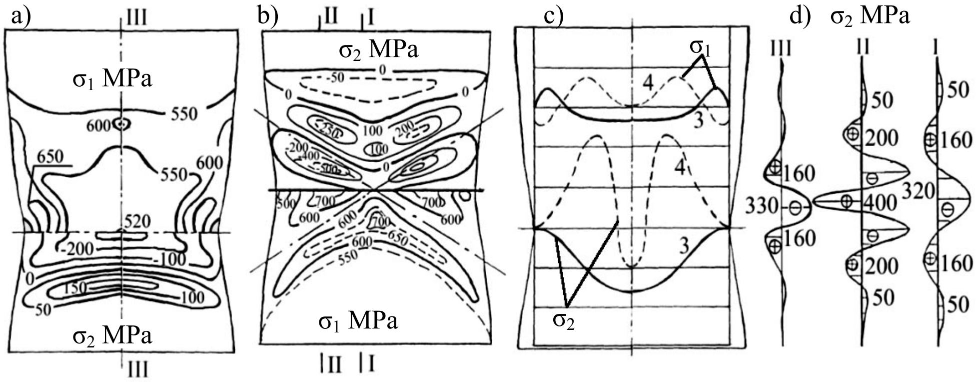


Figure 6: Stresses σ_1 и σ_2 obtained on the basis of co-axial hypothesis.

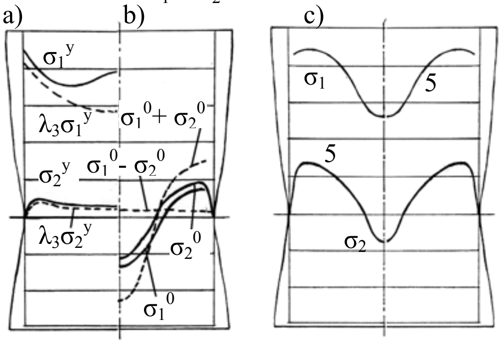


Figure 7: Determination stresses σ_1 and σ_2 by the unloading method.

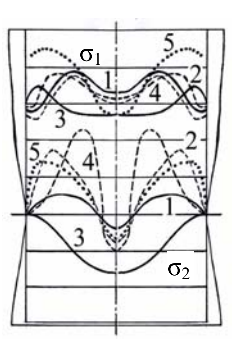


Figure 8: Epures σ_1 and σ_2 obtained by the different methods.

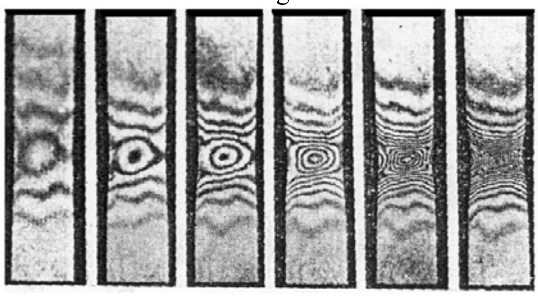


Figure 9: The fringe pictures in the developing necking.

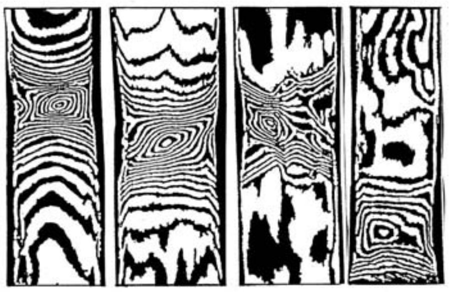


Figure 10: The fringe pictures in the samples with the different kinds of necking.

6c (continuous curves from Figure 6a, dotted from Figure 6b). Epures of stresses in the shown longitudinal sections are given in Figure 6d.

5.3. Method of unloading. In this case complete stresses are obtained by summing: 1) stresses in elastically working model made of sensitive optical organic glass which is similar to the metal deformed specimen (index У in Figure 7a); 2) systems of residual stresses in the unloaded plastically deformed steel specimen (an index O in Figure 7b). The last stresses are determined by optical methods after its cutting into a small strips (an elastic unloading). The complete stresses determined by the method of unloading are given in Figure 7c.

5.4. The analysis of results. The results of stresses determination in necking by five ways are presented in Figure 5÷7, and all graphs on one scheme are given in Figure 8. Theobjective polarization-optical data with the use of various additional conditions: equations or hypotheses of different plasticity theories, additional experimental measures etc lie in the basis of their determination. It is necessary to note some peculiarities in stress distribution.

- Epures σ_1 are similar and differ by value very little.
- The stress σ_2 in all cases is two-signed along a necking field that provides an equilibrium of all forces in vertical section of a specimen if they are projected on a horizontal axis.
- Maximums of epures σ_1 and σ_2 in cross section of the necking are displaced in both sides from its center approximately by a quarter of width of cross-section, except for σ_2 only.

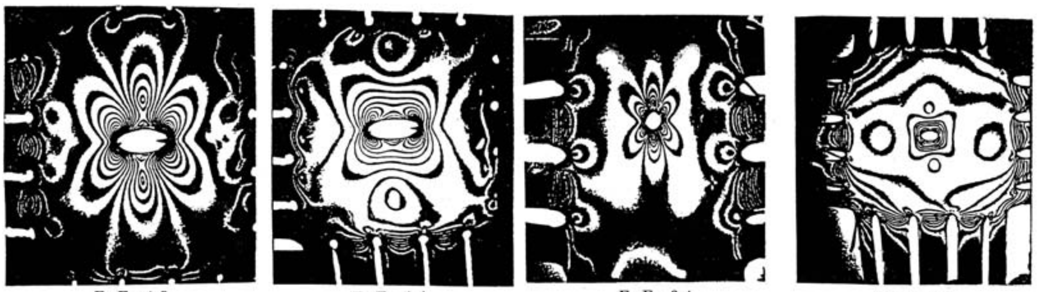


Figure 11: Picture of fringe patterns in rubber plates with crack.

CONCLUSION

The additional information about strains in the plane necking obtained by a method of photoelastic coating is given in Figure 9 and 10. The pictures of interference fringe patterns in the process of necking development in one of steel bar are shown in Figure 9 and the fringe patterns reflecting the complete integral information about strains from the beginning of loading of the bars up to the formation of neckings in four different steel specimens are given in Figure 10.

Several pictures of fringe patterns are given in Figure 11 with information about deformation of thin rubber plates with sharp crack under tensile loading. The cracks transformed into ellipse or circle.

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