CONCEPT OF CONTINUOUS DAMAGE DEACTIVATION IN MODELLING OF LOW CYCLE FATIUGUE

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ABSTRACT

The aim of presented paper is an adaptation of continuous damage deactivation in modelling of low cycle fatigue of AISI 316L stainless steel. On basis of kinetic law of damage evolution, two models are compared: sampling one of discontinuous damage deactivation and the one of continuous damage deactivation. The problem is described by a system of original differential equations derived for a case of uniaxial stress preceding the neck formation as well as for a case of three dimensional state of stress accompanying the strain localisation. Detail quantitative and qualitative analysis of obtained solutions confirms necessity and correctness of an application of continuous damage deactivation.

1 INTRODUCTION

When a material is subjected to a cyclic loading at high values of stress or strain, damage develops together with cyclic plastic strain. If the material is strain loaded, the damage induces a drop of the stress amplitude and the decrease of the elasticity modulus. In order to understand this phenomenon often the concept developed in damage mechanics called damage deactivation is used. In order to demonstrate damage deactivation let us consider the simplified case of uniaxial stress and damage described by scalar parameter. If stress is possitive then microdefects are active and consequently effective modulus of elasticity takes the form

$$\widetilde{E}^{+} = E(1-D) \qquad (1)$$

Under compression microdefects close, therefore the effective modulus of elasticity is given by another formula

$$\widetilde{E}^{-} = E(1-Dh) \qquad (2)$$

where the crack closure parameter $h \in [h_c, 1]$ controls the microdefects closure. Application of this model for description of cyclic fatigue leads to discontinuous bilinear path characterised by different modulae of elasticity, $\widetilde{E}^{+}$ for tension and $\widetilde{E}^{-}$ for compression, respectively (see Fig.1a).

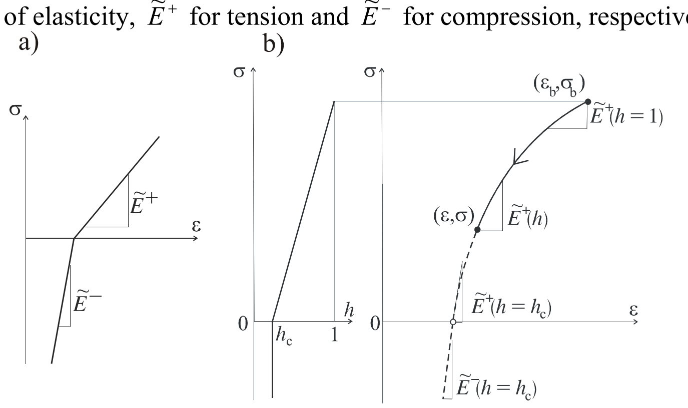

Fig.1 Concept of discontinuous a) and continuous b) damage deactivation

The simplest method for overcoming the discontinuity discussed above is to replace of crack closure parameter by smooth stress dependent function [2, 4]

$$h(\sigma) = h_{\mathrm{c}} + (1 - h_{\mathrm{c}})\frac{\langle\sigma\rangle}{\sigma_{\mathrm{b}}} \tag{3}$$

The use of a continuous function can be interpreted as a physical manifestation of the observation that all microdefects do not close instantaneously but gradually (Fig. 1b). Other words, from a micromechanical point of view this implies that the roughness of microcrack surfaces cause the deactivation to not occur instantaneously. It is worth to notice that the concept of continuous crack deactivation (3) defines a new hypoelastic material being an extension of the classical elastic one.

2 ELASTOPLASTICITY COUPLED WITH DAMAGE

2.1 Two-scalar variable effective stress

Analysis of subsequent strain-stress loops reveals existence of two softening processes corresponding to damage growth [6]. In the first period (till 2000 cycles) damage growth manifested by transcristalline microcracks is isotropic and exhibits decreasing tendency. For higher number of cycles (form 2000 to 2400 cycles) another unilateral damage process associated with cavities on the grain boundaries throughout the volume is activated. The drop of stress amplitude and elasticity modulus is faster for tensile stress than for compressive one. Hence, existence of two separate damage mechanisms requires application of two independent scalar damage variables: the first isotropic damage variable D_{s} that acts on the deviatoric stress and the second unilateral damage variable D_{v} that acts on the volumetric stress [6]

$$\tilde{\sigma} = \frac{\sigma_{\mathrm{s}}}{1 - D_{\mathrm{s}}} + \frac{\sigma_{\mathrm{v}}}{1 - D_{\mathrm{v}} h} \tag{4}$$

The above definition of the effective stress turns out to be the one dimensional simplification of the general damage effect tensor of two-scalar variables. Hence, from view of point of thermodynamic the positive definiteness of the quadratic form associated with the strain energy must be guaranteed which is equivalent the condition $(1 - D_{\mathrm{s}})/(1 - D_{\mathrm{v}} h) > \sqrt{(1 - 2\nu)/(1 + \nu)}$ analysed by authors [3].

