

ON THREE-DIMENSIONAL VIRTUAL CRACK CLOSURE-INTEGRAL METHOD (VCCM) FOR ARBITRARY SHAPED HEXAHEDRON FINITE ELEMENTS

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ABSTRACT

A three-dimensional Virtual Crack Closure-Integral Method (VCCM) for arbitrary shaped hexahedron finite elements is presented in this paper. Conventional three-dimensional VCCM [1] generally assumes the use of hexahedron finite elements that are arranged in an orthogonal manner at the crack front. Furthermore, it is generally required that the sizes of elements across the crack front be the same. However, when crack front has some curvature, it is almost impossible to build a model that completely satisfies such conditions.

1 INTRODUCTION

In this paper a new development of three-dimensional virtual crack closure-integral method (3DVCCM, see [1] for example) is presented. In general, VCCM requires finite elements across the crack tip or crack front to be equally sized and symmetrically placed. These requirements must be followed to evaluate the energy release rate and the stress intensity factor accurately. However, such requirements pose strong constraints on the generations of three-dimensional finite element models. In this paper, a three-dimensional VCCM without such strong constraints is described.

Virtual crack closure-integral method (VCCM), which is based on Irwin's crack closure-integral [2], was first proposed for two-dimensional crack problems by Rybicki and Kanninen [3] and was extended to three-dimensional cases by Shivakumar et al. [1]. Three-dimensional VCCM in its original form assumes that the faces of finite element meshes at the crack front be orthogonally arranged, as shown in Figure 1. Also, their sizes across the crack front must be the same. However, in many cases, it is very difficult to place finite elements in such a way. For example, for the case of semi-elliptical surface flow, it is very convenient to generate a finite element model as shown in Figure 2. In this case, the elements are skewed and the sizes of their faces are not the same across the crack front. Fawas [4,5] proposed a method that allows the finite element arrangement at the crack front to be somewhat skewed and some useful results were obtained [6]. Abdel Wahab and De Roeck [7] presented a method to correct the width change for the problem of semi-elliptical surface flaw. However, the methods [4, 5, 7] seem to be slightly modified versions of the original three-dimensional VCCM which was proposed as an extension of two-dimensional one. Present authors think that the reason that the problems arose is in the original development of three-dimensional VCCM. That is, VCCM for three-dimensional problem was derived by adding thickness to that for two-dimensional case.

In this paper, VCCM for skewed/non-symmetric finite element arrangement is derived and some numerical results are presented.

2 THREE-DIMENSIONAL VCCM FOR SKEWED/NON-SYMMETRICALLY ARRANGED CRACK FRONT ELEMENTS

We assume that the arrangement of finite elements at the crack front is as shown in Figure 2. On the plane of crack, the faces of finite elements are arranged as shown in Figure 3. We introduce two conditions on the mesh arrangement across the crack front. They are (i) the edge lines crossing the crack front must be straight (lines A-A'-A'' and B-B'-B'' in Figure 3) and (ii) the widths Δ of the elements in the radial direction are the same as shown in Figure 3. We adopt a polar coordinate system whose origin is at point O where the extended lines of A-A'-A'' and of B-B'-B'' meet each other. The second condition can be restated that distances between points C-C' and C'-C'' are the same. However, the width Δ may vary place to place. Thus, we express Δ as a function of angle θ . To describe the stresses and the displacements, a local Cartesian coordinates along the crack front are introduced, as depicted in Figure 3. x_1 and x_2 local coordinate axes are perpendicular to and parallel to the crack front. The direction of x_3 coordinate is perpendicular to the plane of crack.

When mode I component is considered without loss of any generalities, we can evaluate energy δG_I spent when crack opens for the area S_1^J which is shown in Figure 4, as:

$$\delta G_I = \frac{1}{2} \int_{S_1^J} \sigma_{33}(r) v_3(\Delta - r) dS_1^J = \int_{\theta_1}^{\theta_2} \int_0^{\Delta} \frac{K_I}{\sqrt{2\pi r \cos \phi}} 2 \sqrt{\frac{2(\Delta - r) \cos \phi}{\pi}} \frac{K_I}{E'} (R + r) dr d\theta \quad (1)$$

