# A micromechanics inspired damage model for initially transversely isotropic materials

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**Abstract**

A new general constitutive model for describing damage and its effect on the overall properties of transversely isotropic solids is proposed. The formulation of this macroscopic model is based on a mixed approach combining tools of representation theory and results from micromechanics. It allows to reduce substantially the number of the model parameters that have to be determined. The ability of the model to describe the loss of symmetry resulting from the interaction between initial and damage-induced anisotropy in a brittle matrix composite is demonstrated.

## 1. Introduction

The basic mechanisms of deformation of a large class of materials ranging from man-made materials to geologic materials is microcracking. Since the orientation distribution of the crack arrays depends on the loading direction, cracks-induced damage is generally anisotropic. Continuum damage mechanics (CDM) (Krajcinovic [1], Chaboche [2]) as well as micromechanics ([1], Nemat-Nasser and Horii [3], Pensée et al. [16] etc..) have been successfully applied to the description of damage-induced anisotropy in isotropic materials. In contrast, few damage models (see for example, Talreja [5], Ladeveze and Letombe [6]; Biegler and Mehrabadi [7]; Chaboche [20]; Halm et al. [8]) for initially anisotropic materials have been proposed in the literature, the modelling of the interaction between primary and damage-induced anisotropy remaining a debated issue.

In this paper, a new damage model for initially transversely isotropic solids is proposed. The only dissipation mechanism considered is growth of distributed microcracks leading to fracture without significant inelastic deformation. The adopted methodology consists first to use representation theorems for deriving an expression of the enthalpy associated to a medium weakened by a set of parallel cracks. This representation shows the interaction between initial and damage-induced anisotropy (cracks orientation). Then, using known results from micromechanics, an identification of some parameters of the model is proposed, the model reduces to the micromechanical one when this model is known. Finally, we adopt an approximation of damage orientation distribution by a second-order symmetric tensor which allows to generate a simple CDM model which can be easily analyzed in the frame of Thermodynamics of irreversible processes. A damage criterion and the corresponding anisotropic damage evolution law are formulated.

We start with some conventions and notations. Intrinsic summation convention on repeated indices is adopted. The dyadic product of either vectors or second-order tensors is denoted by "$\otimes$" whereas "$\overline{\underline{\otimes}}$" is the symmetrized dyadic product; componentwise $(\underline{u} \otimes \underline{v})_{ij} = u_i v_j$, for any two vectors $\underline{u}$ and $\underline{v}$ and

$$(\underline{\underline{A}} \otimes \underline{\underline{B}})_{ijkl} = A_{ij} B_{kl}; \qquad (\underline{\underline{A}} \,\overline{\underline{\otimes}}\, \underline{\underline{B}})_{ijkl} = \frac{1}{2}(A_{ik} B_{jl} + A_{il} B_{jk}) \tag{1}$$

$\underline{\underline{\delta}}$ denoting the second order unit tensor, the fourth order symmetric unit tensor is defined by $\mathbb{I} = \underline{\underline{\delta}} \,\overline{\underline{\otimes}}\, \underline{\underline{\delta}}$

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## 2. Principle of homogenization of anisotropic materials with parallel penny shaped cracks

### 2.1. *Introduction*

The representative elementary volume (r.e.v.) $\Omega$ is constituted of an transversely isotropic linear elastic solid matrix whose stiffness tensor is $\mathbb{C}^0$ and of parallel planar microcracks. From a geometrical point of view, a crack is considered as an oblate spheroid. The classical idealization of a penny-shaped crack corresponds to an aspect ratio $X = \frac{c}{a} \ll 1$ where $a$ is the crack radius and $c$ the half opening. From a mechanical point of view, a crack can be represented as an inhomogeneity with an stiffness tensor $\mathbb{C}^c$ which depends on its opening/closure status [16]. Classically, the choice of $\mathbb{C}^c = 0$ is made for opened cracks.

