FRACTURE CRITERION FOR BRITTLE ROCK-LIKE MATERIALS

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ABSTRACT

The aim of the paper is to present experimental and theoretical study of fracture of brittle materials. To this end new and more complete experimental data on deformability and fracture of brick subjected to tri-axial state of stress were discussed. Such experimental data are necessary to formulate theoretical models capable to describe the mechanical behaviour of concrete, cementitious composites, ceramics and rocks. The program of experiments consisted of two different tests performed under tri-axial loading and also of uni-axial compression that supplied preliminary data necessary to calibrate the material. The second objective of this study is to present the potentialities of own phenomenological model, based on continuum damage mechanics and on theory of tensor function representation, that can be used to describe the response of brittle rock-like material subjected to multi-axial state of stress. Comparison of experimental results for stresses at fracture obtained for tri-axial compression of brick with respective theoretical predictions showed satisfactory agreement. It means that phenomenological model used is accurate enough to describe the phenomena observed in tri-axial loading of brittle rock-like materials.

1 INTRODUCTION

The progress in mechanics of solids and structures requires mutually interrelated extensive theoretical and experimental studies of mechanical properties of structural materials necessary to formulate theoretical models capable to describe the physical processes observed in solids subjected to multi-axial state of stress. Strong motivation for such experiments exists in the case of brittle rock-like materials because of complexity of the phenomena that affect their mechanical response. Some rather incomplete experimental data for rocks subjected to confined pressure presented by Cristescu and Hunche [6], Derski et al. [7] and Goodman [8] and similar but very limited results shown for concrete by Chen [5] and Neville [15] could give only preliminary information on behaviour of brittle rock-like materials tested under tri-axial state of stress. Wide utilisation of ceramics and cementitious composites requires deeper studies of such phenomena like load or deformation induced anisotropy that develops in loading process due to internal oriented damage growth. Simultaneously, new approach based on the methods of continuum damage mechanics was used to formulate some phenomenological models capable to describe the mechanical behaviour of brittle rock-like materials in presence of damage induced anisotropy. Such theoretical descriptions presented by Chaboche et al. [4], Litewka et al. [11], Murakami and Kamiya [14] and Halm and Dragon [9] are based on limited experimental data, particularly for tri-axial state of stress and were verified for some specific cases of loading only. To obtain more realistic theoretical description of overall material response the further extensive experimental studies are needed.

The aim of this note is to supply new and more complete experimental data on mechanical response of brittle materials subjected to tri-axial state of stress. To obtain more information on deformability, damage growth and fracture of these materials two different tri-axial tests were used. The second objective of the study presented here is to show the potentialities of own phenomenological model (Litewka and Dębiński [12]), based on continuum damage mechanics and on theory of tensor function representation, that can be used to describe the response of brittle rock-like material subjected to tri-axial state of stress.

2 THEORETICAL BACKGROUND

Theoretical description used in this paper is based on the methods of the damage mechanics and on assumption of tensorial nature of the material damage. That is why the symmetric second rank damage tensor was used as a variable responsible for deterioration of the material internal structure. Explicit form of the relevant constitutive equations was found by employing the theory of tensor function representations as applied in solid mechanics by Betten [1] and Boehler [2]. Some results of possible application of this mathematical approach to describe a non-linear behaviour of a concrete and rocks, including the experimental verification, were presented by Litewka et al. [11], Litewka and Dębiński [12] and Litewka and Szojda [13]. The theoretical model used here consists of the stress strain relations expressed in the form of the following tensor function

$$\varepsilon_{ij} = -\frac{\nu_0}{E_0}\delta_{ij}\sigma_{kk} + \frac{1+\nu_0}{E_0}\sigma_{ij} + (\delta_{ij}D_{kl}\sigma_{kl} + D_{ij}\sigma_{kk}) + 2D(\sigma_{ik}D_{kj} + D_{ik}\sigma_{kj}) \qquad (1)$$

that describes the anisotropic elastic non-linear response of the damaged material. Equation (1) contains the strain tensor ε_{ij}, stress tensor σ_{ij}, Kronecker delta δ_{ij}, two constants C and D to be determined experimentally, initial Young modulus E_0 and Poisson ratio ν_0 and the second order symmetric damage effect tensor D_{ij} responsible for the current state of internal structure of the material. This material state variable was defined by Litewka [10] to substitute the classical damage tensor Ω_{ij}, which is not sufficient to describe the strength and stiffness reduction of the damaged material. The relation between Ω_{ij} and D_{ij} is given in the form

$$D_i = \frac{\Omega_i}{1-\Omega_i}\ , \quad i = 1,\ 2,\ 3 \qquad (2)$$

were Ω_1, Ω_2 and Ω_3 are the principal values of the damage tensor Ω_{ij} and D_1, D_2 and D_3 are the principal components of the damage effect tensor D_{ij}.

