SIZE EFFECT OF CONCRETE UNDER UNIAXIAL AND FLEXURAL COMPRESSION

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ABSTRACT

This paper presents an analytical and a numerical approach to evaluate the size (slenderness) effect on the post-peak behavior of concrete in compression. The analytical approach takes into account the specimen height in the calculation of the ductility of plain concrete under uniaxial compression and the uniform moment zone length in the calculation of the ductility of reinforced concrete beams. The uniaxial compressive response of concrete is considered using a strain localization-based approach that accounts for the size-dependent stress-strain response. The type of modeling used in this work is conceptually similar to the one considered previously for localization of deformations under tensile loading. The compressive response of concrete is modelled by dividing the response into bulk and damaged sections. These different responses are used in the formulation of a simple approach for predicting the size-dependent moment-strain response of reinforced beams under pure bending. The decrease in post-peak ductility of longer reinforced concrete beams is captured by accounting for the effects of damage localization in the compression zone of the beam. A numerical modeling using the computer program DIANA was carried out to evaluate the size dependence for concrete in compression. The non-linear hardening model of *Thorenfeldt* and the non-linear softening model of *Hordijk* had been used together with smeared crack models. In addition, recent experimental results of beams with different sizes have been found to correlate reasonably well with the ones predicted by both the analytical approach and the numerical modelling.

1 INTRODUCTION

The flexural response of reinforced concrete beams depends upon several factors such as concrete strength, location and quantity of the reinforcement, and the section geometry. In addition, the stress-strain response of concrete in compression (particularly the post-peak) has a strong influence on the behavior of reinforced concrete elements. To accurately predict the response of reinforced concrete beams, sufficient knowledge of the stress-strain curve of the concrete, including the post-peak behavior (i.e., descending branch of the stress-strain curve) is required.

Unfortunately, to date the compressive response of concrete and its contribution to the post-peak response of a reinforced concrete beam has still not been clearly understood. As in the case of tensile fracture, compressive failure involves post-peak strain-softening. However, unlike tensile failure in which a thin crack forms, compressive failure results in the development of a localized damage zone as shown in Figure 1a (JANSEN, SHAH 1997). This damage zone has a finite length, which may occupy a portion of a large specimen, or the entire specimen for small-sized specimens. As a result, it is generally accepted that the post-peak behavior of concrete (following localization of damage) subjected to uniaxial compression is influenced by specimen size and boundary conditions (VONK 1992; VAN VLIET, VAN MIER 1996; JANSEN, SHAH 1997). This implies that the stress-strain response of concrete after localization is dependent on specimen size (Figure 1b), larger specimens exhibiting a more brittle post-peak response.

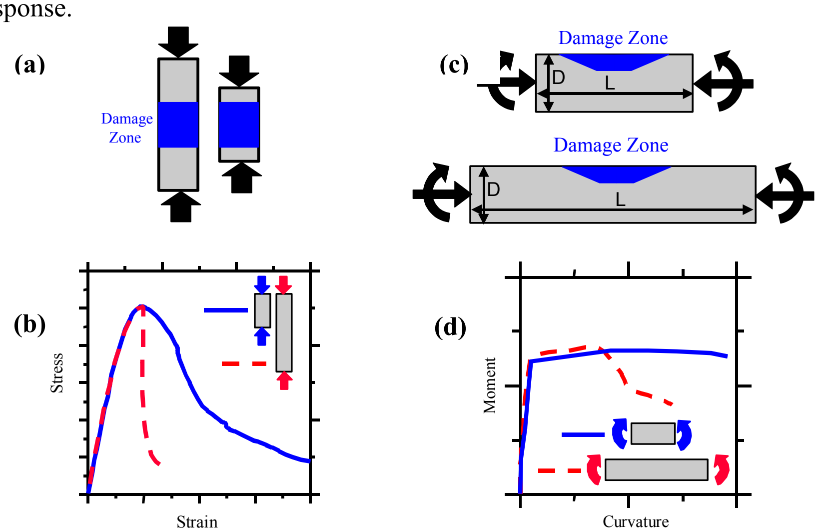

FIG. 1. Localization and size dependence in uniaxial and flexural compression (after JANSEN, SHAH (1997) for Fig. 1a and 1b and WEISS *et al.* (1999) for Fig. 1c and 1d)