### 2.2. *Transversely isotropic solid matrix weakened by parallel opened cracks*

We assume non interaction between cracks; this assumption allows to consider the dilute scheme for the determination of the homogenized properties of the material. Moreover, uniform stress boundary conditions are considered on the boundary $\partial\Omega$ of the r.e.v.. Following Laws [9], the macroscopic free enthalpy reads :

$$W^* = \frac{1}{2}\underline{\underline{\Sigma}} : \left[\mathbb{S}^0 + \mathbb{S}^d\right] : \underline{\underline{\Sigma}} \tag{2}$$

with $\mathbb{S}^0$ the elastic compliance tensor of the undamaged material (solid matrix). $\mathbb{S}^d$ describes the effect of cracks on the overall compliance tensor; it's determination requires to evaluate $\lim_{X \to 0} X\mathbb{Q}^{-1}$.
$\mathbb{Q}$ is related to the more usual Hill's tensor $\mathbb{P}$ (see Mura [10]) by : $\mathbb{Q} = \mathbb{C}^0 - \mathbb{C}^0 : \mathbb{P} : \mathbb{C}^0$. The main difficulty here lies in the anisotropic nature of the solid matrix. Indeed, the analytical determination of $\mathbb{P}$ (or $\mathbb{Q}$) is possible only for cracks in the isotopic plane, i.e. when cracks normal $\underline{n}$ coincides with the direction of the material symmetry axis. Since we aim at constructing an analytical damage model, numerical investigations of $\mathbb{P}$ tensor is not reported here.
Let us introduce now the structural tensor $\underline{\underline{A}} = (\underline{m} \otimes \underline{m})$, $\underline{m}$ representing the vector parallel to the symmetry axis. Considering the classical cracks density parameter $\rho(\underline{m}) = \mathcal{N}a^3$ ($\mathcal{N}$ being the number of cracks by unit volume), the macroscopic free enthalpy reads in this case :

$$W^*(\underline{\underline{\Sigma}}, \underline{\underline{A}}) = \frac{1}{2}\underline{\underline{\Sigma}} : \mathbb{S}^0 : \underline{\underline{\Sigma}} + \frac{\rho(\underline{m})}{2}\left\{\frac{1}{k_1}[(\underline{\underline{A}} : \underline{\underline{\Sigma}})^+]^2 + \frac{1}{k_2}[(\underline{\underline{\Sigma}}.\underline{\underline{\Sigma}}) : \underline{\underline{A}} - \underline{\underline{\Sigma}} : (\underline{\underline{A}} \otimes \underline{\underline{A}}) : \underline{\underline{\Sigma}}]\right\}. \tag{3}$$

$k_1$ and $k_2$ depend only on the elastic coefficients of the transversely isotropic material :

$$\begin{cases} \dfrac{1}{k_2} = \dfrac{64(E_1 - E_3\nu_{13}^2)[E_1 - 2\nu_{13}G_{23}(1+\nu_{12})]}{3E_1E_3G_{23}(1-\nu_{12}^2)} \\[2ex] \dfrac{1}{k_1} - \dfrac{1}{k_2} = \dfrac{3}{32}\left[\sqrt{\dfrac{2E_1G_{23}}{1+\nu_{12}}} + \dfrac{2E_1G_{23}\left(E_1 + E_3(2\nu_{12}^2 - \nu_{13}^2 - 2)\right)}{(E_1 - E_3\nu_{13}^2)[E_1 - 2\nu_{13}G_{23}(1+\nu_{12})]}\right] \end{cases} \tag{4}$$

The term $(\underline{\underline{A}} : \underline{\underline{\Sigma}})^+$ denotes the positive part of $(\underline{\underline{A}} : \underline{\underline{\Sigma}})$. Since the presence of this term in (3) cancels for closed cracks, it accounts for unilateral effects. Expression (3) extends then the results given in [9] to closed cracks. For simplicity, only opened cracks are considered in what follows, analysis of cracks closure being performed out elsewhere.

