DAMAGE DETECTION USING WAVELET TRANSFORM

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ABSTRACT

The paper is concerned in the effectiveness of the discrete wavelet transform depending on the type and parameters of wavelet and depending on the type of experiment and measured response signals. Static and dynamic response of beams and wave propagation and heat conduction in plates is considered. The structural response signals were obtained from the computer simulated experiments accounting for measurement errors by introduction of noise. The key problem of minimum number of measurement points required for successful damage identification is taken up, too.

1 INTRODUCTION

Damage detection emerged as an extremely important engineering problem. It has also focused much attention in the literature in the last three decades. This issue belongs to a class of identification problems, where system parameters are determined from experimental tests. A profound review of damage identification techniques was presented in (Alvin [1]).

One of the modern identification tools is wavelet transform. It allows efficient analysis of non-stationary signals and the real-time processing. Moreover, wavelets are simultaneously localized in time and frequency domains. This feature makes wavelet transform a powerful tool in applications to signal analysis, signal and image compression, image recognizing, signal denoising and solving boundary value problems.

One of the first applications of wavelet transform to damage detection was discussed in (Wang [2]). The method of defect localization using spatial wavelets was presented in (Wang [3]), where the authors considered beam and plate structures using analytically evaluated displacements. At the point of localized damage in the beam a discontinuity of rotation was assumed. In the paper (Quek [4]) the application of different wavelet functions in damage detection structures subjected to static load was discussed and the effects of crack characteristics and boundary conditions were analyzed. Application of continuous wavelet transform to dynamic discrete data for damage detection in beams was discussed in (Gentile [5]). In (Douka [6]) an attempt of quantitative estimation of damage degree using wavelet analysis was presented.

In the present paper the effectiveness of wavelet transformation in damage identification is studied by the way of several examples with the scope to gather information about the recommended types and parameters of wavelets, preferable types of experiments and required measurements. We are also interested in the effectiveness of damage localization depending on a number of measurement points and measurement noise level.

The first part of the paper is devoted to beam structures. We discuss the effectiveness of damage detection for various models of damage, types of actions (static, dynamic) and measured structure response. In the second part we study by the way of several numerical examples the damage identification in plate structures. Two types of experiments are considered namely elastic wave propagation and thermal fields. In the first case the usefulness of 1D dynamic data to 2D structural problems is studied. In the latter case stationary and non-stationary thermal problems are considered. We focus attention on the influence of boundary conditions and manage with undesirable disturbances in the transformed signal. The aim of the study includes not only damage detection but also evaluation of its degree. This issue will be discussed in the presentation.

2 FORMULATION OF THE PROBLEM

Let us consider a structure in which a certain level of damage is expected. Our task is to detect the damage if its level is sufficiently large and to estimate this level. In real engineering practice, different experimental tests can be carried out and different structural responses can be measured. In the present study we use numerical models of structures and simulate numerically experiments. A noise is added to the numerical structural response to account for measurement errors. In principle, it was assumed that we do not know the response of the undamaged structure. In the sequential subsections detailed formulations of mechanical problems are presented. The structural response signals are analyzed using discrete wavelet transform DWT. Theoretical background of this transformation was published in the literature e.g. (Chui [7], Newland [8]). However, for better understanding we provide basic information on DWT.

WT is a method of decomposition of arbitrary signal $f(x)$ into an infinite sum of wavelets at different scales according to the expansion:

$$f(x)=\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}c_{jk}W\left(2^{j}x-k\right) \quad , \tag{1}$$

where $W(x)$ is a wavelet (mother function). Integers j and k are dilation (scale) and translation (position) indices, respectively. The terms c_{jk} are numerical constants called wavelet coefficients. Limiting the range of the independent variable x to one unit interval (here x is non-dimensional) and assuming that $f(x)$ is one period of a periodic signal, the wavelet expansion can be written in the form

$$f(x)=a_0\phi(x)+\sum_{j}\sum_{k}a_{2^{j}+k}W\left(2^{j}x-k\right) \quad , \tag{2}$$

where $\phi(x)$ is a scaling (father) function. The coefficients a represent the amplitudes of subsequent wavelets. The integers j specify different levels of wavelets. The DWT is an algorithm for computing coefficients a when $f(x)$ is sampled at uniformly spaced intervals over $0 \le x \le 1$. Since the number of sampled values is limited, every 1D-function $f(x)$ is approximated using $N=2^J$ discrete values:

$$f(x)=S_J+D_J+D_{J-1}+\ldots+D_j+\ldots+D_2+D_1 \quad , \tag{3}$$

where D_j is the signal representation at the level j. The term D_1 corresponds to the most detailed representation of the signal (high frequency oscillations). The term S_J is called smooth signal representation.

