A MULTI-SCALE METHOD FOR MASONRY COMPUTATIONS

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ABSTRACT

This contribution presents a multi-scale framework for the representation of the non-linear behaviour of planar masonry structures based on computational homogenization techniques. In order to avoid the troublesome formulation of closed-form constitutive equations, the first-order multi-level finite element scheme is enhanced to capture the non-linear macroscopic behaviour of brick masonry in the presence of quasi-brittle damage. This multi-scale technique relies on the formulation of mesoscopic constitutive laws for the individual brick and mortar materials. A periodic unit cell with specific periodicity requirements is used to deduce the average response of the masonry material. The scale transitions formulated to extract the average response of the material make use of the initial periodicity of the material structure. At the macroscopic scale, the deduced material response is used in the frame of a standard continuum approach. An enhanced first-order computational multi-scale solution scheme is outlined, allowing to include mesostructurally based damage localization in structural computations. The model enhances the first-order computational homogenization technique by introducing finite width damage localization bands, each corresponding to a pair of weak discontinuities. The size of this localization band is directly deduced from the initial periodicity of the material. As a result of the use of homogenization techniques on finite volumes and of the presence of quasi-brittle constituents, a mesostructural snap-back may occur in the homogenized material response deduced by the scale transition. A methodology to introduce this type of response in the originally strain driven multi-scale technique is proposed. Its impact on the implementation of the framework is detailed. The results obtained by the framework are illustrated by means of a structural computation example.

1 INTRODUCTION

Ensuring the safety of historical buildings requires careful analysis of the residual strength of the (possibly damaged) structures and of the effect of repair operations. Finite element modelling of the failure process may be extremely useful in such analyses. Conventional finite element analyses require a constitutive model of the building material. For masonry, however, the formulation of closed-form constitutive relations which can accurately describe the aggregate degradation behaviour of bricks and mortar joints is a formidable challenge. Not only may both individual constituents and their bonding be degraded, these degradation processes also strongly influence each other, resulting in a range of possible failure mechanisms [1]. These failure modes and the mechanical responses associated with them are dominated by the mesostructure of the material, i.e. by the geometric arrangement of the bricks and mortar and by their individual properties. Most notably, cracks often follow the mortar joints and thus follow preferential directions which are set by the mesostructure. This results in the possible appearance of complex damage-induced anisotropy effects.

2 MULTI-SCALE MODELLING OF MASONRY

2.1 Multi-scale modelling of heterogeneous materials

Realistic predictions of strength and failure modes of masonry may be obtained from mesoscopic modelling, in which the geometry of the bricks and mortar joints is explicitly modelled, and homogeneous material behaviour is assumed for each of the phases [2]. Even if relatively simple constitutive relations are used for the brick and for the mortar materials, such models show a complex overall behaviour which agrees well with experimental observations. Modelling the full mesostructure of entire walls or structures, however, may quickly become too expensive.

A compromise between computational cost and mesostructural detail can be reached by using an approach in which the mesoscopic and macroscopic scales are coupled. Structures are then modelled using an homogenised continuum description, the constitutive behaviour of which is determined at runtime by mesoscopic computations. In a finite element context, a sample of the mesostructure is used to determine the material response in each Gauss point of the macroscopic finite element discretisation. The local macroscopic strain is applied in an average sense to the mesostructure and the resulting mesostructural stresses are determined by a finite element analysis. The averaging of these mesostructural stresses and the condensation of the mesostructural tangent stiffness to the homogenised tangent stiffness then yield the macroscopic material response associated with the Gauss point. This concept based on computational homogenisation has been used before to model heterogeneous polymers, see e.g. [3]. Its added value in the context of masonry resides in the fact that no complex closed form constitutive relation needs to be postulated for the representation of the overall material behaviour. The complexity associated with the damaging mesostructure of the material is naturally accounted for by scale transitions. Also, material identification issues are shifted to the level of mortar joints and brick constituents. It is emphasized that this multi-scale framework makes use of a strain-based scale transition. It thus heavily relies on the availability of a solution of the mesostructural boundary value problem for the macroscopic strain increment prescribed by the macroscopic computation.