## 3. Representation theorems applied for initially transversely isotropic materials

### 3.1. *Macroscopic energy for transversely isotropic materials with parallel cracks*

We recall that in the undamaged state, the material is transversely isotropic, its symmetry group $G$ being the group of rotations about a preferred direction, say, the unit vector $\underline{m}$:

$$G = \{ \underline{\underline{Q}} \in O(3) \mid \underline{\underline{Q}} .\underline{m} = \underline{m} \quad \text{or} \quad \underline{\underline{Q}} .\underline{m} = -\underline{m} \}. \tag{5}$$

or equivalently $G = \{ \underline{\underline{Q}} \in O(3) \mid \underline{\underline{Q}} .\underline{\underline{A}} .\underline{\underline{Q}}^T = \underline{\underline{A}} \}$, with $\underline{\underline{A}} = \underline{m} \otimes \underline{m}$.
We emphasize that the tensor $\underline{\underline{A}}$ characterizes the symmetry of the material in the undeformed state. Liu [12] proved that any scalar, vector, or second-order tensor valued anisotropic function of vectors and second order tensors, can be expressible as an isotropic function of the original arguments, and of structural tensors as additional arguments . Thus, any scalar function, say, the elastic energy $W^*$ can be represented relative to $G$ by an isotropic function of $\underline{\underline{\Sigma}}$, the cracks orientation $\underline{n}$, and the tensor $\underline{\underline{A}}$.
For a transversely isotropic material weakened by a set of parallel cracks of normal $\underline{n}$, the representation theory (see for instance also Boehler [13]) indicates that the macroscopic free enthalpy is a polynomial isotropic scalar function of the symmetric tensors ($\underline{\underline{\Sigma}}, \underline{\underline{A}}, \underline{n} \otimes \underline{n}$). The presence of $\underline{n} \otimes \underline{n}$ accounts for the fact that the energy must be objective, radially symmetric with respect to $\underline{n}$ (see Welemane and Cormery [17]). $W^*$ takes then the form given in Appendix.

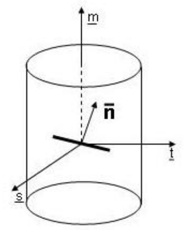


Figure 1. The solid matrix with a slit crack

### 3.2. *Identification based on micromechanics results*

Observing that representation (20) contains too many parameters we aim now at identifying some coefficients of the model by using known results from micromechanics. We consider first the cracks configuration studied in subsection 2.2. In this case tensor $\underline{\underline{N}} = \underline{n} \otimes \underline{n}$ coincides with the structural tensor $\underline{\underline{A}} = \underline{m} \otimes \underline{m}$ and the general representation given in appendix must reduce to the micromechanics one (3). This procedure leads to following relations:

$$\begin{cases} b_2 = 0; \quad b_{14} = -(b_4 + b_{11}); \quad b_{19} = \dfrac{1}{4k_2} - \dfrac{1}{4k_1} - (b_5 + b_{10} + b_{15} + b_{17} + b_{18}) \\ b_7 = 0; \quad b_{16} = \dfrac{1}{4k_1} - (b_8 + b_9) \end{cases} \tag{6}$$

A second identification is done by constraining the general representation to coincide also with the dilute scheme estimation when the solid matrix is isotropic (see [1], [4]). It follows that :

$$b_9 = 0; \qquad b_{11} = b_5 + b_{15} = -b_{10} \tag{7}$$

To summarize, the identification procedure proposed here allows to reduce the model parameters to the 5 elastic constant of the solid matrix and 6 remaining parameters which have to be determined from standard experiments.

## 4. Generalization for a randomly distributed cracks system

### 4.1. *Principle*

In this section we extend the previous results to randomly oriented cracks. The generalized macroscopic free enthalpy can be formulated as the integral of the free enthalpy $W^*(\underline{\underline{\Sigma}}, \underline{\underline{A}}, \underline{n}\otimes\underline{n})$ associated to all cracks families and performed over all orientations (over the unit sphere $S$):

$$\psi^* = \frac{1}{2}\underline{\underline{\Sigma}} : \mathbb{S}^0 : \underline{\underline{\Sigma}} + \frac{1}{4\pi}\int_{S^{2+}} W^*(\underline{\underline{\Sigma}}, \underline{\underline{A}}, \underline{n} \otimes \underline{n}) dS \tag{8}$$