2.2 Elastoplastic constitutive law

The strain-stress relation is approximated by the nonlinear Ylinen's model [9] with influence of damage taken into account by replacement of nominal" stress σ by the effective stress $\tilde{\sigma}$. The differentiatial form of the damage modified Ylinen approximation for loading cycles takes, therefore, the following form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon} = \begin{cases} \tilde{E}^{\pm} \dfrac{\tilde{\sigma}_0^{\pm} - |\sigma|}{\tilde{\sigma}_0^{\pm} - c|\sigma|} & \text{loading} \\ \tilde{E}^{\pm} & \text{unloading} \end{cases} \tag{5}$$

where c denotes a material constants ($0 \le c \le 1$), the effective damage modulus of elasticity and the effective asymptotic yield stress of Ylinen's model become functions of damage

$$\frac{\tilde{E}^{\pm}}{E} = \frac{\tilde{\sigma}_0^{\pm}}{\sigma_0} = \frac{(1 - D_{\mathrm{s}})(1 - D_{\mathrm{v}} h)}{1 - \frac{2}{3} D_{\mathrm{v}} h - \frac{1}{3} D_{\mathrm{s}}} \tag{6}$$

2.3 Damage evolution law

The thermodynamically based theory taking advantage of assumption that the damage is related to the accumulated plastic strain is due to Lemaitre and Chaboche [6, 7]. It is assumed that the potential of dissipation F is a sum of two parts, the first referring to Mises yield condition f, where damage coupling acts only by the effective stress, and the other F_D associated with the kinetic law of damage evolution. In case of uniaxial state of stress the dissipation potential takes the following form

$$\begin{aligned} F &= f(\sigma, X, R, D) + F_\mathrm{D}(Y, D) \\ f &= |\tilde{\sigma} - X| - R - \sigma_\mathrm{y} \end{aligned} \tag{7}$$

where the back stress X may be neglected as constant amplitude of strain causes that the strain-hardening is saturated at first cycle whereas the isotropic hardening variable R is included into Ylinen's model. Application of the classical formalism of associated plasticity leads to the uniaxial plastic strain

$$\mathrm{d}\varepsilon^\mathrm{p} = \frac{\partial F}{\partial \sigma}\mathrm{d}\lambda = \frac{\partial F}{\partial \sigma_\mathrm{s}}\mathrm{d}\lambda = \mathrm{sign}(\sigma_\mathrm{s})\frac{\mathrm{d}\lambda}{1 - D_\mathrm{s}} \tag{8}$$

and the accumulated plastic strain

$$\mathrm{d}p = \left|\mathrm{d}\varepsilon^\mathrm{p}\right| = \frac{\mathrm{d}\lambda}{1 - D_\mathrm{s}} \tag{9}$$

The form of damage dissipation potential is assumed as follows

$$F_\mathrm{D} = \frac{Y_\mathrm{s}^2}{2S_\mathrm{s}(1 - D_\mathrm{s})} + \frac{Y_\mathrm{v}^2}{2S_\mathrm{v}(1 - D_\mathrm{v})} \tag{10}$$

where the strain energy density release rate associated to the unilateral damage is

$$Y = Y_\mathrm{s} + Y_\mathrm{v} = \frac{\frac{2}{3}\sigma^2}{2E(1 - D_\mathrm{s})^2} + \frac{\frac{1}{3}\sigma^2}{2E(1 - D_\mathrm{v}h)^2} \tag{11}$$

Since

$$\mathrm{d}D_\mathrm{s} = \frac{\partial F_\mathrm{D}}{\partial Y_\mathrm{s}}\mathrm{d}\lambda, \quad \mathrm{d}D_\mathrm{v} = \frac{\partial F_\mathrm{D}}{\partial Y_\mathrm{v}}\mathrm{d}\lambda \tag{12}$$

hence taking advantage of (8), final form of the kinetic damage law in one dimension is written as

$$\frac{\mathrm{d}D_\mathrm{s}}{\mathrm{d}p} = \frac{\frac{2}{3}\sigma^2}{2ES_\mathrm{s}(1 - D_\mathrm{s})^2}g(p), \quad \frac{\mathrm{d}D_\mathrm{v}}{\mathrm{d}p} = \frac{\frac{1}{3}\sigma^2}{2ES_\mathrm{v}(1 - D_\mathrm{v}h)^2}\left(\frac{1 - D_\mathrm{s}}{1 - D_\mathrm{v}h}\right)H(p - p_\mathrm{D}) \tag{13}$$