The assumed damage evolution equation has a form of the following tensor function representation

$$\Omega_{ij} = As_{kl}s_{kl}\left(1+\frac{227\det\sigma_{pq}}{200\left|\det\sigma_{rs}\right|+\left|\sigma_{ll}^{3}\right|}\right)^{F}\delta_{ij} + B\sqrt{\sigma_{kl}\sigma_{kl}}\left(1+\frac{227\det\sigma_{pq}}{200\left|\det\sigma_{rs}\right|+\left|\sigma_{ll}\right|}\right)^{F}\sigma_{ij} \qquad (3)$$

formulated in [13], where s_{kl} is the stress deviator and A, B, F are three material parameters to be determined experimentally. Five constants A, B, C, D and F seen in eqns (1) and (3) can be identified by using various methods discussed in [11, 12]. The numerical values of these constants shown in Table 1 were obtained for brick by employing the specific method explained in [12].

Convenient method of theoretical or experimental study of material fracture is to construct the respective limit surface. Possible form of such a surface that describes the combinations of principal components σ_1, σ_2 and σ_3 of the stress tensor applied at material fracture is shown in

Table 1: Experimental data and constants used in analysis of tri-axial state of stress of brick

E_0	ν_0	f_c	A	B	C	D	F
MPa	-	MPa	MPa^{-2}	MPa^{-2}	MPa^{-1}	MPa^{-1}	-
2420	0.105	-11.49	1064×10^{-5}	100.0×10^{-5}	-1.500×10^{-5}	3.508×10^{-5}	0.6300

Figure 1. The analysis shown in this paper concerns to two specific cases of the tri-axial loading defined by the stress tensor

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} = p & 0 & 0 \\ 0 & \sigma_{22} = p & 0 \\ 0 & 0 & \sigma_{33} = \sigma_V + p \end{bmatrix} \tag{4}$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} = \sigma_H + p & 0 & 0 \\ 0 & \sigma_{22} = \sigma_H + p & 0 \\ 0 & 0 & \sigma_{33} = p \end{bmatrix} \tag{5}$$

and referred to as State I and State II, respectively. The loading paths for these two states of tri-axial compression shown in Figure 1 consisted of two stages. The Stage 1 was the same in both cases and consisted in a monotonic increase of hydrostatic pressure up to prescribed value p. In the Stage 2 of the first tri-axial state of stress (State I) the vertical normal stress σ_V was increased up to material failure that occurs for $\sigma_{3f} = p + \sigma_V$. The second tri-axial state of stress (State II) is a combination of hydrostatic pressure p and uniform bi-axial compression. In the Stage 2 of this case