Such procedure is somewhat similar to techniques classically used for microplane models [18]. However, it is worth noticing that $W^*(\underline{\underline{\Sigma}}, \underline{\underline{A}}, \underline{n} \otimes \underline{n})$ is constructed by combining representation theorems and micromechanics arguments. Following Lubarda and Krajcinovic [11], we choose to approximate the crack density distribution $\rho$ by a second-order tensor $\underline{\underline{d}}$, such as $\rho(\underline{n}) = \underline{\underline{d}} : (\underline{n} \otimes \underline{n})$. It is possible to introduce at the place of $\underline{\underline{d}}$ a macroscopic tensor $\underline{\underline{D}}$ defined by : $\underline{\underline{D}} = \frac{1}{4\pi}\int_{S^2} \rho(\underline{n})(\underline{n} \otimes \underline{n}) dS$. This replacement gives :

$$\rho(\underline{n}) = \frac{3}{2}\ [5\underline{\underline{D}} : (\underline{n} \otimes \underline{n}) - \mathrm{tr}(\underline{\underline{D}})]. \tag{9}$$

Therefore, in this new representation of microcracking, the damage is characterized by an internal variable, denoted $\underline{\underline{D}}$, which is related to the distribution (density, orientation) of defects. Taking into account (9), the integration (8) on the unit sphere can now be performed analytically using the following identities :

$$\frac{1}{4\pi}\int_{|\underline{n}|=1} \underline{n}\otimes\underline{n}\, dS = \frac{1}{3}\underline{\underline{\delta}} \quad ; \quad \frac{1}{4\pi}\int_{|\underline{n}|=1} \underline{n}\otimes\underline{n}\otimes\underline{n}\otimes\underline{n}\, dS = \frac{1}{3}\mathbb{J} + \frac{2}{15}\mathbb{K} \quad ; \quad \mathbb{J} = \frac{1}{3}\underline{\underline{\delta}}\otimes\underline{\underline{\delta}}\ ;\ \mathbb{K} = \mathbb{I} - \mathbb{J} \tag{10}$$

$$\frac{1}{4\pi}\int n_i n_j n_k n_l n_\alpha n_\beta \ d\varphi = \frac{1}{7}\ \mathbb{A}_{ijkl\alpha\beta} \tag{11}$$

with $\mathbb{A}_{ijkl\alpha\beta} = \frac{1}{5}\ (\delta_{ij}\mathbb{A}_{kl\alpha\beta} + \delta_{ik}\mathbb{A}_{jl\alpha\beta} + \delta_{il}\mathbb{A}_{jk\alpha\beta} + \delta_{i\alpha}\mathbb{A}_{jkl\beta} + \delta_{i\beta}\mathbb{A}_{jkl\alpha})$
The free enthalpy of the damaged medium takes the form :

$$\psi^* = \frac{1}{2}\underline{\underline{\Sigma}} : \mathbb{S}^{hom} : \underline{\underline{\Sigma}}, \tag{12}$$

where the overall compliance tensor $\mathbb{S}^{hom}$ takes the form :

$$\mathbb{S}^{hom} = \left\{ \begin{array}{c} c_1(\underline{\underline{\delta}}\otimes\underline{\underline{\delta}}) + c_2(\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}}) + c_3(\underline{\underline{D}}\otimes\underline{\underline{\delta}} + \underline{\underline{\delta}}\otimes\underline{\underline{D}}) + c_4(\underline{\underline{D}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}} + \underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{D}}) + \\ c_5(\underline{\underline{\delta}}\otimes\underline{\underline{A}} + \underline{\underline{A}}\otimes\underline{\underline{\delta}}) + c_6(\underline{\underline{\delta}}\,\overline{\underline{\otimes}}\,\underline{\underline{A}} + \underline{\underline{A}}\,\overline{\underline{\otimes}}\,\underline{\underline{\delta}}) + c_7(\underline{\underline{A}}\otimes\underline{\underline{A}}) + c_8(\underline{\underline{A}}\otimes\underline{\underline{D}} + \underline{\underline{D}}\otimes\underline{\underline{A}}) + \\ c_9\left[\underline{\underline{\delta}}\otimes(\underline{\underline{A}}\cdot\underline{\underline{D}} + \underline{\underline{D}}\cdot\underline{\underline{A}}) + (\underline{\underline{A}}\cdot\underline{\underline{D}} + \underline{\underline{D}}\cdot\underline{\underline{A}})\otimes\underline{\underline{\delta}}\right] + \\ c_{10}\left[\underline{\underline{A}}\otimes(\underline{\underline{A}}\cdot\underline{\underline{D}} + \underline{\underline{D}}\cdot\underline{\underline{A}}) + (\underline{\underline{A}}\cdot\underline{\underline{D}} + \underline{\underline{D}}\cdot\underline{\underline{A}})\otimes\underline{\underline{A}}\right] + c_{11}\left[\underline{\underline{\delta}}\,\overline{\underline{\otimes}}(\underline{\underline{A}}\cdot\underline{\underline{D}} + \underline{\underline{D}}\cdot\underline{\underline{A}}) + (\underline{\underline{A}}\cdot\underline{\underline{D}} + \underline{\underline{D}}\cdot\underline{\underline{A}})\overline{\underline{\otimes}}\,\underline{\underline{\delta}}\right] \end{array} \right\} \tag{13}$$