According to derived evolution law the damage is related to irreversible strain, proportional to accumulated plastic strain. Moreover evolution of isotropic damage D_s is additionally controlled by the exponent function $g(p) = a\exp(-bp)$ in order to take into account its decreasing character. Also unilateral damage D_v is activated when accumulated plastic strain p reaches damage threshold plastic strain p_D.

2.4 Approximate description of damage evolution after strain localisation

The proper description of the stage after strain localisation requires analysis of three dimensional state of stress. The simplest approximate solution of this problem is presented in [1]. Assuming material incompressibility, equality of hoop and radial logarithmic strain at the minimal cross section, curvature of an eigenstress trajectory at the minimal cross section in the form $1/\rho = r/(r_1\rho_1)$, authors derive following formulas for stress in the point belonging to axis of symmetry at the minimal cross section

$$\sigma_\mathrm{r} = \sigma_\mathrm{i}\frac{r_1}{2\rho_1}, \quad \sigma_\mathrm{z} = \sigma_\mathrm{i}\left(1 - \frac{r_1}{2\rho_1}\right), \quad \sigma_\mathrm{i} = \left(\frac{R}{r_1}\right)^2 \frac{\sigma}{1 + r_1/(4\rho_1)} \tag{14}$$

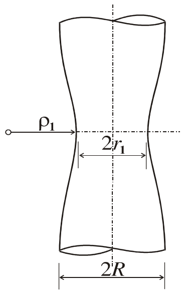

Fig.2. Neck geometry

where $\sigma_\mathrm{i} = \sigma_\mathrm{z} - \sigma_\mathrm{r}$ stands for Mises stress whereas σ denotes uniaxial stress beyond the neck. The ratio of radius of specimen to radius of neck (Fig.2) results from assumption of incompressibility and definition of logarithmic strain $R/r_1 = \sqrt{1 + \langle\varepsilon\rangle}$ and it is approximated by unity for applied amplitude of strain $\varepsilon_{\max} = 2.1\times10^{-3}$. On the contrary, the ratio of radius of neck to radius of curvature $r_1/\rho_1 \cong R/\rho_1$ cannot be neglected and it is assumed in the form $R/\rho_1 = a_1\sqrt{\langle\varepsilon\rangle}$. Strain under root appears in McAuley brackets since only its positive magnitude induces neck. Substitution of above derived approximations to Eqs (14) gives

$$\sigma_\mathrm{r} = \frac{\sigma}{1 + 0.25a_1\sqrt{\langle\varepsilon\rangle}}, \quad \sigma_\mathrm{z} = \sigma\frac{1 + 0.5a_1\sqrt{\langle\varepsilon\rangle}}{1 + 0.25a_1\sqrt{\langle\varepsilon\rangle}}, \quad \sigma_\mathrm{i} = \frac{\sigma}{1 + 0.25a_1\sqrt{\langle\varepsilon\rangle}} \tag{15}$$

Replacement of the "nominal" stress σ by the stress σ_z in (5) leads to extension of Ylinen constitutive law for 3D state of stress in the minimal cross section of the neck

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon} = \begin{cases} \breve{E}^{\pm}\dfrac{\breve{\sigma}_0^{\pm} - |\sigma|}{\breve{\sigma}_0^{\pm} - c|\sigma|} & \text{loading} \\ \breve{E}^{\pm} & \text{unloading} \end{cases} \tag{16}$$

where the effective damage modulus of elasticity and the effective asymptotic yield stress of Ylinen's model are expressed by following relations

$$\frac{\breve{E}^{\pm}}{\tilde{E}^{\pm}} = \frac{\breve{\sigma}_0^{\pm}}{\tilde{\sigma}_0^{\pm}} = \frac{1 + 0.25a_1\sqrt{\langle\varepsilon\rangle}}{1 + 0.5a_1\sqrt{\langle\varepsilon\rangle}} \tag{17}$$