Similarly, decomposition of the 2D function $f(x,y)$ has the form

$$f(x,y)=S_J(x,y)+\sum_{j=1}^{J}D_j^V(x,y)+\sum_{j=1}^{J}D_j^H(x,y)+\sum_{j=1}^{J}D_j^D(x,y) \quad . \tag{4}$$

In Eq. (4) D^V, D^H and D^D express vertical, horizontal and diagonal detail images, respectively. Each detail element at the level j is the sum of vertical, horizontal and diagonal detail elements at the level j:

$$D_j(x,y)=D_j^V(x,y)+D_j^H(x,y)+D_j^D(x,y) \quad . \tag{5}$$

Due to this feature, the above method of signal representation is called multi-resolution analysis (MRA).

3 DAMAGE DETECTION

3.1 Beam structures

For the sake of simplicity the Bernoulli beam model will be used for the discussion of the conditions which influence the effectiveness of WT in damage detection. First note that, the experimentally measured response signal must contain local disturbance induced by the damage. Then, the signal vector must contain sufficiently large number of components (measurements) to make the DWT possible and effective. Finally, a proper type of wavelet must be used. The DWT is capable to extract extremely small local disturbances from the global response signal. In principle, only the response signal of damaged structure is used. We need neither the response signal of the undamaged structure, nor numerical models of these structures. However, to study the effectiveness of DWT, numerical models of damaged structures are used for computer simulation of the experiment.

We used FEM beam models and the damage was introduced in the form of bending stiffness reduction at a small area or in the form of the elastic hinge. The latter case was much more often used in the literature. Let us consider two types of response signals: vertical displacements and slopes. The regularity of the displacement function is C^1 in the case of stiffness reduction and C^0 in the case of the hinge. The slope function is continuous C^0 and discontinuous, respectively. We expect that the lower is the regularity at the region of damage, the better is the damage detection using DWT. This expectation was confirmed by several examples (Knitter-Piątkowska [9]). Hinge model of damage manifests more distinctly its existence and slope signal is better, too. However, DWT was capable to detect and localize damage from all signals and damage models, provided that the level of noise was limited. To each response signal certain critical noise level is assigned.

Haar wavelets were used in the analyses described above. This is the simplest wavelet and therefore it was very often discussed in the literature. It belongs to a wider class of orthogonal wavelets. The characteristic feature of orthogonal wavelets is that a scaling function is orthogonal to itself with respect to its shifting. In this group Daubechies wavelets, Symmlets and Coiflets should be mentioned. Hitherto experience shows that Daubeschies wavelets are very useful in damage detection. The Daubeschies wavelets are compactly supported, have sharp edges and are highly nonsymmetrical. It helps to expose local disturbances of the analyzed signal. However, the support of this wavelet is larger than *[0, 1]*. It results in strong boundary disturbances in the transform. This problem will be discussed more precisely during the presentation.

Planning an experiment one faces a natural question, if it is better to use static or dynamic structural response in damage identification process. To answer this question a beam model with damage defined as stiffness reduction was analyzed. The response signals in the form of displacements due to a concentrated force were assumed. Two classes of problems were examined: static response and harmonic steady-state vibrations. Various positions of the concentrated force and various frequencies were analyzed. Wavelets named “Daublet 8” were used in signal transformation. Damage detection failed for both: static and dynamic response signals, when in the place of damage very small strain was induced in the experiment. This can be overcome by variation of force position, similarly as in (Dems [10], Mróz [11]). Similar effect can be attained by proper modulation of the frequency of dynamic force. Basing on several numerical examples we can conclude, that damage is detected as well for static as for dynamic structural responses. However, the application of dynamic excitation provides more possibilities in planning the experiment.