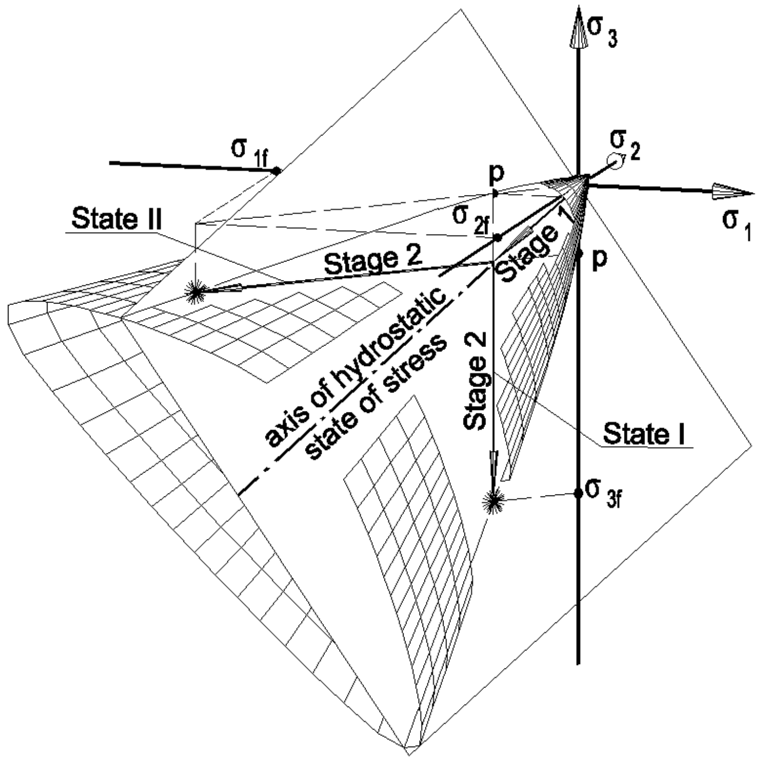

Figure 1: Limit surface at material fracture and loading paths: ∗ point corresponding to material fracture.

of loading two identical horizontal components σ_H of uniform bi-axial state of stress were increased simultaneously up to material failure that corresponds to $\sigma_{1f} = \sigma_{2f} = p + \sigma_H$.

3 TRI-AXIAL MATERIAL TESTS

To obtain more complete information on deformability and fracture of brittle rock-like materials at least two different tri-axial tests and various combinations of the stress tensor components within each test are needed. That is why the classical procedure, in which only confined compression (State I) is used, was improved by one more tri-axial test referred to as State II explained in Fig. 1. The details of the experimental procedure and technique employed can be found in [13]. The experimental results presented here were obtained for specimens of brick subjected to uni-axial compression and to tri-axial loading defined by eqns (4) and (5) and explained in Figure 1. The objective of the test performed under uni-axial compression was to determine the initial Young modulus E_0 and Poisson ratio ν_0 as well as to measure the uni-axial compressive strength f_c for material tested. The numerical values of the above mentioned constants together with the other material parameters included in the theoretical model adopted in this paper are shown in Table 1. The objective of the tests under tri-axial state of stress was to determine the respective stress-strain curves and to measure the stresses at material fracture for prescribed loading programs.

Theoretical model presented in this paper can also be used to determine the maximum stresses that can be sustained by the material subjected to multi- axial state of stress. According to the rules of the damage mechanics the material looses its continuity when at least one of the principal components Ω_1, Ω_2 or Ω_3 of the damage tensor Ω_{ij} determined from eqn (3) reaches the limit value equal to unity. In such a case the stiffness of the material described by eqns (1) and (2) decreases to zero what corresponds to practically horizontal part of the theoretical stress-strain curves. It means that eqn (3) can be used to formulate the fracture criterion for material tested. The

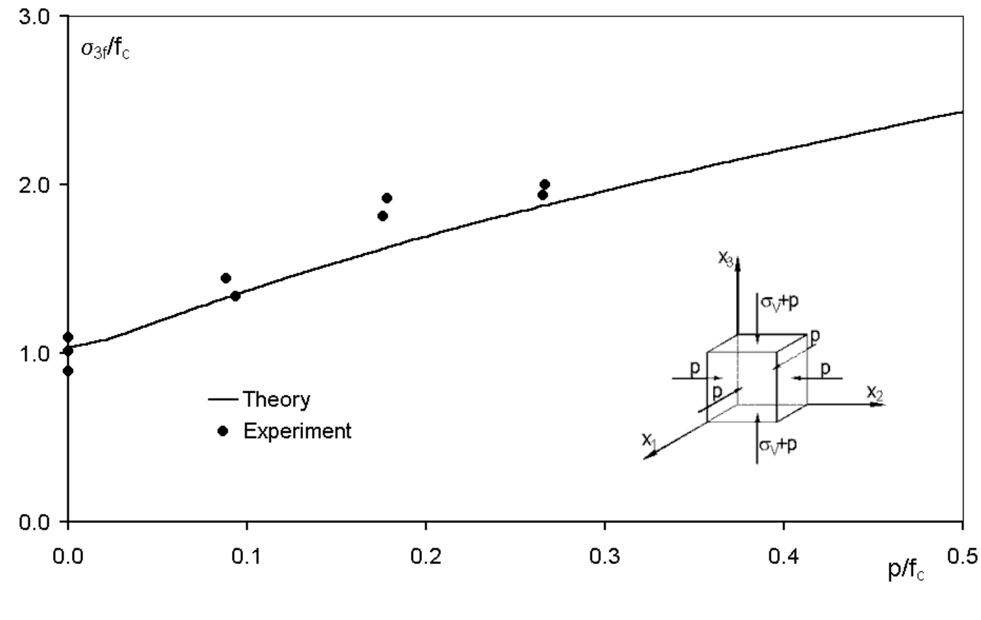

Figure 2: Comparison of experimentally determined stresses at material fracture σ_{3f} with theoretical predictions for tri-axial State I.