Yield condition takes the form

$$f = \frac{\sigma_i}{1 - D_\mathrm{s}} - \sigma_\mathrm{y} \tag{18}$$

whereas plastic strains are

$$\mathrm{d}\varepsilon_\mathrm{z}^\mathrm{p} = \frac{\partial F}{\partial\sigma_\mathrm{z}}\mathrm{d}\lambda = \frac{\mathrm{d}\lambda}{1 - D_\mathrm{s}}, \quad \mathrm{d}\varepsilon_\mathrm{r}^\mathrm{p} = \mathrm{d}\varepsilon_\mathrm{t}^\mathrm{p} = \frac{\partial F}{\partial\sigma_\mathrm{r}}\mathrm{d}\lambda = -\frac{\mathrm{d}\lambda}{1 - D_\mathrm{s}} \tag{19}$$

and the accumulated plastic strain is equal to

$$\mathrm{d}p = \sqrt{\frac{2}{3}\mathrm{d}\varepsilon_{ij}^\mathrm{p}\mathrm{d}\varepsilon_{ji}^\mathrm{p}} = \frac{\sqrt{2}\mathrm{d}\lambda}{1 - D_\mathrm{s}} \tag{20}$$

Since volumetric stress is known

$$\sigma_\mathrm{v} = \frac{2\sigma_\mathrm{r} + \sigma_\mathrm{z}}{3} = \frac{\sigma}{3}\frac{1 + 1.5a_1\sqrt{\langle\varepsilon\rangle}}{1 + 0.25a_1\sqrt{\langle\varepsilon\rangle}} \tag{21}$$

the strain energy density release rate associated to the unilateral damage takes the form

$$Y_{\mathrm{s}}=\frac{\frac{2}{3}\sigma_{\mathrm{i}}^{2}}{2E\left(1-D_{\mathrm{s}}\right)^{2}}=\frac{\sigma^{2}}{3E\left(1-D_{\mathrm{s}}\right)^{2}}\frac{1}{\left(1+0.25a_{1}\sqrt{\langle\varepsilon\rangle}\right)^{2}}$$

$$Y_{\mathrm{v}}=\frac{3\sigma_{\mathrm{v}}^{2}}{2E\left(1-D_{\mathrm{v}}h\right)^{2}}=\frac{\sigma^{2}}{6E\left(1-D_{\mathrm{v}}h\right)^{2}}\left(\frac{1+1.5a_{1}\sqrt{\langle\varepsilon\rangle}}{1+0.25a_{1}\sqrt{\langle\varepsilon\rangle}}\right)^{2} \tag{22}$$

hence the 3D extension of kinetic damage law (13) is written as

$$\frac{\mathrm{d}D_{\mathrm{s}}}{\mathrm{d}p}=\frac{\sigma^{2}}{3\sqrt{2}ES_{\mathrm{s}}\left(1-D_{\mathrm{s}}\right)^{2}}\frac{1}{\left(1+0.25a_{1}\sqrt{\langle\varepsilon\rangle}\right)^{2}}g(p)$$

$$\frac{\mathrm{d}D_{\mathrm{v}}}{\mathrm{d}p}=\frac{\sigma^{2}}{6\sqrt{2}ES_{\mathrm{v}}\left(1-D_{\mathrm{v}}h\right)^{2}}\left(\frac{1-D_{\mathrm{s}}}{1-D_{\mathrm{v}}h}\right)\left(\frac{1+1.5a_{1}\sqrt{\langle\varepsilon\rangle}}{1+0.25a_{1}\sqrt{\langle\varepsilon\rangle}}\right)^{2}H\left(p-p_{\mathrm{D}}\right) \tag{23}$$

3 RESULTS

A system of three ordinary differential equations (5, 13) or (16, 23) is numerically integrated for strain from $\varepsilon_{\min}=-2.45\times10^{-3}$ to $\varepsilon_{\max}=2.1\times10^{-3}$ by odeint.for routine using fourth-order Runge-Kutta technique with adaptive step size control [8]. The material data of AISI 316L stainless steel at room temperature is presented in Tab.1.