The asymptotic solutions for stress and displacement at the crack face is given by (see [8], for example):

$$\sigma_{33}(\bar{r}) = \frac{K_I}{\sqrt{2\pi \bar{r}}}, \quad v_3(\bar{r}) = 4 \sqrt{\frac{2\bar{r}}{\pi}} \frac{K_I}{E'} \quad (2)$$

where $E' = E$ for plane stress and $E' = E/(1 - \nu^2)$ for plane strain case. \bar{r} is the distance from the crack front. The direction of r and normal direction of the crack front make an angle ϕ (see Figure 4). After some algebraic calculations on eq. (1), we derive a simple expression.

$$\delta G_I = \frac{K_I^2}{E'} \int_{\theta_1}^{\theta_2} \left(\Delta R + \frac{\Delta^2}{4} \right) d\theta = \frac{K_I^2}{E'} \left[S_1^J - \frac{1}{4} (S_1^J - S_2^J) \right] \quad (3)$$

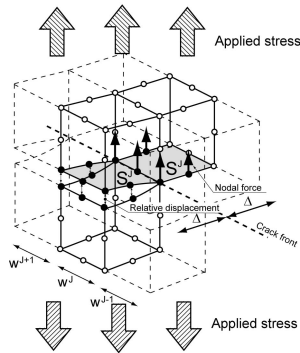


Figure 1: Three-dimensional VCCM with 20 node serendipity elements which are arranged symmetrically across the crack front

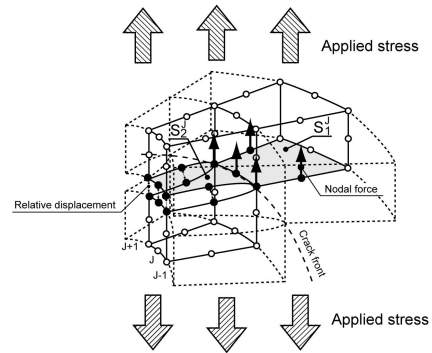


Figure 2: Three-dimensional VCCM applied to curved crack front with skewed/non-symmetrical mesh arrangement across the crack front

where areas S_1^J and S_2^J , as depicted in Figure 4, are expressed by:

$$S_1^J = \int_{\theta_1}^{\theta_2} \left(\Delta R + \frac{\Delta^2}{2} \right) d\theta \quad \text{and} \quad S_2^J = \int_{\theta_1}^{\theta_2} \left(\Delta R - \frac{\Delta^2}{2} \right) d\theta \quad (4)$$

We then define the energy release rate G_I based on its relationship with the stress intensity factor [2, 8], by:

$$G_I = \frac{K_I^2}{E'} = \frac{1}{2 \left[S_1^J - \frac{1}{4} (S_1^J - S_2^J) \right]} \int_{S_1^J} \sigma_{33}(r) v_3 (\Delta - r) dS_1^J \quad (5)$$

The same can be applied to the cases of Mode II and III components. The area integral on S_1^J in the right hand side of eq. (5) can be computed by the nodal forces and nodal displacements.

$$\int_{S_1^J} \sigma_{33}(r) v_3 (\Delta - r) dS_1^J = \sum_{I=1}^5 v_3^I (P_3^I)^J \quad (6)$$

where v_3^I are the crack opening displacements at nodes on S_2^J and $(P_3^I)^J$ are the corresponding consistent nodal forces arising from the cohesive stress on S_1^J .

3 VCCM FOR UNEQUAL ELEMENT WIDTHS ACROSS THE CRACK FRONT

As depicted in Figure 4, we assume that the widths of the elements across the crack front are different. We let the width of the element ahead the crack front be Δ and that behind the crack front be $\alpha\Delta$, as shown in Figure 4. Here the constant α represents the ratio of element widths across the crack front. Though Figure 4 implies that α be equal or smaller than one, α can be greater than one. We then slightly modify the right hand side of eq. (1) by using the constant α , as:

$$\delta G_I = \frac{1}{\sqrt{\alpha}} \int_{\theta_1}^{\theta_2} \int_0^{\Delta} \frac{K_I}{\sqrt{2\pi r \cos \phi}} 2 \sqrt{\frac{2\alpha(\Delta - r) \cos \phi}{\pi}} \frac{K_I}{E'} (R + r) dr d\theta \quad (7)$$

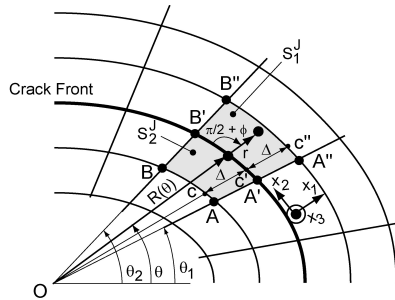


Figure 3: Skewed/non-symmetric finite element face arrangement at the crack front