The definition of a multiscale scheme for masonry thus essentially requires the definition of four ingredients: (i) a mesoscopic constitutive setting for the brick and mortar materials, (ii) the definition of a representative mesostructural sample, (iii) the choice of a macroscopic continuum representation, and (iv) the set-up of scale transitions linking macroscopic and mesoscopic quantities. These features should carefully be selected in order to allow a proper incorporation of the localisation behaviour, both at the mesoscopic and macroscopic scales.

2.2 Mesoscopic modelling and unit cell definition

Masonry constituents are quasi-brittle materials exhibiting low fracture energies and high sensitivities to tensile stresses. A scalar implicit gradient damage framework is therefore used at the mesoscopic scale, involving the solution of a non-local averaging equation in addition to the mesoscopic equilibrium [4].

Based on the periodicity of the initial mesostructure of masonry, periodic homogenisation concepts are used in order to build scale transitions between the mesoscopic and macroscopic scales. In this contribution, the smallest periodic mesostructural sample is selected as the representative volume element in order to limit the computational effort at the mesoscopic scale. For running bond masonry, the smallest possible unit cell is represented in Figure 1.

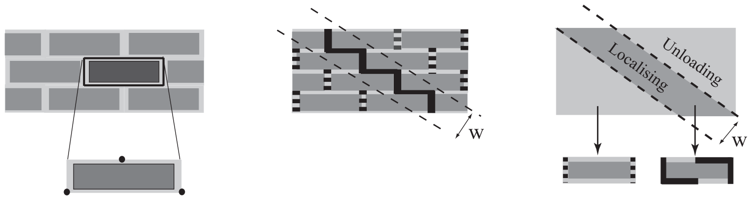

Figure 1: Identification of a mesostructural unit cell (left), and of a macroscopic localisation band (center and right)

2.3 Localisation enhanced scheme

In order to deal with localisation at the macroscopic scale, embedded localisation bands surrounded by unloading material are introduced in a standard first order continuum description. A material bifurcation analysis based on the homogenised acoustic tensor eigenspectrum analysis as proposed in [6] allows to detect the appearance of localisation along orientations which are consistent with the unit cell mesostructural damage patterns. The band width is deduced from this orientation and the initial periodicity of the mesostructure, as illustrated in Figure 1 for a staircase crack pattern. Due to the presence of the band width, the resulting overall energy dissipation becomes sensitive to the ratio between mesostructural and structural sizes.

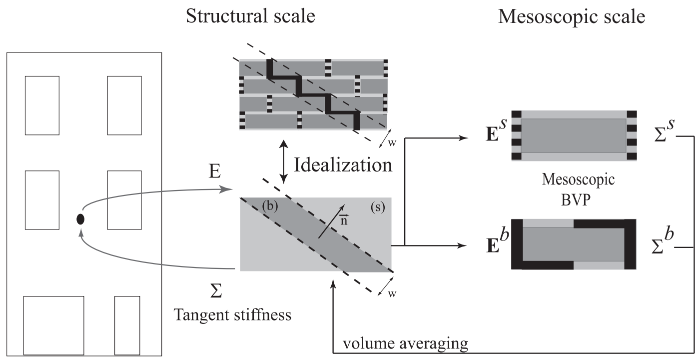

Figure 2: Enhanced first-order multi-scale scheme with embedded strain discontinuity for localised behaviour.