2 BEHAVIOR OF CONCRETE SUBJECTED TO UNIAXIAL COMPRESSION

The analytical approach adopted herein is similar in concept to the one proposed previously for damage localization in concrete under tensile loading (HILLERBORG *et al.* 1976; BAŽANT 1989). In this idealization, the response of the specimen is homogeneous up to the peak load and the damage is assumed to localize into a band of finite height along the length of the specimen at the peak load. After localization, the responses of the damaged and bulk concrete (i.e., unloading zone) are considered separately, and each of these behaviors are treated as material properties. After peak load, the damage zone continues to accrue damage and exhibit an increase in deformation while the remainder of the specimen unloads elastically. After the peak load, the total displacement of the specimen is obtained by adding the displacements inside and outside the damage zone. A linear unloading path is assumed for the bulk concrete. This paper uses this approach to predict the flexural behavior of reinforced members with different constant moment zone lengths. Specific details can be found elsewhere (BORGES 2002; BORGES *et al.* 2002; BORGES *et al.* 2004).

The length of the damage zone has to be estimated accurately for proper implementation of this type of model. ROKUGO, KOYANAGI (1992), MARKESET (1994) and JANSEN, SHAH (1997) estimated this value to be 2 to 3 times the width of the specimen at the end of the tests. The length of the damage zone in uniaxial compression is taken equal to 1.5 times the specimen diameter throughout the loading process (BORGES *et al.* 2002). In flexure, this length is assumed to be equal to 4 times the depth of the neutral axis at peak moment.

For any specimen containing a localized damage zone, the post-peak displacement of the overall specimen is given by

$$\delta = \varepsilon L = \varepsilon_u L + \varepsilon_D L_D \tag{1}$$

where ε is the overall strain measured along the entire length of the specimen, ε_u is the strain in the bulk concrete outside the damage zone that unloads elastically, L is the length of the specimen, L_D is the length of the damage zone, and ε_D is the additional inelastic strain within the damage zone. It is assumed that ε_D is a characteristic material parameter depending only on the type of concrete. According to the linear bulk unloading path, i.e, the path along which the bulk concrete unloads, the unloading strain is given as

$$\varepsilon_u = \varepsilon_0 - \frac{f'_c - \sigma}{E} \tag{2}$$

where ε_0 is the strain corresponding to peak stress, f'_c is the concrete compressive strength, E is the elastic modulus of the concrete and σ is the stress at a given point on the curve.

Combining equations 1 and 2, the overall post-peak strain for a specimen containing a localized damage zone can be written as

$$\varepsilon = \varepsilon_0 - \frac{f'_c - \sigma}{E} + \frac{\varepsilon_D L_D}{L} \tag{3}$$

The parameter ε_D for a given material can be determined from uniaxial compression test results of specimens of different lengths.
According to equation 3, the post-peak inelastic strain within the damage zone can be expressed as

$$\varepsilon_D = (\varepsilon - \varepsilon_0 + \frac{f'_c - \sigma}{E})\frac{L}{L_D} \tag{4}$$

3 BEHAVIOR OF REINFORCED CONCRETE BEAMS SUBJECTED TO PURE BENDING

In flexure, the length L_D of the damage zone is assumed to be proportional to the neutral axis depth. This was initially suggested by HILLERBORG (1988) and MARKESET (1993). WEISS *et al.* (1999) carried out an experimental investigation with beams of various sizes subjected to four-point bending and found a value equal to four times the neutral axis depth at peak load. This has been confirmed by the test results of BORGES (2002). Using any hypotheses, the post-peak response within the damage zone is determined inserting $L = L_D$ in equation 3. Together with a linear softening curve, this yields the following compressive stress-strain relation within the damage zone

$$\sigma = \frac{\varepsilon_{DC,f} - \varepsilon_D + \varepsilon_0 - \frac{f'_c}{E}}{\frac{\varepsilon_{DC,f}}{f'_c} - \frac{1}{E}} \tag{6}$$