3.2 Two-dimensional structures

3.2.1. Wave propagation

The effects of wave reflection and deflection due to localized damage will be used for damage detection. The boundary displacement induced waves will be considered. We assume that the structural response is measured in 1D-domain, in a cross-section of the plate. Various response signals will be analyzed, namely the vectors of displacements, velocities and accelerations. Since

we have no a priori information on the existence and localization of damage, therefore in numerical examples the measurement points were located in the front and behind assumed damage.
We will study the effectiveness of damage detection by the way of numerical examples using FEM model. Let us consider a steel plate structure shown in Fig 1a. The Young modulus was assumed *E=200 GPa*. The rectangular 4 nodes FEM shell elements were used, with the number of elements 80x40 in horizontal and vertical directions, respectively. The damage was modeled as local stiffness reduction to $E_d=10$ *GPa* at the region shown in Fig. 1a. Wave excitation by displacement of constant velocity *v(t)=10 m/s* (in the form of the Heaviside function) applied to the right edge of the plate was assumed. Three forms of damaged area were considered: 2 horizontal FEM elements x 4 vertical, 3H x1V and 1H x 3V. Horizontal and vertical components of acceleration and velocity vectors were subject to DWT. The wavelet called "Doublet 4" was used. Fig. 1b presents the results of the detail 1 of DWT, which corresponds to the term D_1 in (3).

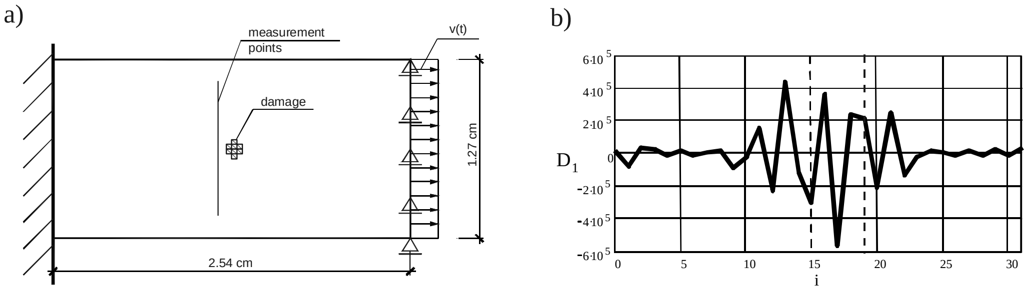

Figure 1: a) plate structure with the damaged area, b) detail D_1 of DWT.

It was assumed that the measurements in all points along vertical cross-section of the plate were registered in the same point in time domain. By the way of several examples it was found that the effectiveness of damage localization strongly depends on that point of time. The best efficiency of damage localization was when the front of the wave was just passing through the line of the measurement points. It appears that the disturbances in the wave are accumulated in the front of the wave.
The influence of the distance between the line of measurement points and the damaged area was examined, too. It was clearly visible, that the larger is this distance the worse is the damage localization. The front of the wave was of course in the same position with respect to measurement points.

3.2.2 Thermal problems
In this Chapter we will check the effectiveness of damage localization in 2-D structures basing on heat transfer experiments. The 2-D image of temperature field can be obtained using thermography. Since small defects or inclusions induce small changes in thermal structural response, wavelet transform will be helpful. Steady-state heat transfer with or without convection and transient heat transfer problems of 2D-structures will be considered. The model of the thermally loaded structure is presented in Fig. 2.

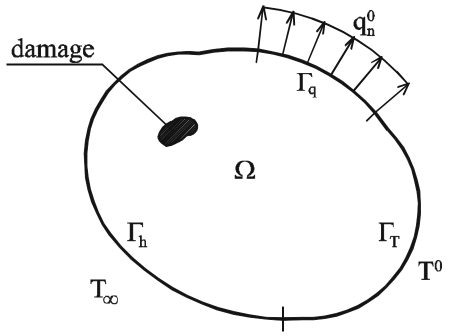

Figure 2: Model of thermally loaded 2D-structure.