Upon detection of localisation in a given material point, a band is inserted and the multi-scale scheme depicted in Figure 2 is initiated. The averaged response of the system composed of the localisation band with its surrounding material is obtained using a relaxed Taylor assumption [5]. The finite volume associated to the macroscopic material point is split into a localising band (b), and its surrounding volume (s). A strain jump between the sub-regions is defined by the normal to the band $\vec{n}$ and a strain jump mode $\vec{m}$. For a given macroscopic strain $\mathbf{E}$, this embedded band model assumes a constant strain in each sub-region, determined according to

$$\begin{aligned} \mathbf{E}^b &= \mathbf{E} + f^s \left(\vec{m}\vec{n}\right)^{sym} \\ \mathbf{E}^s &= \mathbf{E} - f^b \left(\vec{m}\vec{n}\right)^{sym} \end{aligned} \qquad (1)$$

where f^b and f^s are the respective volume fractions. The stress variations in the sub-regions are deduced from unit cell computations, and the macroscopic stress $\mathbf{\Sigma}$ is obtained by volume averaging

$$\mathbf{\Sigma} = f^b \mathbf{\Sigma}^b + f^s \mathbf{\Sigma}^s \tag{2}$$

The strain jump between the band and its surrounding volume is obtained from the traction continuity requirement at the interface between both sub-regions

$$\vec{n}.\left(\mathbf{\Sigma}^b - \mathbf{\Sigma}^s\right) = \vec{0} \tag{3}$$

3 SCALE TRANSITION ISSUES

Mesoscopic damage localisation in weaker zones of the order of a narrow mortar joint may lead to snap-backs in the retrieved homogenised material response used at the macroscopic scale. Note that snap-back effects result from the averaging operations performed on a finite material volume. In the enhanced multi-scale scheme sketched in Figure 2, it may thus appear at the level of the relaxed Taylor averaging or at the level of the localising unit cell. The original strain-driven scale transition is unable to treat such a mesostructural response.

A snap-back stemming from the relaxed Taylor assumption is treated here by searching for positive increments of the strain jump between the localising and the surrounding material. This is achieved by considering this strain jump as a macroscopic unknown, which allows a control by means of path following techniques. The traction continuity requirement (3) is used as an additional equation in the macroscopic solution procedure.

A snap-back effect in the localising unit cell averaging step is more difficult to control. The presence of a snap-back in the unit cell response implies that a solution does not necessarily exist for any prescribed macroscopic strain increment. Since the unit cell equilibrium problem is strain-driven in the original multi-scale scheme, an adaptation of the framework is introduced to handle such snap-backs. This enhancement consists in steering the unit cell computation on the snap-back path. This is achieved where needed by forcing further mesoscopic energy dissipation through the increase of selected mesoscopic non-local strain unknowns. Similarly to path following techniques, the unit cell boundary value problem is then controlled by the prescription of the macroscopic averaged strain complemented by a non-local strain increment acting as a control variable. In the same spirit as for the relaxed Taylor model snap-back, the selected non-local strain unknowns are included in the macroscopic solution procedure. The prescription of the non-local strain as an additional boundary condition on the unit cell problem leads to the appearance of a conjugate residual $f_{\bar{\varepsilon}}$. This residual has to vanish in order to obtain an equilibrium configuration of the unit cell, and this equation may be used as an additional equation in the macroscopic solution procedure.

These snap-back enhancements are summarised in Figure 3. The equations solved in each solution procedure are mentioned at the related scale. The macroscopic equilibrium, the traction continuity for localising bands and the non-local residual equations are solved in the macroscopic solution procedure (full box), while the mesoscopic equilibrium and the non-local averaging equations are solved at the mesoscopic solution procedure (dashed box). Details on the selection of the controlling mesoscopic non-local strain, on the extraction of the homogenised tangent stiffness and of the non-local residual are available in [7].