Table 1. The material data for AISI 316L stainless steel at room temperature

a	a_1	b	c	E	ν
0.3	2.5	5.0	0.5773	2.0×10^{5} MPa	0.3
h_{c}	p_{D}	S_{v}	S_{s}	σ_0	σ_{y}
0.2	3.2	0.135MPa	0.075MPa	280MPa	260MPa

Numerical simulation with sampling model of discontinuous damage deactivation is presented in Fig. 3a. Considered model properly maps unilateral nature of damage softening, however, the model exhibits $\partial\sigma/\partial\varepsilon$ discontinuity for $\sigma=0$. In contrast to above model the numerical simulation with model of continuous damage deactivation exhibits both quantitative and qualitative excellent correctness in comparison with experimental results (Fig. 3b). The essential defect of the previous model, an existence of non smooth, bilinear material characteristic separating tensile and compression, manifested specially at point of zero ordinate lying on tensile unloading to compression reloading brunch of hysteresis loop, is successfully eliminated. At the instant of failure the magnitudes of damage variables are equal to $D_{\mathrm{s}}=0.1$ and $D_{\mathrm{v}}=0.91$ respectively, hence the ratio $\left(1-D_{\mathrm{s}}\right)/\left(1-D_{\mathrm{v}}h\right)=1.1$ is essentially bigger than the critical magnitude $\sqrt{\left(1-2\nu\right)/\left(1+\nu\right)}=0.554$ in order to eliminate physically impossible deformation [3].

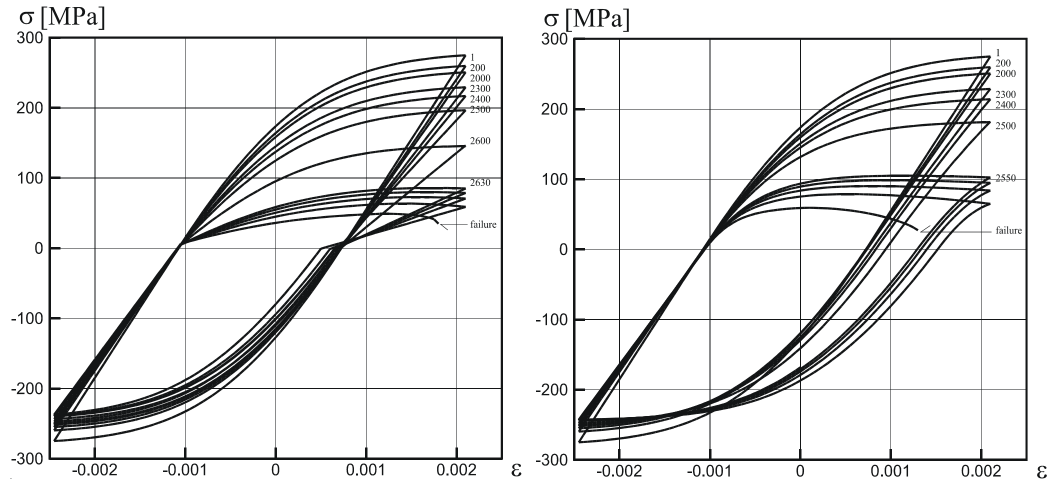

Fig.3 Numerical simulation with model of discontinuous a) and continuous b) damage deactivation

4 CONCLUSIONS

Two models presented in this paper demonstrate necessity and correctness of the application of continuous damage deactivation concept in kinetic law of damage evolution in order to model low cycle fatigue of AISI 316L stainless steel.

REFERENCES

[1] Davidenkov N.N., Spiridonova N.E. Analysis of stress state in the neck of tensioned specimen, Zavodskaja Laboratorja, XI, 6, 1945 (in Russian).

[2] Foryś P., Ganczarski A. Modelling of microcrack closure effect. Proc. *Int. Symp. ABDM*, 2003 (on CD).

[3] Ganczarski A., Barwacz L. Notes on damage effect tensors of two-scalar variables, *Int. J. Damage Mech.*, 2004 (in print).

[4] Hansen N.R., Schreyer H.L. Damage deactivation. *Trans. ASME*, 62: 450-458, 1995.

[5] Ladeveze P. On an anisotropic damage theory, in Failure criteria of structural media, CNRS Int. Coll. No 351, Villard-de-Lans, ed. Boehler, Balkema, Rotterdam, 1983.

[6] Lemaitre J. *A course on damage mechanics*. Springer-Verlag, 1992.

[7] Lemaitre J., Chaboche J.-L. *Mécanique des matériaux solides*. Bordas, Paris, 1985.

[8] Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P. *Numerical Recipes in Fortran*, Cambridge Press, 1993.

[9] Ylinen A. A method of determining the buckling stress and required cross-sectional area for centrally loaded straight columns in elastic and inelastic range, Mémoires, Association Internationale des Ponts et Charpents, Zürich, 16, 529-550, 1956.