with $c_i$:

$$\begin{array}{l} c_1 = g_1 + g_2\,\mathrm{tr}(\underline{\underline{D}}) + g_3\,\mathrm{tr}(\underline{\underline{A}}.\underline{\underline{D}}), \qquad c_2 = g_4 + g_5\,\mathrm{tr}(\underline{\underline{D}}) + g_6\,\mathrm{tr}(\underline{\underline{A}}.\underline{\underline{D}}), \qquad c_3 = g_7, \\ c_4 = g_8, \qquad c_5 = g_9 + g_{10}\,\mathrm{tr}(\underline{\underline{D}}) + g_{11}\,\mathrm{tr}(\underline{\underline{A}}.\underline{\underline{D}}), \qquad c_6 = g_{12} + g_{13}\,\mathrm{tr}(\underline{\underline{D}}), \\ c_7 = g_{14} + g_{15}\,\mathrm{tr}(\underline{\underline{D}}) + g_{16}\,\mathrm{tr}(\underline{\underline{A}}.\underline{\underline{D}}), \qquad c_8 = g_{17}, \qquad c_9 = g_{18}, \qquad c_{10} = g_{19}, \qquad c_{11} = g_{20}. \end{array} \tag{14}$$

Again, it is worth noticing that coefficients $g_i$ in (14) depend only on the 5 elastic coefficients of the solid matrix and the remaining 6 parameters.

### 4.2. *Damage evolution law*

Given the expression of the energy $\psi^*$, the thermodynamic force $\underline{\underline{Y}}$, associated to damage is obtained by partial derivation : $\underline{\underline{Y}} = \frac{\partial \psi^*}{\partial \underline{\underline{D}}}$. The damage surface is expressed in terms of $\underline{\underline{Y}}$, (also called the damage energy-release rate), and $\underline{\underline{D}}$ as:

$$f(\underline{\underline{Y}}, \underline{\underline{D}}) = \|\underline{\underline{Y}}\| - [h_0 + h_1 tr\underline{\underline{D}} + h_2 tr(\underline{\underline{DA}})]. \qquad (15)$$

In (15) $h_0$, $h_1$, and $h_2$ are model parameters: $h_0$ defines the initial damage threshold while $h_1$ and $h_2$ describe the manner in which the surface evolves with damage. In particular, $h_2 tr(\underline{\underline{DA}})$ accounts for the interaction between initial and damage-induced anisotropy. The evolution of $\underline{\underline{D}}$ is assumed to follow the normality rule:

$$\dot{\underline{\underline{D}}} = \begin{cases} 0, & \text{if } f < 0 \quad \text{or} \quad f = 0 \text{ and } \dot{f} < 0 \\ \dot{\lambda} \dfrac{\partial f}{\partial \underline{\underline{Y}}}, & \text{if } f = 0 \text{ and } \dot{f} = 0 \end{cases} \qquad (16)$$

The positive scalar $\dot{\lambda}$ (damage multiplier) is given by the classical Kuhn-Tucker condition $\dot{f} = 0$, i.e.:

$$\dot{\lambda} = \frac{tr(\underline{\underline{Y}} \cdot \dot{\underline{\underline{Y}}})}{k_1 tr(\underline{\underline{Y}}) + k_2 tr(\underline{\underline{Y}}.\underline{\underline{A}})} \qquad (17)$$