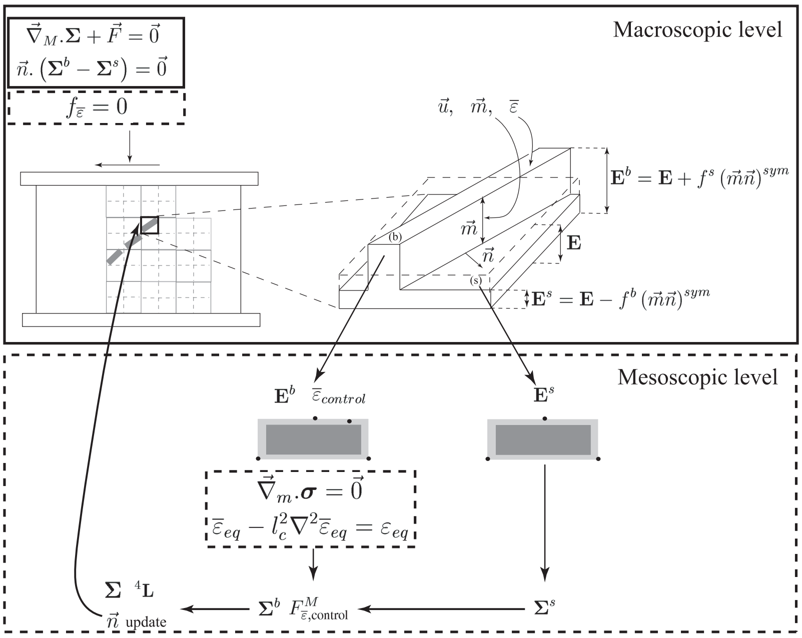

Figure 3: Enhanced multi-scale scheme with snap-back enhancements.

4 APPLICATION

The proposed multilevel scheme was implemented using parallel computing facilities. The capacities of the proposed approach are shown by means of a structural computation. A typical structural application consists in a confined sheared wall with an opening as illustrated in Figure 4. The wall is first vertically compressed. In a second phase, a shear load is applied in a confined way, i.e. keeping the vertical position of the top boundary fixed at its value at the end of the first phase. The totality of bricks consistent with the mesoscopic and structural dimensions is drawn in Figure 4 in order to emphasize the costly character of a complete fine scale modeling of this structure, and the benefit of a multi-scale scheme. The damage localisation bands before the final collapse are given in Figure 5 together with the associated mesoscopic damage patterns in unit cells. This figure clearly shows damage localisation from corners of the opening towards the top-right and bottom-left corners of the wall, accompanied with failure of bed joints at the external face of the wall. This overall crack pattern with staircase cracks was also experimentally observed for smaller walls of similar shape.

5 CONCLUSIONS

The multi-scale methodology proves to be a valuable tool for the investigation of masonry structures. In particular, it allows to account for the strong coupling between the structural response and the underlying mesostructural features of the material. Specific enhancements are however needed in order to account properly for the consequences of the quasi-brittle nature of the constituents. Snap-back responses are treated in this contribution by means of an enhanced scale transition which makes use of mesostructural quantities as controlling variables.

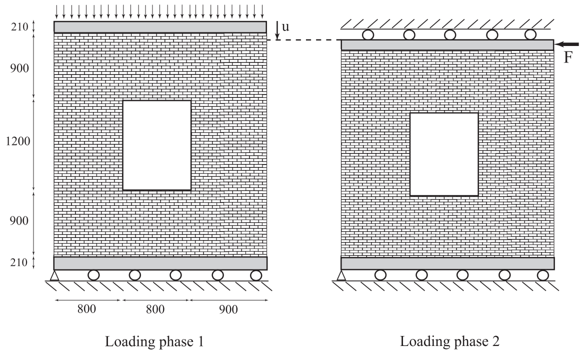

Figure 4: Application: Confined shearing of a masonry wall with opening.

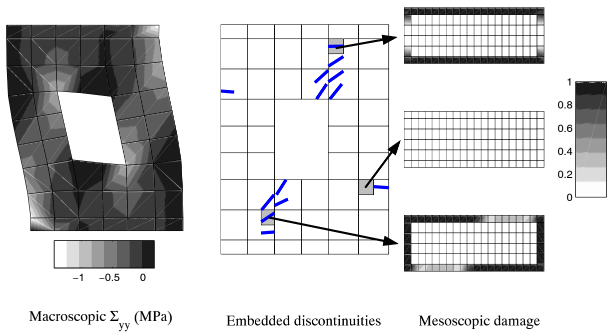

Figure 5: Application of the multi-scale method to a masonry wall segment with opening.

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