where $\varepsilon_{DC,f}$ is the critical damage strain in flexure and ε_D is the post-peak inelastic strain within the damage zone. To take the confinement effect in bending, the value of the critical damage strain in flexure is assumed to be two times that in uniaxial compression, i.e., $\varepsilon_{DC,f} = 2\ \varepsilon_{DC}$. For a given value of bending moment, the overall top fiber strain is determined by

$$\varepsilon = \frac{\varepsilon_u\ (L_M - L_D) + \varepsilon_D\ L_D}{L_M} \tag{7}$$

4 NUMERICAL ANALYSIS

This work is concentrated in the evaluation of the size effect by means of smeared crack models in the concrete. In the numerical simulations the finite element-based code DIANA is used. The non-linear hardening model of *Thorenfeldt* and the non-linear softening model of *Hordijk* was adopted.
The non-linear hardening model of *Thorenfeldt* presented in Figure 2 presents a equation between the tensions of compression and the deformations based in the adoption of diverse parameters in agreement the equation 8.

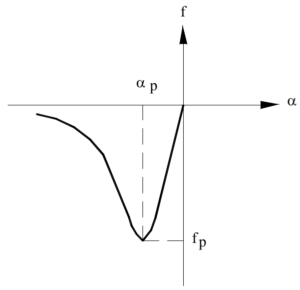

Fig. 2 *Thorenfeldt* hardening curve

$$f = -f_p \frac{\alpha_j}{\alpha_p} \left(\frac{n}{n - 1 + \left(\frac{\alpha_j}{\alpha_p} \right)^{nk}} \right) \quad n = 0{,}80 + \frac{f_c}{17}\,; \quad k = \begin{cases} 1 & \text{if} \quad 0 > \alpha > \alpha_p \\ 0{,}67 + f_c / 62 & \text{if} \quad \alpha \le \alpha_p \end{cases} \tag{8}$$

The non-linear softening model of *Hordijk* presented in Figure 3 and equation 9 uses an exponential relation between the normal stress of traction and the deformations, with "$c_1 = 3$" and "$c_2 = 6{,}93$".

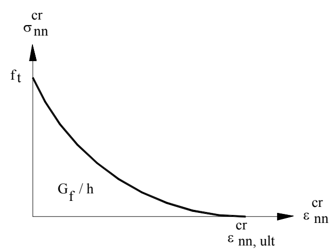

Fig. 6 *Hordijk* non-linear softening curve

$$\frac{\sigma_{nn}^{cr}\left(\varepsilon_{nn}^{cr}\right)}{f_t} = \begin{cases} \left(1 + \left(c_1 \frac{\varepsilon_{nn}^{cr}}{\varepsilon_{nn,ult}^{cr}}\right)^3\right) \exp\left(-c_2 \frac{\varepsilon_{nn}^{cr}}{\varepsilon_{nn,ult}^{cr}}\right) - \frac{\varepsilon_{nn}^{cr}}{\varepsilon_{nn,ult}^{cr}}\left(1 + c_1^{\,3}\right)\exp\left(-c_2\right) \xrightarrow{\text{IF}} 0 < \varepsilon_{nn}^{cr} < \varepsilon_{nn,ult}^{cr} \\ 0 \xrightarrow{\text{IF}} \varepsilon_{nn,ult}^{cr} < \varepsilon_{nn}^{cr} < \infty \end{cases} \tag{9}$$

5 EXPERIMENTAL RESULTS

The preliminary experimental results indicated that:

The method of separating the post-peak behaviors for cross sections within and outside the damage zone can provide a way to perform simple incremental cross-sectional analyses for the beams to simulate the length effect on the post-peak ductility. Due to strain localization, there is a remarkable length effect on the post-peak behavior of specimens under uniaxial compression as well as reinforced beams under bending. This effect is primarily manifested in ductility behavior as opposed to changes in load carrying capacity. The analytical and numerical analysis had provided resulted that they had indicated the same trend observed in the experimental program.

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