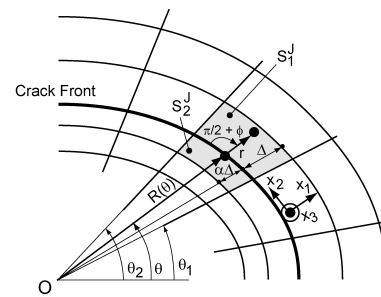


Figure 4: VCCM computation for non-equal element widths across the crack front

Thus, the energy release rate can be computed by:

$$G_I = \frac{K_I^2}{E'} = \frac{1}{2\sqrt{\alpha} \left[S_1^J - \frac{1}{2(1+\alpha)} \left(S_1^J - \frac{S_2^J}{\alpha} \right) \right]} \sum_{I=1}^5 \bar{C}^I v_3^I P_3^I \quad (8)$$

where v_3^I are the crack opening displacements at nodes on S_2^J and $(P_3^I)^J$ are the corresponding consistent nodal forces arising from the cohesive stress on S_1^J . The Mode II and III energy release rates can be given in the same fashion.

4 NUMERICAL RESULTS

The problem of through crack in a large panel subject to tension, as depicted in Figure 5 is presented. Young's modulus and Poisson's ratio are set to be 210 GPa and 0.3. All the boundaries, except for the top and bottom surfaces, are free from any stresses. We compare the results of present three-dimensional VCCM with analytical solution [9]. Also, we evaluated the stress intensity factors from the energy release rates which were computed in three different ways as listed below.

$$G_I|_{S_1^J} = \frac{1}{2S_1^J} \sum_{I=1}^5 \bar{C}^I v_3^I P_3^I, G_I|_{S_2^J} = \frac{1}{2S_2^J} \sum_{I=1}^5 \bar{C}^I v_3^I P_3^I, G_I|_{\frac{1}{2}(S_1^J+S_2^J)} = \frac{1}{S_1^J + S_2^J} \sum_{I=1}^5 \bar{C}^I v_3^I P_3^I \quad (9)$$

The stress intensity factors are then computed by $K_I = \sqrt{E'G_I}$.

The problem was first solved by an orthogonal mesh arrangement. The elements are symmetrically placed across the crack front, as shown in Figure 6 and the mode I stress intensity factor is evaluated along the crack front. When the stress intensity factor is evaluated from the energy release rate G_I , the plane stress condition is assumed since the plate is very thin. The thickness ratio t/L is assumed to be 0.025 in this case. The widths of elements at the crack front is set to be

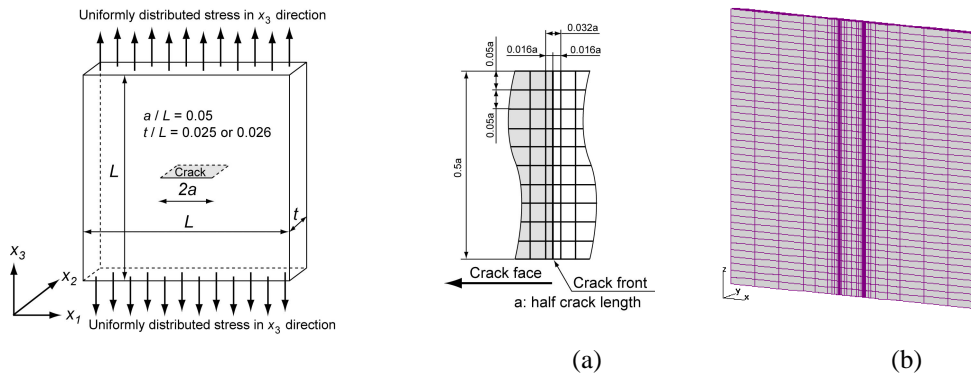


Figure 5: Thin flat panel with a through crack

Figure 6: Finite element model used to analyze the through crack problem (an orthogonal mesh pattern) [(a) Mesh arrangement at the vicinity of crack front and (b) Overall view]

$e=16/1000$ of half crack length a . The total numbers of elements and nodes are 16800 and 75376, respectively.