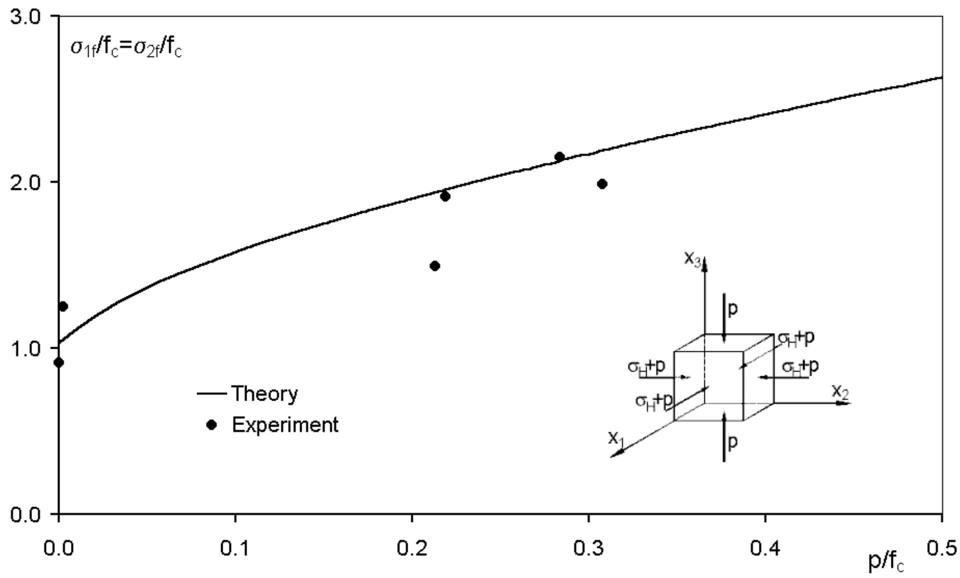

Figure 3: Comparison of experimentally determined stresses at material fracture $\sigma_{1f} = \sigma_{2f}$ with theoretical predictions for tri-axial State II.

specific form of this criterion was obtained for State I and State II from eqn (3) expressed in terms of the stress tensors components shown in eqns (4) and (5). The respective relations have a form

$$\Omega_1 = \Omega_2 = \left(\frac{2}{3}A\sigma_V^2 + Bp\sqrt{\sigma_V^2 + 2\sigma_V p + 3p^2}\right)\left[1 + \frac{227(\sigma_V + p)p^2}{200\left|(\sigma_V + p)p^2\right| + \left|(\sigma_V + 3p)^3\right|}\right]^F = 1 \qquad (6)$$

for State I and

$$\Omega_3 = \left(\frac{2}{3}A\sigma_H^2 + Bp\sqrt{2\sigma_H^2 + 4\sigma_H p + 3p^2}\right)\left[1 + \frac{227(\sigma_H + p)^2 p}{200\left|(\sigma_H + p)^2 p\right| + \left|(2\sigma_H + 3p)^3\right|}\right]^F = 1 \qquad (7)$$

for State II. Comparison of the stresses at material fracture determined experimentally with the theoretical prediction obtained from eqns (6) and (7) is shown in Figures 2 and 3. The maximum difference between theoretical and experimental results seen in Figures 2 and 3 for some specimens is equal approximately to 20%. The discrepancies obtained for other specimens are much smaller and the average difference determined for all the specimens tested does not exceed 0.3%. This relatively good agreement shows that overall accuracy of the theoretical model used in this paper is satisfactory.

4 CONCLUSIONS

Theoretical model used in this paper made it possible to describe experimentally determined mechanical properties of brick. It was found that experimental data obtained for the stresses at material fracture show fairly good agreement with theoretical predictions. Increasing compressive strength of brittle rock-like materials detected earlier in experiments for specimens subjected to confined axial compression was also observed for brick in new tri-axial test used here. This phenomenon can be explained theoretically within the mathematical model proposed. Thus, the experimental technique adopted and phenomenological model used in this paper proved to be accurate enough to obtain information necessary to understand the phenomena observed in tri-axial loading of brittle rock-like materials.

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