The governing equations for the transient heat transfer problem are:

$$\left.\begin{array}{r} -\,div\,\mathbf{q}(\mathbf{x},t)+f=c(\mathbf{x})\dot{T}(\mathbf{x},t) \\ \mathbf{q}(\mathbf{x},t)=-\Lambda(\mathbf{x})\cdot\nabla T(\mathbf{x},t) \end{array}\right\}\ in\ \Omega \qquad \begin{array}{ll} T(\mathbf{x},t)=T^0(\mathbf{x},t) & on\ \Gamma_T \\ q_n(\mathbf{x},t)=\mathbf{n}\cdot\mathbf{q}=q_n^0(\mathbf{x},t) & on\ \Gamma_q \\ q_n(\mathbf{x},t)=\mathbf{n}\cdot\mathbf{q}=h\left[T(\mathbf{x},t)-T_\infty(\mathbf{x},t)\right] & on\ \Gamma_h \\ T(\mathbf{x},0)=T_0(\mathbf{x}) & in\ \Omega\cup\Gamma \end{array}, \qquad (6)$$

where **q**, *f*, **Λ**, *T*, *h* are heat flux vector, heat generated per unit volume, material conductivity matrix, temperature field and film (convection) coefficient, respectively. The dot above the symbol denotes time derivative and *c(***x***)* is specific heat. On the boundary portions Γ_T, Γ_q and Γ_h the Dirichlet, Neumann and Henkel condition are specified, respectively. In the steady-state case the term $c(\mathbf{x})\dot{T}(\mathbf{x},t)$ vanishes and the initial condition $T(\mathbf{x},0)$ need not be specified.

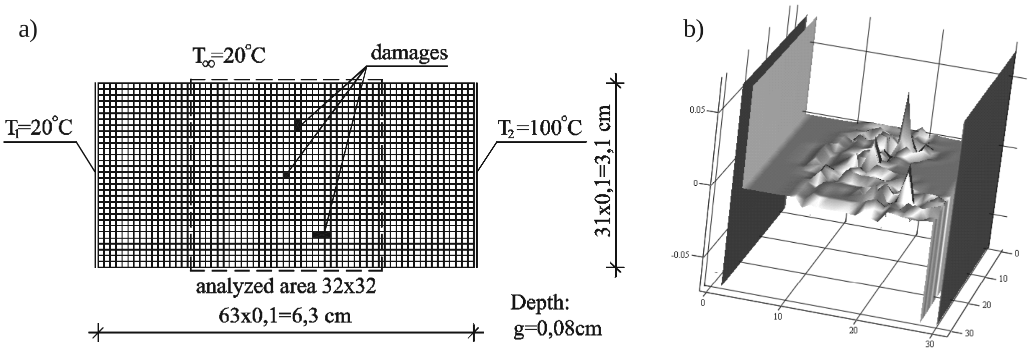

Figure 3: a) FEM model of the structure b) DWT (detail D_1) of temperature field in steady-state process without convection.

Several numerical examples with various structure parameters were analyzed using FEM. One of representative examples is a steel plate structure illustrated in Fig.3a. The following material properties were assumed: thermal conductivity $\lambda=50\ W/(m\cdot K)$, film coefficient $h=9.76\ W/(m^2\cdot K)$, density $\rho=7850\ kg/m^3$, specific heat $c=450\ J/(kg\cdot K)$. The damage is modeled as local reduction of thermal conductivity to $\lambda_d=45\ W/(m\cdot K)$. Fig. 3a illustrates the FEM model of the structure with three damage zones. Fig. 3b presents DWT - detail D_1 of temperature field in steady-state problem without convection. The wavelet called "Daublet 4" was used in the transform. Fig. 3b proves that

DWT satisfactorily identifies the position of damage and also provides information on its shape and magnitude. The effects of convection and measurement errors take the role of disturbances spread throughout the transformed signal. Damage identification becomes more difficult, but in normal conditions it is still possible. In the example, the admissible noise level was about ± 0.01 *˚C*.

5 CONCLUDING REMARKS

The effectiveness of wavelet transform in damage identification was studied by the way of several numerical examples. Damage in beams and plate structures was considered. In the latter case two types of experiments were discussed, namely wave propagation and thermal problems. Wavelet transforms 1D and 2D were implemented. The examples demonstrated that WT effectively identified defects even in case of measurement noise. It was proved that effectiveness of damage identification strongly depends on type of wavelet, structural response signal and number of measurement points. The wavelet transform of response signal can also be used to estimation of damage intensity.

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