## 5. Conclusions

A general model for describing damage in initially transversely isotropic solids is proposed in the framework of irreversible thermodynamics. Assuming a moderate density of defects, a linear dependence of the elastic energy on cracks density parameter was considered. Representation theorems were used to obtain a general form of the enthalpy of a solid matrix containing a set of parallel cracks. By using an identification procedure based on results provided by micromechanics, it is shown that the number of model parameters is substantially reduced. Next, a second-order symmetric tensor defined as an approximation of cracks density distribution is introduced for the continuum damage model. A damage criterion and anisotropic damage evolution law were also formulated. The model is now applied to the description of the tensile behavior of a ceramic matrix composite. A procedure for identification of the model parameters based on the experimental variation of the stiffness components with the applied load will be presented. First comparison with data obtained by Baste and Aristegui [14] show the ability of the model to describe the loss of elastic symmetry due to anisotropic damage as well as damage effect on the overall stress-strain response (see [19]). Study of cracks closure process is also under progress.

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## Appendix : Free enthalpy of a transverse isotropic materials weakened by parallel cracks

$$W^*(\underline{\underline{\Sigma}}, \underline{\underline{A}}, \underline{n} \otimes \underline{n}) = W_0^* + \rho(\underline{n}) W_d^* \qquad with \tag{18}$$

$$W_0^* = b_1 \operatorname{tr}^2(\underline{\underline{\Sigma}}) + b_3 \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{A}}) \operatorname{tr}(\underline{\underline{\Sigma}}) + b_6 \operatorname{tr}(\underline{\underline{\Sigma}}^2) + b_{12} \operatorname{tr}^2(\underline{\underline{\Sigma}}.\underline{\underline{A}}) + b_{13} \operatorname{tr}(\underline{\underline{\Sigma}}^2.\underline{\underline{A}}) \tag{19}$$

$$W_d^* = \left\{ \begin{array}{c} b_2 \operatorname{tr}(\underline{\underline{A}}.\underline{\underline{N}}) \operatorname{tr}^2(\underline{\underline{\Sigma}}) + b_4 \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{N}}) \operatorname{tr}(\underline{\underline{\Sigma}}) + b_5 \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{A}}) \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{N}}) + b_7 \operatorname{tr}(\underline{\underline{A}}.\underline{\underline{N}}) \operatorname{tr}(\underline{\underline{\Sigma}}^2) + b_8 \operatorname{tr}(\underline{\underline{\Sigma}}^2.\underline{\underline{N}}) + \\ b_9 \operatorname{tr}(\underline{\underline{A}}.\underline{\underline{N}}) \operatorname{tr}(\underline{\underline{\Sigma}}^2.\underline{\underline{A}}) + b_{10} \operatorname{tr}(\underline{\underline{A}}.\underline{\underline{N}}) \operatorname{tr}^2(\underline{\underline{\Sigma}}.\underline{\underline{A}}) + b_{11} \operatorname{tr}(\underline{\underline{A}}.\underline{\underline{N}}) \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{A}}) \operatorname{tr}(\underline{\underline{\Sigma}}) + b_{14} \operatorname{tr}(\underline{\underline{\Sigma}}) \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{A}}.\underline{\underline{N}}) + \\ b_{15} \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{A}}) \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{A}}.\underline{\underline{N}}) + b_{16} \operatorname{tr}(\underline{\underline{\Sigma}}^2.\underline{\underline{A}}.\underline{\underline{N}}) + b_{17} \operatorname{tr}^2(\underline{\underline{\Sigma}}.\underline{\underline{N}}) + b_{18} \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{N}}) \operatorname{tr}(\underline{\underline{\Sigma}}.\underline{\underline{A}}.\underline{\underline{N}}) + b_{19} \operatorname{tr}^2(\underline{\underline{\Sigma}}.\underline{\underline{A}}.\underline{\underline{N}}) \end{array} \right\} \tag{20}$$

in which $\underline{n}$ denotes the unit normal to the crack plane and $\underline{\underline{N}} = \underline{n} \otimes \underline{n}$. Note that, the linear dependence of the elastic energy on cracks density is justified by the consideration of moderate cracks density parameter. Note also that, since they do not bring any other modification of the material symmetry, some extra terms implying $\operatorname{tr}(\underline{\underline{A}}.\underline{\underline{N}})$ are not considered here.