In Figure 7, the computed stress intensity factor is compared with the analytical solution [9]. Difference between present solution and analytical one is 0.86% at the mid-thickness of the plate. At the free surface of the plate, the value of the stress intensity factor drops slightly. The same behavior was reported in Fawaz [4]. The stress intensity factor is computed accurately and the crack front element size $e=16/1000$ of a is verified to be small enough. We use the same size in the following analyses.

We then arrange the elements on the plane of crack as shown in Figure 8. We chose two kinds of arrangements on the plane of crack. One has the ratio S_1^J/S_2^J or S_2^J/S_1^J of areas of element faces across the crack front to be about 0.86 and the other has the ratio 0.6.

The results are presented in Figures 9 and 10 for the ratios of element faces 0.86 and 0.6, respectively. In Figures 9 and 10, the stress intensity factor which was calculated by present method is designated by $K_I [0.75 S_1 + 0.25 S_2]$. Stress intensity factors computed from the energy release rates calculated by the first, second and third of eq. (9) are designated to be $K_I [S_1]$, $K_I [S_2]$ and $K_I [0.5 S_1 + 0.5 S_2]$, respectively. It is seen in Figures 9 and 10, except for $K_I [0.75 S_1 + 0.25 S_2]$ which

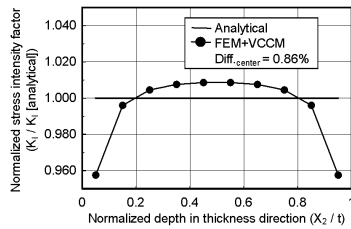


Figure 7: Comparison between the computed stress intensity factor by using the orthogonal mesh arrangement and the analytical solution

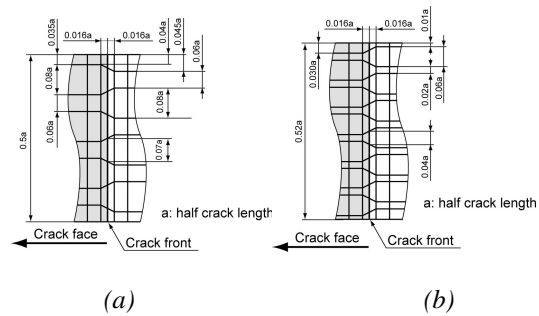


Figure 8: Non-symmetrical mesh arrangements for the through crack problem. (a) ratio S_1^J/S_2^J or S_2^J/S_1^J is 0.86 and (b) ratio S_1^J/S_2^J or S_2^J/S_1^J is 0.6

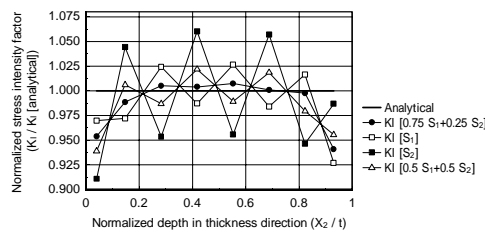


Figure 9: Distributions of the stress intensity factors along the crack front normalized by the analytical solution, for the case that the ratio S_1^J/S_2^J or S_2^J/S_1^J is 0.86

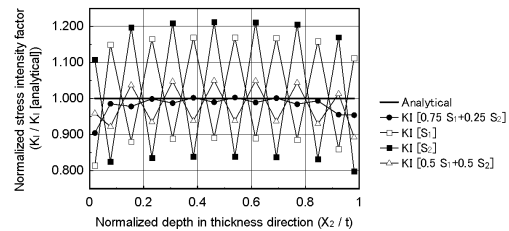


Figure 10: Distributions of the stress intensity factors along the crack front normalized by the analytical solution, for the case that the ratio S_1^J/S_2^J or S_2^J/S_1^J is 0.6

is computed by present method, large magnitudes of oscillations are seen. $K_I [0.75 S_1 + 0.25 S_2]$ is very close to the analytical solution.

5 CONCLUSION

In this paper, a new development in VCCM technique for three-dimensional fracture analysis is presented. The proposed method does not pose strong constraints in finite element mesh arrangement at the crack front. Thus, processes in finite element model generation became much simpler.

Though the results for more complex problems such as elliptical embedded crack and semi-elliptical surface flaw problems were analyzed, they are not presented in this paper due to the page limitation. Good agreements with analytical solutions [9] and existing numerical solutions [10] were obtained even when the elements are skewed at and not symmetric across the crack front.

6 ACKNOWLEDGEMENT

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